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The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\ }$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(\pi x/2b)}{\cos(\pi a/2b)} \text{ is preferable to } \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1}\sin u\,du \text{ is preferable to } \int \frac{\sin u}{u}\,du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx)\cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n]\cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n)\cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials,* vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions,* Phil. Mag. (5)36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids,* but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten.*

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

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—BOOKS RECEIVED—

Computer solution of ordinary differential equations: the initial value problem. By Lawrence F. Shampine and Marilyn K. Gordon, W. H. Freeman and Co., San Francisco, 1975. x + 318 pages. $13.95.

This book concentrates on the Adam's family of numerical methods and develops a series of algorithms and Fortran programs based on it. There are detailed studies of realistic problems, their solutions and representative computations; program listings are given in full, including implementation instructions for various computer systems. The programs are available on tape or cards from Argonne National Laboratory.

Vorlesungen über nichtlineare Operatorengleichungen. By Th. Riedrich. B. G. Teubner Verlagsgesellschaft, Leipzig, 1976. 182 pages. 16.00M.

This monograph presents basic facts necessary for an understanding of the present state of nonlinear functional analysis. It includes the classical finite-dimensional existence theorems of Brouwer, Barsuk-Ulam and Krasnoselskij, and their generalizations to infinite-dimensional topological vector spaces. Topics such as the Leroy-Schauder theory, mapping degree, simple approximation methods and perturbations of nonlinear operator equations are emphasized.

An introduction to the mathematical theory of finite elements. By J. T. Oden and J. N. Reddy. John Wiley and Sons, New York, 1976. xii + 429 pages. $24.95.

The book is divided into two parts, of four chapters each: mathematical foundations and the theory of finite elements. It begins with basic mathematical concepts and builds on these the elements of approximation theory, Sobolev and Hilbert spaces and partial differential equations. The central topic of the book is finite-element approximations of elliptic boundary-value problems and time-dependent problems, which are treated rigorously using the mathematical tools developed earlier.

Module theory: an approach to linear algebra. By T. S. Blyth. Oxford University Press, New York, 1977. ix + 400 pages. $21.00.

The aim of this text is to develop the basic properties of modules and to show their importance in the theory of linear algebra. The first part deals with modules and vector spaces and is suitable as an introductory undergraduate course, and the second part deals with advanced linear algebra (including multilinear, tensor, and exterior algebra) and is suitable as a final-year undergraduate or graduate text. It would also be useful as preliminary reading for graduate courses on category theory, homological algebra, algebraic topology, algebraic number theory, group representations, Lie algebras, etc. There is a good supply of exercises.

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Springer-Verlag, Berlin, Heidelberg, New York: *Lecture Notes in Mathematics*

Alternative problems, coincidence degree and nonlinear differential equations. No. 568. By Robert E. Gaines and Jean L. Mawhin. 1977. v + 262 pages. $11.00.

The emphasis is on the use of topological degree techniques in studying alternative problems, i.e. problems which can be written as operator equations of the form $Lx = Nx$ in a suitable abstract space, with N linear and non-invertible. Problems of this type are studied in a unified way and applications of the theorems discussed in the case of boundary-value problems for ordinary differential equations, including approximations of the solutions of $Lx = Nx$ by Galerkin-type methods.

Cohomologie etale. No. 569. SGA $4\frac{1}{2}$. Edité par P. Deligne. Springer-Verlag, Berlin, Heidelberg, New York, 1977. v + 312 pages. $13.70. Séminaire de Géometrie Algébrique du Bois-Marie.

Differential geometrical methods in mathematical physics. No. 570. Edited by K. Bleuler and A. Reetz. 1977. viii + 576 pages. $19.40.

These are the Proceedings of a symposium held at the University of Bonn, July 1–4, 1975. The 26 lectures are grouped into five chapters: geometric quantization; graded Lie algebras—super-symmetry; connections—gauge theories; symplectic structures—mechanics; Riemannian space—general relativity.

Constructive theory of functions of several variables. No. 571. Edited by W. Schempp and K. Zeller. 1977. vi + 290 pages. $11.00.

These are the Proceedings of a Conference held at the Oberwolfach Mathematical Research Institute, April 25–May 1, 1976. The topics treated cover problems on multivariate approximation theory (such as new results in the theory of spline approximations), multivariate interpolation, cubature, positive operators, intermediate spaces, special functions of mathematical physics, etc.

Sparse matrix techniques. No. 572. Edited by V. A. Barker. 1977. v + 184 pages. $8.00.

These notes were written for the "Course in Advanced Sparse Matrix Techniques" held at the Technical University of Denmark, Copenhagen, on August 9–12, 1976. It consists of lectures by Owe Axelsson on iterative methods, J. Alan George on direct methods for finite-element problems, John K. Reid on general direct methods, and Axel Ruhe on computation of eigenvalues and eigenvectors.

Springer-Verlag, Berlin, Heidelberg, New York: *Lecture Notes in Computer Science*

ECI conference 1976. No. 44. Edited by K. Samelson. 1976. viii + 322 pages. $12.30.

These are the Proceedings of the First Conference of the European Cooperation in Informatics, held at Amsterdam, August 9–12, 1976. The papers are grouped under the headings: computer architecture and computer systems structure (5 papers), concepts and techniques of data base management (7 papers), and program development and program verification (6 papers).

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Mathematical foundations of computer science 1976. No. 45. Edited by A. Mazurkiewicz. 1976. xi + 606 pages. $18.90.

This Symposium, held at Gdansk, Poland, September 6–10, 1976, was the fifth in the series of annual Mathematical Foundations of Computer Science Symposia organized in turn in Poland (in even years) and Czechoslovakia (in odd years). Proceedings of the third and fourth symposia were published as volumes 28 and 32 of these Lecture Notes, respectively. This volume contains invited papers and short communications presented at the fifth Symposium, concerning mathematical results motivated by practical problems and related to: programs and computations, programming languages, data bases and information retrieval systems, analysis and complexity of algorithms, and formal languages and automata.

Springer-Verlag, Berlin, Heidelberg, New York: *Lecture Notes in Economics and Mathematics Systems*

Integer programming and related areas: a classified bibliography. No. 128. Edited by C. Kastning. 1976. xii + 495 pages. $15.20.

This bibliography was compiled at the Institut für Ökonometrie und Operations Research, University of Bonn. It contains 4704 items by 6767 authors, classified by 11839 descriptor entries. Besides papers on the theory and methods of integer programming and combinatorial and graph-theoretical optimization problems, applications in economics, business administration, engineering and science are also included, as are algorithmic approaches and descriptions of computer programs for combinatorial optimization. The bibliography extends to the end of 1975, and an updated version is planned. It is in three parts: 1. an alphabetical bibliography by first authors; 2. alphabetical bibliographies by first authors according to 41 subject headings; 3. an alphabetical catalogue of all authors and coauthors, with references to part 1.

Multiple criteria decision making. No. 130. Edited by H. Thiriez and S. Zionts. 1976. vi + 409 pages. $14.40.

These are the Proceedings of a Conference held at Jouy-en-Josas, France, May 21–23, 1975. There are 31 papers discussing the current state of the art with respect to both theory and practice. The conference considered such points as recent theoretical developments in terms of models, the behavioral aspects of multiple-criteria decision making, as well as practical applications, problems of the implementation of multiple-criteria methods and the interface between theory and practice.

Mathematical systems theory. No. 131. Edited by G. Marchesini and S. K. Mitter. 1976. x + 408 pages. $14.40.

This volume is the record of lectures delivered at the Conference on Mathematical Systems Theory, held at Udine, Italy, June 16–27, 1975. The papers are grouped under the following headings: 1. Automata theory (1 paper); 2. Linear systems theory (6 papers); 3. Bilinear and nonlinear systems (5 papers); 4. Infinite-dimensional systems (6 papers); 5. Coding and filtering for sequential systems (4 papers); 6. General dynamical systems and categorical approach to systems (5 papers).

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Computing methods in applied science and engineering. No. 134. Edited by R. Glowinski and J. L. Lions. 1976. viii + 390 pages. $15.20.

This book contains some of the lectures which were presented during the Second International Symposium on Computing Methods in Applied Sciences and Engineering, December 15–19, 1975, at the Institut de Recherche l'Informatique et d'Automatique. (The remainder of these Proceedings are published as Lecture Notes in Physics, volume 58.) The contents are divided into the following parts: numerical algebra (5 papers), finite elements (4 papers); dynamical problems (4 papers); identification and inverse problems (6 papers); integral methods (3 papers).

A disequilibrium-equilibrium model with money and bonds. No. 135. By Hanjiro Haga. 1976. vi + 119 pages. $7.40.

Chapter headings: 1. Introduction; 2. Households and firms; 3. The general equilibrium system under the "dual-decision hypothesis"; 4. The existence proof; 5. Conclusions.

Expectations and stability in oligopoly models. No. 138. By Koji Okuguchi. 1976. vi + 103 pages. $8.00.

Chapter headings: 1. Existence and stability of the Cournot oligopoly solution (or equilibrium); 2. Uniqueness of the Cournot oligopoly solution; 3. Entry in the Cournot model: Quasi-competitiveness vs. perfect competition; 4. Revenue maximizing duopoly; 5. Stackelberg duopoly models reconsidered; 6. Extrapolative expectations and stability of oligopoly equilibrium. 7. Adaptive expectations and stability of oligopoly equilibrium; 8. Unknown demand function and stability; 9. Probability models.

Theory of the price index. No. 140. By Wolfgang Eichhorn and Joachim Voeller. 1976. vii + 95 pages. $8.00.

This monograph focuses on the theory of the price index starting with Irving Fisher's contributions in the early twenties. It aims to develop considered sets of properties in order to define price indices and so-called price levels; to characterize special classes of price indices as well as price levels; to derive the general solution of the inconsistency problem of Fisher's tests; and to evaluate the validity of Fisher's famous equation of exchange.

Mathematical economics and game theory. No. 141. Edited by R. Henn and O. Moeschlin. 1977. xiv + 709 pages. $22.50.

Published in honor of Oskar Morgenstern's 75th birthday—and serving, alas, to mark his recent death—this volume reviews Morgenstern's work under the headings of: game theory (13 papers), utility theory (6 papers), economic models (5 papers), economic theory (10 papers), econometrics and statistics (8 papers), different topics (6 papers), with an addendum (4 papers, including one by Kenneth Arrow on dynamic systems) and a bibliography of Morgenstern's publications, consisting of 301 items. There is an introductory essay by R. Hena and O. Moeschlin entitled "The scientific work of Oskar Morgenstern".

Springer-Verlag, Berlin, Heidelberg, New York: *Lecture Notes in Physics*

Group analysis of classical lattice systems. No. 60. By C. Gruber, A. Hinermann, and D. Merlini. 1977. ix + 326 pages. $13.70.

This monograph presents a systematic analysis of classical spin systems on a lattice. The mathematical structure consists essentially of two Albelian groups associated respectively with the configuration space and the interactions. There are three parts: Spin 1/2 lattice systems without constraints; spin 1/2 lattice systems with constraints; arbitrary spin lattice systems.

Harmonic analysis. No. 63. By V. K. Dobrev, G. Mack, V. B. Petkova, S. G. Petrova, and I. T. Todorov. 1977. x + 280 pages. $11.00.

The full title reads: *Harmonic analysis on the n-dimensional Lorentz group and its application to conformal quantum field theory.* Work on conformal quantum field theory and partial wave expansions led the authors to a systematic study of representations and intertwining operators for the pseudoorthogonal group, placing the subject in the context of modern harmonic analysis. This led to Part I of the book. Part II consists of recent results on tensor product expansions and related physical applications.

Combinatorial optimization: network and metroids. By Eugener Lawler. Holt, Rinehart & Winston, New York, 1976. x + 374 pp. $25.00.

Chapter headings: 1. Introduction; 2. Mathematical preliminaries; 3. Shortest paths; 4. Network flows; 5. Bipartite matching; 6. Nonbipartite matching; 7. Matroids and the greedy algorithm; 8. Matroid intersections; 9. The matroid parity problem. Chapters 3–5 comprise the material in the usual one-term network flow theory courses, and chapters 6–9 are suitable for a second course, presumably on the graduate level. The book is also suitable as a text or reference for courses on combinatorial computing and concrete computational complexity.

Introduction to number theory. By W. W. Adams and L. J. Goldstein. Prentice-Hall, Inc., New Jersey, 1976. xiii + 362 pages. $13.95.

An introduction, focusing on the theory of Diophantine equations, which is partly intended as a text for a standard elementary number theory course. Chapters 1–6 are appropriate for such a one-semester course and chapters 7–11 give an introduction to algebraic number theory by studying quadratic Diophantine equations, which lead to the study of quadratic fields. There are well over a thousand problems.

Stochastic processes in queueing theory. By A. A. Borovkov. Springer-Verlag, New York, 1976. xi + 280 pages. $29.80.

This book, translated by Kenneth Wickwire, was first published in Russian in 1972. Chapter headings: 1. Systems with queues and service of type one; 2. Some boundary problems for processes continuous from below with independent increments; 3. Boundary problems for sequences with independent increments and factorization identities; 4. Properties of the supremum of sums of independent random variables and related problems of queueing theory; 5. Multi-channel queueing systems; 6. The system $\langle G, G, G/\infty, 1\rangle$ with an infinite number of service channels; 7. Systems with refusals; 8. Systems with autonomous service.

On systems analysis: an essay concerning the limitations of some mathematical methods in the social, political, and biological sciences. By D. Berlinski. The MIT Press, Cambridge, Mass., 1976. xi + 186 pages. $15.00.

Chapter headings: 1. General systems theory; 2. Dynamical systems; 3. Mathematical systems theory. The book attempts to show that systems analysis is largely a sham.