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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Manuscripts should be typewritten double-spaced on one side only. Marginal instructions to the typesetter should be written in pencil to distinguish them clearly from the body of the text. The author should keep a complete copy.

The papers should be submitted in final form. Only typographical errors should be corrected in proof; composition charges for any major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he/she prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line following his/her name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the average typewriter should be inserted using either instant lettering or by careful insertion in ink. Manuscripts containing pencilled material other than marginal instructions to the typesetter will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts, and exponents to exponents should be clearly indicated.

Single embellishments over individual letters are allowed, the only embellishment allowed above groups of letters is the overbar.

Double embellishments are not allowed. These may be replaced by superscripts following the symbols.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\left[(a^2+b^2)^{1/2}\right] \text{ is preferable to } e^{[a^2+b^2]^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(x/2b)}{\cos(a/2b)} \text{ is preferable to } \frac{\cos\dfrac{x}{2b}}{\cos\dfrac{a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1}\sin u\,du \text{ is preferable to } \int \frac{\sin u}{u}\,du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in typeset formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a+bx)\cos t \text{ is preferable to } \cos t(a+bx).$$

Figures: Figures should be drawn in black ink with clean, unbroken lines; do not use ball point pen. The paper should be of a nonabsorbant quality so that the ink does not spread and produce fuzzy lines. If the figures are intended for reduction, they should be drawn with heavy enough lines so that they do not become flimsy at the desired reduction. The notation should be of professional quality and in proportion for the expected reduction size. Figures which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying figures should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References in text to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372 (1893)). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow them.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Stromung zaher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details such as ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations such as Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c." even if this special abbreviation is defined somewhere in the text.

CONTENTS

NEW BOOKS

Introduction to ordinary differential equations. By Roger C. McCann. Harcourt Brace Jovanovich, Inc., New York, 1982. xi + 451 pp.

It is the goal of this text not only to show how to solve elementary differential equations, but also to show how they are used to model real-world phenomena: ocean water circulation, silica concentration in the sediment of the North Sea floor, stabilization of production in a closed economy, relationship between rainfall and runoff in a watershed, the Lancaster war model, the Ross model for the way malaria affects a community, the water level in a canal that empties into the open sea, a nonparametric description of a cycloid.

Elements of statistical reasoning. By Edward W. Minimum and Robert B. Clarke. John Wiley & Sons, New York, 1982. xv + 400 pp. $18.95.

This is a text intended primarily for students in psychology, social science, and education, using only the most elementary mathematics, stressing conceptual development and the logic of statistics.

Mechanics of solids—The Rodney Hill 60th anniversary volume. Edited by H. G. Hopkins and M. J. Sewell. Pergamon Press, Oxford, 1982. viii + 693 pp. $100.00.

This volume provides a structured and many-sided account of solid mechanics. It contains 18 substantial research and survey articles by leading international authorities which describe new research and recent advances in the field. The choice of articles was made from topics which Rodney Hill had sought to illuminate, or from certain topics immediately adjacent thereto in either physical or mathematical or engineering terms. There is also a general biographical sketch of Rodney Hill and a complete current bibliography. The following is a list of the authors of the papers in the volume: Geoffrey Hopkins, Michael Sewell, J. F. W. Bishop, B. Budiansky, J. W. Hutchinson, S. Slutsky, P. Chadwick, G. D. Smith, D. R. J. Chillingsworth, I. F. Collins, J. D. Eshelby, J. H. Gittus, K. S. Havner, W. Johnson, R. J. Knops, P. B. Mellor, F. Milstein, A. Needleman, V. Tvergaard, R. W. Ogden, J. R. Rice, A. J. M. Spencer, J. R. Willis.

Applications of variational inequalities in stochastic control. By A. Bensoussan and J. L. Lions. North-Holland Publishing Co., New York, 1982. xii + 564 pp. $81.50

This is volume 12 in the series Studies in Mathematics and its Applications. The volume is devoted to the study of stopping time and control problems for systems governed by stochastic differential equations, which play an important role in numerous applications. It uses intensively the theory of partial differential equations and variational inequalities to solve problems of stochastic control; these methods are also useful in fields like free boundary problems in mechanics and physics. The work deals with branches of mathematics which may at first sight appear totally different and which have developed along quite independent lines, fertilizing each other. The fundamental link lies in the interpretation of the solutions of certain partial differential equations. This interpretation is an extension of the method of characteristics which allows the solution of a linear first-order hyperbolic equation to be expressed explicitly as a functional defined along the characteristic trajectories. The presentation is self-contained and broadly covers problems and methods. In particular, the authors have endeavored to provide, when possible, two proofs of a single result: an analytic proof and a probabilistic proof. There are four chapters: 1. General introduction to optimal stopping-time problems. 2. Stochastic differential equations and linear partial differential equations of second order. 3. Optimal stopping-time problems and variational inequalities. 4. Stopping-time and stochastic optimal control problems.

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Mathematical methods for mathematicians, physical scientists and engineers. By J. Dunning-Davies. John Wiley & Sons, New York, 1982. 416 pp. $57.95.

This is a volume in the Ellis Horwood Series Mathematics and its Applications. It is a collection of mathematical methods useful in applications. Only a knowledge of algebra, trigonometry and geometry is assumed.

Numerical analysis of variational inequalities. By R. Glowinski, J. L. Lions and R. Trémolières. North-Holland Publishing Co., New York, 1981. xv + 776 pp. $109.75.

This is volume 8 in the series Studies in Mathematics and its Applications. The volume is concerned with the numerical treatment of variational inequalities for partial differential operators. Such inequalities play a fundamental role in the modeling of various phenomena in physics and mechanics, as well as in engineering and applied sciences. A detailed analysis of various iterative methods is given for solving the finite-dimensional problems resulting from various types of approximations. Emphasis is placed in particular on finite difference and finite element approximations and also on iterative techniques such as relaxation, conjugate gradient, and duality methods. A large number of numerical tests are given to illustrate the efficiency of the methods described in the work. The work features a large, updated appendix which contains a section showing how variational inequality concepts and methods may help in the numerical solution of transonic flow problems. The volume gives a complete treatment of variational inequality problems, proceeding from a rigorous mathematical analysis to the results of computational experiments.

Quantum mechanics of atoms and molecules. By Walter Thirring. Springer-Verlag, New York, 1981. vi + 300 pp. $27.70.

This is volume 3 of Walter Thirring's Course in Mathematical Physics. He has attempted, here, not simply to introduce axioms and derive quantum mechanics from them, but also to progress to relevant applications. There are four chapters: 1. Introduction. 2. The Mathematical Formulation of Quantum Mechanics. 3. Quantum Dynamics. 4. Atomic Systems.

Turtle geometry—The computer as a medium for exploring mathematics. By Harold Abelson and Andrea A. di Sessa. The MIT Press, Cambridge, MA, 1981. xx + 475 pp. $20.00.

Turtle Geometry is an innovative program of mathematical discovery that demonstrates how the effective use of inexpensive personal computers can profoundly change the nature of a student's contact with mathematics. Based on ten years' work at MIT with high-school students and university undergraduates, Turtle Geometry proceeds from a novel "procedural" view of the elements of plane geometry to such central ideas in modern mathematics as symmetry groups and topological invariance. From the beginning, geometric figures are regarded not as static entities but as tracings of an imaginary "turtle", a view that culminates in a treatment of the ultimate synthesis of geometry and motion, Einstein's General Theory of Relativity.

Probability and statistics with reliability, queueing, and computer science applications. By Kishor Shridharbhai Trivedi. Prentice-Hall, Inc., Englewood Cliffs, NJ, 1982. x + 624 pp. $27.50.

The aim of this book is to provide an introduction to probability, stochastic processes and statistics for students of computer science, electrical/computer engineering, reliability engineering, and applied mathematics. The prerequisites are two semesters of calculus, a course on introduction to computer programming, and, preferably, a course on computer organization.

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Introduction to real analysis. By Robert G. Bartle and Donald R. Sherbert. John Wiley & Sons, New York, 1982. xii + 370 pp. $28.95.

The aim of this text is to provide an accessible, reasonably paced textbook dealing with the fundamental concepts and techniques of real analysis, which develops the material at a deliberate pace, careful to provide examples to illustrate each idea.

New quantitative techniques for economic analysis. Edited by Giorgio P. Szegő. Academic Press, New York, 1982. xiv + 319 pp. $49.50.

The papers in this book are divided into four parts: 1. Models and reality, with general papers concerning models and model making by G. P. Szegő, R. E. Kalman and S. Lombardini. 2. New quantitative techniques at work, with four papers by J. Ford, P. Allen, F. J. Evans and G. Fradellos, and T. Ho and A. Saunders. This part is concerned with the presentation of the main results and of various interesting economic applications of promising quantitative techniques such as ergodic theory, irreversible thermodynamics and catastrophe theory. 3. A reappraisal of established methods, four papers by M. J. Beckmann, G. Gambarelli and G. P. Szegő, K. Malinkowski, and P. Mazzoleni, on subjects such as dynamic programming, dynamical systems and game theory. 4. Special problems, with five papers by G. Barozzi, C. Corradi, F. Cugno and L. Montrucchio, P. Montesano and P. Mazzoleni.

An introduction to programming and problem solving with Pascal. By G. Michael Schneider, Steven W. Weingart and David M. Perlman. John Wiley & Sons, New York, 1982. xi + 468 pp. $21.95.

This is the second edition of a text first published in 1978. It has three goals, in order of importance: 1. Introducing all aspects of the programming and problem-solving process, including problem specification and organization, algorithms, coding, debugging, testing, documentation, and maintenance. 2. Introducing what constitutes good programming style and how to produce a high-quality finished product. These points are brought out in numerous Style Clinics throughout the text. 3. Teaching the syntax of the Pascal programming language.

Optimization of distributed parameter structures: volume II. Edited by Edward J. Haug and Jean Cea. Sijthoff & Nordhoff, Rockville, MD, 1981. xvii + 841 pp.

This is Volume 50, Series E (Applied Sciences) in the NATO Advanced Study Institutes Series. It is the second volume of proceedings of a NATO-NSF Advanced Study Institute, Iowa City, 21 May–4 June 1980. In this volume, the following main topics are presented: optimization of structures under nonconservative loading and other special problems of distributed parameter structural optimization; the shape optimal design problem, and a thorough treatment of design sensitivity analysis of structural systems. Included are examples and applications.

Pattern recognition with fuzzy objective function algorithms. By James C. Bezdek. Plenum Press, New York, 1981. xv + 256 pp. $35.00.

This is a volume in the series Advanced Applications in Pattern Recognition. It presents a method for handling problems of pattern recognition within imprecisely defined categories. Chapter headings: 1. Models for Pattern Recognition. 2. Partitions and Relations. 3. Objective Function Clustering. 4. Cluster Validity. 5. Modified Objective Function Algorithms. 6. Selected Applications in Classifier Design.

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A grammar of English on mathematical principles. By Zellig Harris. John Wiley & Sons, New York, 1982. xvi + 429 pp. $43.50.

This new theory of English grammar organizes the syntax of the language in a manner that also yields its weak semantics. The relationships between word formation, sentence structure, and information content are explored by way of grammatical derivation. There are two mechanisms of the derivation. One combines words according to a formal relation of partial order between operator and argument words. This is a purely logical and historically rather stable mechanism that first produces simple sentences, then complex ones. The meaning of a sentence is created from the meaning of its components and from the syntactic relations between them. The sentences so obtained are simplified by the second mechanism, the reductions, whereby much of the redundancy is eliminated. Reductions are not absolutely prescribed, because they depend on the likelihoods of usage. Therefore, they are subject to historic change and to variation depending on the shared informational background that underlies the usage. From this, differences between the sublanguages of various communities and disciplines arise. Thus, the partial order relation defines a subset of English sentences, which are not always colloquially customary, but in which all information possible to convey in the language can be expressed. The reductions provide a paraphrastic mapping of that subset onto the set of remaining English sentences—colloquial, technical and metaphoric. There are nine chapters: 1. A grammatical theory with mathematical properties; 2. The base sublanguage; 3. Reductions producing all other sentences; 4. Word clauses; 5. The noun phrase; 6. The verb phrase; 7. Forms due to metalinguistic operators; 8. Transformed sentences; 9. Conjunctional constructions.

Stochastic processes. By J. Medhi. John Wiley & Sons, New York, 1982. xi + 387 pp. $17.95.

This text aims at a level between elementary texts and advanced works on stochastic processes. The prerequisites are a course on elementary probability theory and statistics and one on advanced calculus. Chapter headings: 1. Introduction. 2. Stochastic Processes. 3. Markov Chains. 4. Markov Processes with Discrete State Space: Poisson Process and its Extensions. 5. Markov Processes with Continuous State Space. 6. Renewal Processes and Theory. 7. Markov Renewal and Semi-Markov Processes. 8. Stationary Processes and Time Series. 9. Branching Processes. 10. Stochastic Processes in Queuing and Reliability.

Optimization of distributed parameter structures: volume I. Edited by Edward J. Haug and Jean Cea. Sijthoff & Nordhoff, Rockville, MD, 1981. xvii + 840 pp.

This is volume 49, Series E (Applied Sciences) in the NATO Advanced Studies Institutes series. It is Volume I in a series of two volumes, containing lectures and contributed papers presented at the NATO-NSF Advanced Study Institute, Iowa City, 21 May–4 June 1980. One of the objectives was to promote interaction by engineers and applied mathematicians who have, in the past, taken rather different approaches to structural optimization. Shape optimal design was emphasized in the Institute, to provide a forum for the study of mathematical techniques of shape optimization and to consider their applicability for structural optimization. In this volume, the following main topics are presented: optimality criteria methods for structural optimization; numerical optimization methods; optimization of structures under earthquake loads, and finite dimensional structural optimization. For each important class of problems, applications and examples of problem-solving methods are included.

A primer in data reduction. By A. S. C. Ehrenberg. John Wiley & Sons, New York, 1982. xviii + 305 pp. $51.95.

This is an introductory text in statistics for non-specialists. It avoids mathematics, emphasizing numeracy and relevance.

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Numerical methods—A software approach. By R. L. Johnston. John Wiley & Sons, New York, 1982. ix + 276 pp. $24.95.

This book is intended as an introductory text, oriented towards learning how to use mathematical software–preprogrammed, reliable computer subroutines or packages–effectively, i.e. how to select the most appropriate routine available for solving the problem at hand and how to interpret the results returned by it.

Numerical solutions of partial differential equations. Edited by John Noye. North-Holland Publishing Co., New York, 1982. xii + 648 pp. $93.00.

This book is the outcome of the 1981 Conference on Numerical Solutions of Partial Differential Equations held at Queen's College, Melbourne University, Australia, 23–27 August, 1981. The aim of the conference was to present reviews, capable of being easily understood by practicing scientists, of the most commonly used methods of numerically solving partial differential equations and to give research workers the opportunity to present their latest work on improving or applying these methods. The work contains surveys of finite difference, finite element, Galerkin and boundary integral equation methods, all under one cover. The methods described are illustrated with solutions to commonly occuring partial differential equations. There are also two survey articles on the numerical solution of sets of large sparse linear algebraic equations, one describing iterative techniques and the other describing direct methods. In addition, twenty papers are included in the work in which recent refinements of the various numerical techniques used to solve partial differential equations are described, or experiences in applying these methods to solve problems in engineering physics, oceanography, hydrology or meteorology were reported.

Current issues in quantum logic. Edited by E. Beltrametti and Bas C. van Fraassen. Plenum Press, New York, 1981. ix + 492 pp. $59.50.

This is volume 8 of the Ettore Majorana International Science Series. It constitutes the proceedings of a workshop held in Erice (Sicily), December 2–9, 1979. The 34 pages are divided into 6 topics: 1. Classification of Different Areas of Quantum Logic, and Open Problems. 2. Comparison and Unification of Different Approaches to Quantum Theories; Problems of Interpretation. 3. Formal Quantum Logic; Axiomatics. 4. Modal Interpretations of Quantum Logic. 5. Quantum Set Theory. 6. Advances Concerning Mathematical Structure; other Problems.

Theory of technical change and economic variance—Application of Lie groups. By Ryuzo Sato, Academic Press, New York, 1982. xv + 439 pp. $56.00.

This book deals with a variety of topics in economic theory, ranging from the analysis of production functions to the general recoverability problem of optimal dynamic behavior. The various selected topics are treated from the unifying point of view of transformation and invariance. In general, the book is concerned with the economic invariance problems of observable behavior under general transformations such as technical change and/or taste change. It is fundamentally a study of market behavior and economic invariance under "Lie types of technical change" (the exact definition being given in the text). The book is intended for economists, and hence emphasis is placed on economic interpretation rather than mathematical rigor. The book assumes a knowledge of the basic elements of modern economic theory as well as some amount of elementary mathematics used in economics. Beyond this it is self-contained: the reader who is not familiar with even the elementary aspects of Lie's theory of transformation groups can read this book by first studying the brief survey of Lie group theory presented in the appendix. Chapter headings: 1. An overview. 2. Holotheticity of a technology. 3. A theory of endogenous technical progress. 4. "G-neutral" technical change, comparative statistics, and integrability conditions. 5. Holotheticity of an implicit technology. 6. Self-dual preferences and technologies. 7. Dynamic symmetries and economic conservation laws. 8. A Lie group approach to the index number problems. 9. The group structure and the theory of observable market behavior.

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Case studies in mathematical modelling. Edited by R. Bradley, R. D. Gibson, and M. Cross. John Wiley & Sons, New York, 1981. ii + 167 pp. $37.95.

There are seven case studies: analyzing blast furnace performance, minimization of sound distortion in a record player, performance of hydrolic buffers, comminution of particulate solids, roller spacing in the float glass process, bond graph models, the freezing and thawing of meat.

Concepts and applications of finite element analysis. By Robert D. Cook. John Wiley & Sons, New York, 1981. xix + 537 pp. $34.95.

This is the second, revised edition of an introductory, practically oriented text first published in 1947.

Lagrangian analysis and quantum mechanics: A mathematical structure related to asymptotic expansions and the Maslov index. By Jean Leray, with English translation by Carolyn Schroeder. The MIT Press, Cambridge, MA, 1981. xvii + 271 pp. $35.00.

This work might have been entitled The Introduction of Planck's Constant into Mathematics, in that it introduces quantum conditions in a purely mathematical way in order to remove the singularities that arise in obtaining approximations to solutions of complex differential equations. In 1967 the Russian mathematician V. P. Maslov improved the resolution of differential equations through asymptotic expansions, using Fourier transforms to show that singularities are only apparent. He defined his index—later clarified by V. I. Arnold—and introduced his quantum conditions. Arnold then asked Jean Leray—one of this century's most prominent masters of both pure and applied mathematics—what the mathematical implications of Maslov's procedure might be. This book represents Leray's answer to Arnold's question. The book's first chapter develops the necessary mathematical apparatus: Fourier transforms, metaplectic and symplectic groups, the Maslov index and Lagrangian varieties. The second chapter orders Maslov's conceptions in a manner that avoids contradictions and creates step by step an essentially new structure: the Lagrangian analysis. Unexpectedly and strangely the last step requires the datum of a constant, which in applications to quantum mechanics is identified with Planck's constant. Lagrangian analysis is directly applied to the Schrödinger and the Klein-Gordon equations in the third chapter and to the Dirac equation in the final chapter, leading to a new interpretation of these equations and simplifying the computation of energy levels. Magnetic field effects and even the Paschen-Back effect are taken into account.

Differential equations and applications in ecology, epidemics, and population problems. Edited by Stavros N. Busenberg and Kenneth L. Cooke. Academic Press, New York, 1981. xv + 359 pp. $34.50

These are the proceedings of a conference held in Claremont, California, January 10–11, 1981. The contents are collected in four groups. The first of these deals with aspects of population dynamics that involve the interaction between spatial and temporal effects. The second group treats other questions in population dynamics and some other areas of biomathematics. The third group deals with a number of topics in differential and functional differential equations that are continuing to find important applications in mathematical biology. The last group contains the abstracts of papers that were presented at the research conference but do not appear in this volume.

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The numerical solution of nonlinear problems. Edited by Christopher T. H. Baker and Chris Phillips. Clarendon Press, Oxford, 1981. viii + 369 pp. $39.00.

This volume is based upon the lectures presented at a Summer School held in Liverpool in July 1980, the fifth in a sequence, all published by Oxford University Press. The editors suggest the volumes should be read in conjunction with each other. Contributions to the present volume, twenty-three in all, are grouped into seven parts: 1. Algebraic and transcendental equations. 2. Initial-value problems in ordinary differential equations. 3. Boundary-value problems in ordinary differential equations. 4. Partial differential equations. 5. Integral equations. 6. Approximation. 7. Programming techniques.

Solitons and the inverse scattering transform. By Mark J. Ablowitz and Harvey Segur. Society for Industrial and Applied Mathematics, Philadelphia, PA, 1981. x + 425 pp. $54.50.

This is a volume in the monograph series SIAM Studies in Applied Mathematics. Its basic aim can be stated thus: Certain nonlinear problems have a surprisingly simple underlying structure, and can be solved by essentially linear methods. Typically, these problems are in the form of evolution equations, which describe how some variable evolved in time from a given initial state. The equations may take a variety of forms, including partial differential equations, differential-difference, partial difference, integro-differential, as well as coupled ordinary differential equations. Even though these problems are nonlinear, one may obtain the general solution that evolves from arbitrary initial data without approximation. Some of these exactly solvable problems arise naturally as models of physical phenomena. One of the viewpoints about these problems identifies the general solution of an appropriate initial value problem as the objective of the analysis. This solution is obtained by the inverse scattering transform, described in chapters 1 and 2. Some other perspectives are examined in chapter 3 and a number of physical applications are discussed in chapter 4.

Quantum gravity 2, *A second Oxford symposium.* Edited by C. J. Isham, R. Penrose, and D. W. Sciama. Clarendon Press, Oxford, 1981. xiv + 669 pp. $39.95.

The union of quantum theory with Einstein's general theory of relativity remains one of the major unattained goals of present-day theoretical physics. In February 1974, the first Oxford Symposium on Quantum Gravity was held, and in April 1980 the second. This second set of lectures is published in the present volume. Some of these provide review articles covering general or specific areas of interest, whilst others are of a more technical nature, reflecting the increasing complexity and sophistication of current mathematical techniques. Among significant recent advances reported here is the renaissance in quantum field theory following the discovery that Yang-Mills theory is renormalizable, and the consequent unifications of weak and electromagnetic interactions by the Salam-Ward, Weinberg and Glashow theories, raising hopes for a grand unified theory, perhaps through the theory of supergravity. These and other topics are discussed in 29 papers by the foremost authorities in the field.

Mathematics and physics. By Yu. I. Manin. Translated by Ann and Neal Koblitz. Birkhauser Boston, Inc., Cambridge, MA, 1981. xii + 99 pp. $10.00.

This is volume 3 of the series Progress in Physics. The author, a mathematician, is a member of the Steklov Mathematical Institute, Moscow. He has tried in this book to select several important abstractions from mathematics and from physics and make them confront one another. He wishes to show how mathematics associates new mental images with physical abstractions; these images are almost tangible to the trained mind but are far removed from those that are given directly by life and physical experience. Chapter headings: 1. A Bird's Eye View of Mathematics. 2. Physical Quantities, Dimensions and Constants: The Source of Numbers in Physics. 3. A Drop of Milk: Observer, Observation, Observable and Unobservable. 4. Space-time as a Physical System. 5. Action and Symmetry.

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The application of mathematics in industry. Edited by R. S. Anderson and F. R. de Hoog. Martinus Nijhoff Publishers, The Hague, The Netherlands, 1982. xiv + 202 pp. $32.50.

These are the proceedings of a one-day seminar held at the Australian National University, Canberra, December 3, 1980. There are thirteen papers, covering industrial applications of mathematics such as: lasers, aircraft component reinforcement, ironmaking blast furnaces, urban drainage systems, non-life insurance, cutting stock problems, acceptance sampling in the peanut industry, grinding of contact lenses, sheet-metal bending, fabrication of optical fibers, annealing. An attempt was made to structure the program to the extent that (a) a major aim in organizing the seminar was to bring together the academic mathematician with little first-hand experience with the application of mathematics to industrial problems, and non-academics with an ongoing industrial responsibility; (b) the seminar itself aimed to illustrate the different reasons why mathematics should be, is or must be used in the solution of industrial problems; and (c) all speakers were requested to organize their talks so that they first discussed an application, then showed why mathematics was necessary for the solution process, and only then discussed the mathematics itself.

Problem solving principles: Programming with Pascal. By Ronald E. Prather. Prentice-Hall, Englewood Cliffs, NJ, 1982. xiii + 350 pp. $19.95.

This is a textbook designed for use in a first course in programming.

VLSI systems and computations. Edited by H. T. Kung, Bob Sproull and Guy Steele. Computer Science Press, Rockville, MD, 1981. xi + 415 pp. $29.95.

This is an edited collection of papers presented at a conference at Carnegie-Mellon University, October 19–21, 1981. The papers focus on the theory and design of computational systems using VLSI (Very Large Scale Integration). The book is divided into nine sections: 1. Invited survey papers (by R. F. Lyon, F. Baskett, J. G. Peterson, A. Sawni, N. R. Powell, J. Rattner); 2. Models of computation: papers that deal with abstracting the properties of VLSI circuits into models that can be used to analyze the chip area, time or energy required for a particular computation. 3. Complexity theory. This section shows how computations can be analyzed to obtain bounds on the resources (chip area, time, energy) required to perform some computation. 4. Layout theory and algorithms. Papers in this section describe ways to route wires that connect together different circuits on a chip. 5. Languages and verification. This section presents several results on the specification and verification of circuits and of entire systems. 6. Special-purpose architectures. This section deals with systolic computing architectures and their application to areas such as signal processing. 7. Multiplier designs. The problem of designing an efficient multiplier is of both practical and theoretical interest. 8. Processors. Two papers in this section describe new designs for single-chip general-purpose computers whose architecture is influenced by VLSI design opportunities. 9. Systems and processors. This section contains papers describing frameworks for entire systems, such as parallel processing arrays and content-addressable memories.

The second workshop on grand unification. Edited by Jacques P. Leveille, Lawrence R. Sulak, and David G. Unger. Birkhauser Boston, Inc., Cambridge, MA, 1981. ix + 321 pp. $19.95.

These are the proceedings of a workshop held at the University of Michigan, April 24–26, 1981. Its purpose was to discuss the physics beyond the framework of the standard $SU(3) \times SU(2) \times U(1)$ model—within which there has recently been rapid progress towards understanding the separate theories of the strong, weak and electromagnetic interactions—and the major topic was grand unified theories which unify these three sectors. Such theories are presently used to calculate experimentally accessible quantities, which are being measured by new, dedicated experiments. Reports on these experimental and theoretical activities occupied much of the workshop, but the cosmological implications of the grand unified theories and the constraints on these theories from cosmology were also of great interest at the workshop. There were 24 papers and a keynote address by S. L. Glashow.