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OF
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QUARTERLY OF APPLIED MATHEMATICS

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Manuscripts (two copies) submitted for publication in the QUARTERLY OF APPLIED MATHEMATICS should be sent to the Editorial Office, Box F, Brown University, Providence, RI 02912, either directly or through any one of the Editors. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to scientists or engineers. Authors will receive galley proof only. The author's institution will be requested to pay a publication charge of \$30 per page which, if honored, entitles the author to 100 free reprints. Detailed instructions will be sent with galley proofs.

The current subscription price per volume (April through January) is \$40. Single issues can be purchased, as far as they are available, at \$11 and back volumes at \$40 per volume. Subscriptions and orders for back volumes must be addressed to the American Mathematical Society, P. O. Box 1571, Providence, RI 02901. All orders must be accompanied by payment. Other subscription correspondence should be addressed to the American Mathematical Society, P. O. Box 6248, Providence, RI 02940.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Manuscripts should be typewritten double-spaced on one side only. Marginal instructions to the typesetter should be written in pencil to distinguish them clearly from the body of the text. The author should keep a complete copy.

The papers should be submitted in final form. Only typographical errors should be corrected in proof; composition charges for any major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he/she prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line following his/her name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the average typewriter should be inserted using either instant lettering or by careful insertion in ink. Manuscripts containing pencilled material other than marginal instructions to the typesetter will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter *O*, between the numeral one (1), the letter *l* and the prime ('), between alpha and *a*, kappa and *k*, mu and *u*, nu and *v*, eta and *n*.

The level of subscripts, exponents, subscripts to subscripts, and exponents to exponents should be clearly indicated.

Single embellishments over individual letters are allowed, the only embellishment allowed above groups of letters is the overbar.

Double embellishments are not allowed. These may be replaced by superscripts following the symbols.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol *exp* should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{[a^2 + b^2]^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(x/2b)}{\cos(a/2b)} \text{ is preferable to } \frac{\cos \frac{x}{2b}}{\cos \frac{a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in typeset formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

Figures: Figures should be drawn in black ink with clean, unbroken lines; do not use ball point pen. The paper should be of a nonabsorbant quality so that the ink does not spread and produce fuzzy lines. If the figures are intended for reduction, they should be drawn with heavy enough lines so that they do not become flimsy at the desired reduction. The notation should be of professional quality and in proportion for the expected reduction size. Figures which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying figures should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References in text to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372 (1893)). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow them.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Stromung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details such as ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations such as Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c." even if this special abbreviation is defined somewhere in the text.

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 — NEW BOOKS —

Probability Theory and Computer Science. Edited by G. Louchard and G. Latouche. Academic Press, London, England, 1983. 204 pp. \$27.50.

This is a volume in the International Lecture Series in Computer Science. It is based on three lectures presented at the Universite Libre de Bruxelles in 1980/81. These are: 1. Stochastic modeling: ideas and techniques, by Donald P. Gaver; 2. Stochastic modeling: queueing models, by Hisashi Kobayashi; 3. Mathematical analysis of combinatorial algorithms, by Robert Sedgewick.

Random Fields: Analysis and Synthesis. By Erik Vanmarcke. The MIT Press, Cambridge, Massachusetts, 1983. 363 pp. \$45.00.

The aim of this book is to develop a new synthesis of methods to describe and analyze and, where appropriate, predict and control random fields. Chapters 2 and 3 serve mainly as a review of the classical theory of multidimensional random processes. Chapter 4 presents a synthesis of results (many new) on level excursions and extremes of Gaussian and related random fields. Spectral moments and associated measures of degree of disorder are introduced and interpreted for different types of stochastic variation. The main new developments are based on a proposal to treat the correlation structure of one-dimensional random processes in terms of the variance function and the scale of fluctuation. This treatment extends elegantly to multidimensional situations and opens the way to considerable expansion of present capabilities to deal in a practical and relatively simple way with problems involving one- and two-dimensional random variation (chapters 5 and 6, respectively) and general n -dimensional homogeneous fields (chapter 7). Chapter 8 proposes new methodology in the areas of estimation and prediction, and provides an application-oriented review of new results.

The Cauchy Problem, Encyclopedia of Mathematics and Its Application, Volume 18. By H. O Fattorini. Addison-Wesley Publishing Co., Reading, Massachusetts, 1983. xxii + 509 pp. \$69.95.

This is volume 18 of the Encyclopedia of Mathematics and Its Applications, edited by Gian-Carlo Rota. It is a volume in the section Analysis, edited by Felix E. Browder. This volume is devoted to an extensive development and expansion of the application of the concept of the abstract Cauchy problem to partial differential equations. Chapter 0 presents the elements of functional analysis. Chapter 1 consists almost exclusively of examples drawn from problems of mathematical physics; the resulting equations are treated by ad hoc methods (Fourier series and transforms) and the results provide motivation for the definition of the well-posed Cauchy problem. The resulting theory is examined in Chapter 2. Chapter 3 discusses the particular case corresponding to dissipative operators and some related facts (such as semigroups in Banach lattices) with applications to second order ordinary differential operators and symmetric hyperbolic equations. Chapter 4 is on abstract parabolic equations. Chapter 5 deals with perturbation theory; the applications include the neutron transport equation and the Schrödinger and Dirac equations with potentials. Other topics include continuous and discrete approximations to abstract differential equations, among them finite difference methods, which are illustrated with a parabolic initial-boundary value problem. Further considerations on the idea of the well posed problem are found in Chapter 6, where formulations different from that of the Cauchy problem are introduced in several examples. The theory of the equation $u'(t) = Au(t), u(0) = u_0$ with the operator A depending on t is the subject of Chapter 7. Finally, the author presents in Chapter 8 a brief account of the theory of the Cauchy problem in the sense of distributions. Here the restriction to purely differential equations of the above type is unreasonable and the author considers instead hereditary equations; relations with system theory are pointed out. In the case of the above equation this formulation is seen to be equivalent to the so-called mildly well-posed Cauchy problem, brought into existence as a tool for the treatment of hyperbolic equations with multiple characteristics. Each chapter ends with miscellaneous comments; then gives historical information, discusses parts of the theory not treated in detail, and provides bibliographical indications. The subject of nonlinear equations is not covered although some references to quasilinear equations are found in Chapter 2.

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Theory of Point Estimation. By E. L. Lehmann. John Wiley & Sons, Inc., New York, 1983. 491 pp. \$36.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. It is concerned with point estimation in Euclidean sample spaces. The first four chapters deal with exact (small-sample) theory, and their approach and organization parallel those of the companion volume, *Testing Statistical Hypotheses*. Optimal estimators are derived according to criteria such as unbiasedness, equivariance, and minimaxity, and the material is organized around these criteria. The principal applications are to exponential and group families, and the systematic discussion of the rich body of (relatively simple) statistical problems that fall under these headings constitutes a second major theme of the book. A theory of much wider applicability is obtained by adopting a large-sample approach. The last two chapters are therefore devoted to large-sample theory, with Chapter 5 providing a fairly elementary introduction to asymptotic concepts and tools. Chapter 6 establishes the asymptotic efficiency, in sufficiently regular cases, of maximum likelihood and related estimators, and of Bayes estimators, and presents a brief introduction to the local asymptotic optimality theory of Hajek and LeCam. Even in these two chapters, however, attention is restricted to Euclidean sample spaces, so that estimation in sequential analysis, stochastic processes, and function spaces, in particular, is not covered. The test is supplemented by numerous problems. These and references to the literature are collected at the end of each chapter.

Statistics in Medical Research. Edited by Valerie Mike and Kenneth E. Stanley. John Wiley & Sons, Inc., New York, 1983. xviii + 541 pp. \$34.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. It is based on the lectures and discussions presented at a conference held at Memorial Sloan-Kettering Cancer Center, June 22–26, 1981. The book offers a comprehensive overview of the field of biostatistics. Although the emphasis of the meeting was on cancer research, nearly everything discussed is applicable to other areas of medical investigation. The seventeen papers are divided into six parts: 1. Introduction—the role of statistics in medical research (Frederick Mosteller); 2. Epidemiology (an overview of both descriptive and analysis epidemiology); 3. Issues in clinical studies (historical perspective, ethical, legal and psychological issues); 4. Practical considerations; 5. Statistical methodology (basic concepts of the analysis of survival data, analysis of categorical data using log-linear and logistic models, assessment of methods for monitoring and stopping clinical trials). The final section contains two panel discussions on communication: the statisticians' interaction with the medical community, and the interpretation and presentation of statistical results.

Understanding Robust and Exploratory Data Analysis. Edited by David C. Hoaglin, Frederick Mosteller and John W. Tukey. John Wiley & Sons, Inc., New York, 1983. xvi + 431 pp. \$37.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. In current statistical practice, both exploratory data analysis and robust and resistant methods have gained important roles. To apply such methods most effectively, the user needs to understand why they are needed and how they work—and can be helped by some insight into how they were devised. This book provides conceptual, logical, and sometimes, mathematical support for the simpler of these new techniques. It also explains and illustrates connections between the concepts of classical statistical theory and the techniques of exploratory data analysis, particularly as expounded by Tukey in a book of that title. The robust and resistant techniques discussed have considerable support in the statistical research literature, both at a highly abstract mathematical level and in extensive Monte Carlo studies. The book provides the basis for an adequate understanding of these techniques using examples and a much reduced level of mathematical sophistication. Chapter headings: 1. Stem-and-Leaf Displays; 2. Letter Values: A Set of Selected Order Statistics; 3. Boxplots and Batch Comparison; 4. Transforming Data; 5. Resistant Lines for y Versus x ; 6. Analysis of Two-way Tables by Medians; 7. Examining Residuals; 8. Mathematical Aspects of Transformation; 9. Introduction to More Refined Estimators; 10. Comparing Location Estimators: Trimmed Means, Medians, and Trimean; 11. M -Estimators of Location: An Outline of the Theory; 12. Robust Scale Estimators and Confidence Intervals for Location.

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UMAP modules 1977–1979: Tools for teaching. Developed by Educational Development Center, Inc., Birkhauser Boston, Inc., Cambridge, MA, 1981. xii + 727 pp.

This represents the first annual collection of instructional modules (pre-1980) from the Undergraduate Mathematics and Its Applications Project. UMAP modules are self-contained, lesson-length, instructional units from which undergraduate students learn professional applications of mathematics and statistics to such fields as biomedical sciences, economics, American politics, numerical methods, computer science, earth science, social sciences, and psychology. The units in this collection were field-tested in college and university classrooms throughout the United States and abroad.

UMAP modules 1980: Tools for teaching. Developed by Educational Development Center, Inc., Birkhauser Boston, Inc., Cambridge, MA, 1981. ix + 690 pp.

This volume contains the 1980 UMAP modules: see preceding notice for description.

Man in competition with the spruce budworm: an application of differential equations. By Philip M. Tuchinsky. Birkhauser Boston, Inc., Cambridge, MA, 1981. viii + 77 pp.

This is a volume in the UMAP Expository Monograph Series. It discusses from a practical standpoint the development and application of a mathematical model for the natural cycle in which the budworm “harvests” the foliage of the balsam fir.

Smallpox: when should routine vaccination be discontinued? By James C. Frauenthal. Birkhauser Boston, Inc., Cambridge, MA, 1981. xii + 50 pp.

This is a volume in the UMAP Expository Monograph Series. It examines the mathematical models that address the problem of risk determination and the usefulness of preventive smallpox vaccination. It builds on the mathematical differences between stochastic model formulation using birth-death, random walk and branching processes.

The role of mathematics in the rise of science. By Salomon Bochner. Princeton University Press, Princeton, NJ, 1981. x + 386 pp. \$6.95.

This is a collection of essays, first published in book form in 1966. It is the first Princeton paperback printing.

Algebraic topology—A first course. By Marvin J. Greenberg and John R. Harper. The Benjamin/Cummings Publishing Company, Reading, MA, 1981. xi + 307 pp.

This is a revision of the first author's *Lectures on algebraic topology*.

Basic mathematics for biochemists. By A. Cornish-Bowden. Chapman and Hall, London and New York, 1981. ix + 144 pp. \$23.00 hardcover, \$12.95 paperback.

There are six chapters in this elementary introduction. 1. The Language of Mathematics. 2. Exponents and Logarithms. 3. Differential Calculus. 4. Integral Calculus. 5. Solving Equations. 6. Partial Differentiation.

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Gauge theory and variational principles. By David D. Bleecker. Addison-Wesley Publishing Co., Inc., Reading, MA, 1981. xviii + 179 pp. \$17.50.

This is a volume in the series Global Analysis: Pure and Applied. It grew out of a seminar stimulated by a visit of I. M. Singer to the University of Hawaii. Owing to a preliminary chapter, the text can be understood by advanced undergraduate mathematics or physics majors. Chapter headings: 0. Preliminaries. 1. Principal fiber bundles and connections. 2. Curvature and G -valued differential forms. 3. Particle fields, Lagrangians, gauge invariance. 4. Lagrange's equation for particle fields. 5. The inhomogeneous field equation. 6. Free Dirac electron fields. 7. Interactions. 8. Calculus on frame bundle. 9. Unification of gauge fields and gravitation. 10. Additional topics.

Linear Programming in Single- and Multiple-Objective Systems. By James P. Ignazio. Prentice-Hall, Englewood Cliffs, NJ, 1982. xvii + 506 pp. \$29.95.

There are four parts, containing twenty papers, in this book: 1. The unified linear model; 2. Single-objective linear programming; 3. Special models in single-objective linear programming. 4. Multiple-objective linear programming. The text is written as an introductory, yet relatively complete exposition of linear programming for the practitioner, and it is also designed to incorporate, in the coverage, a unified presentation of classical, i.e. single-objective, and multiple-objective linear programming.

The theory of spinors. By Elie Cartan. Dover Publications, Inc., New York, 1981. vi + 157 pp. \$4.00.

This is a major work on the mathematical theory of quantum mechanics, first published in France in 1937. Part One is devoted to generalities on the group of rotations in n -dimensional space, the linear representations of groups, the theory of spinors in three-dimensional space, and to linear representations of the group of rotations in that space. Part Two is devoted to the theory of spinors in spaces of any number of dimensions, and particularly in the space of special relativity.

Semigroup and factorization methods in transport theory. By C. V. M. van der Mee. Mathematisch Centrum, Amsterdam, 1981. iii + 167 pp. \$7.80.

This is Mathematical Centre Tract 146. Its scope is indicated by its table of contents: 0. Introduction. 1. Linearization of transfer functions. 2. Wiener-Hopf factorization of transfer functions. 3. Spectral theory of hermitian admissible pairs. 4. The operator differential equation $(T\psi)' = -A\psi + f$. 5. The convolution equation $\psi(t) - \int_0^t H(t-s)B\psi(s) ds = \omega(t)$. 6. Applications to transport theory.

1980 seminar on harmonic analysis. Edited by C. Herz and R. Rigelhof. American Mathematical Society, Providence, RI, 1982. iv + 313 pp. \$15.00.

This is volume 1 of the series Conference Proceedings of the Canadian Mathematical Society. The seminar was held at McGill University, August 4–22, 1980. It was devoted to harmonic analysis with special emphasis on connections with number theory. A substantial part of this volume is taken up by the notes for James Arthur's minicourse on Automorphic Representations and Number Theory. The other talks are divided into three groups: representation theory of semisimple Lie groups, differential and pseudo-differential operators, and harmonic analysis.

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Hardy spaces on homogeneous groups. By G. B. Folland and E. M. Stein. Princeton University Press, Princeton, NJ, 1982. xii + 284 pp. \$13.00.

This is volume 28 of Mathematical Notes. The monograph gives an exposition of the real-variable theory of Hardy spaces. This theory has in recent years led to a better understanding in R^n of such related topics as singular integrals, multiplier operators, maximal functions, and real-variable methods generally. The reasons for the more general setting, an underlying homogeneous group rather than R^n , are described in detail.

Combinatorics. Edited by H. N. V. Temperley. Cambridge University Press, London, 1981. 190 pp. \$24.50.

This is volume 52 in the London Mathematical Society Lecture Note Series. It contains the nine invited lectures given at the 8th British Combinatorial Conference, University College, Swansea, 1981.

Newtonian attraction. By A. S. Ramsey. Cambridge University Press, London, 1982. ix + 184 pp. \$14.95.

This is a volume in the series Cambridge Science Classics. It was first published in 1940 under the title *An introduction to the theory of Newtonian attraction*. This reissue of a classic text on potential theory is identical to the 1940 edition.

The theory of homogeneous turbulence. By G. K. Batchelor. Cambridge University Press, London, 1982. xi + 197 pp. \$13.95.

This is an unaltered reissue of the text first published in 1953. This classic account includes an introduction to the study of homogeneous turbulence, including its mathematic representation and kinematics. Linear problems, such as the randomly-perturbed harmonic oscillator and turbulent flow through a wire gauze are then treated. The author also presents the general dynamics of decay, universal equilibrium theory, and the decay of energy-containing eddies.

Green's functions and transfer functions handbook. By Anatoliy G. Butkovskiy. John Wiley & Sons, New York, 1982. 236 pp. \$65.95.

This is a volume in the Ellis Horwood Series Mathematics and Its Applications. The author is professor and chief of laboratory, institute of control sciences, Academy of Sciences of the USSR, Moscow. Part I of this handbook gives the characteristics of about 500 boundary value problems for ordinary and partial differential equations, and also the Green's functions for certain integral equations. Part II is devoted to a concise presentation of the main features of a structural theory of distributed parameter systems.

Numerical solution of partial differential equations in science and engineering. By Leon Lapidus and George F. Pinder. John Wiley & Sons, New York, 1982. 627 pp. \$44.95.

This book was written to provide a text for graduate and undergraduate students who took the author's courses in numerical methods at Princeton University. Not only finite-difference methods were included, but also finite-element, collocation and boundary-element procedures. Chapter headings: 1. Fundamental Concepts. 2. Basic Concepts in the Finite Difference and Finite Element Methods. 3. Finite Elements on Irregular Subspaces. 4. Parabolic Partial Differential Equations. 6. Hyperbolic Partial Differential Equations.

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Introduction to optimal control theory. By Jack Macki and Aaron Strauss. Springer-Verlag, New York, 1982. xiii + 165 pp.

This is a volume in Undergraduate Texts in Mathematics. It is an introduction to optimal control theory for systems governed by vector ordinary differential equations. It is not intended as a state-of-the-art handbook for researchers. The authors have tried to keep two types of reader in mind: (1) mathematicians, graduate students, and advanced undergraduates in mathematics who want a concise introduction to a field which contains nontrivial interesting applications of mathematics (for example, weak convergence, convexity, and the theory of ordinary differential equations); (2) economists, applied scientists, and engineers who want to understand some of the mathematical foundations of optimal control theory. In general, they have emphasized motivation and explanation, avoiding the "definition-axiom-theorem-proof" approach. They make use of a large number of examples, especially one simple canonical example which is carried through the entire book. In proving theorems, they often just prove the simplest case, then state the more general results which can be proved.

Partial differential equations for scientists and engineers. By S. J. Farlow. John Wiley & Sons, New York, 1982. ix + 402 pp. \$29.95.

This book is organized in 47 semi-independent lessons, in contrast to the more usual chapter-by-chapter approach. Contents: 1. Introduction, 2. Diffusion-type problems (14 lessons), 3. Hyperbolic-type problems (15 lessons), 4. Elliptic-type problems (6 lessons), 5. Numerical and approximate methods (8 lessons). There is an appendix with integral transform tables and one with a PDE crossword puzzle.

An introduction to tensor calculus, relativity and cosmology. By D. F. Lawden. John Wiley & Sons, New York, 1982. xii + 205 pp. \$35.95.

This is the third edition of a text first published in 1962 by Methuen & Co. All sections have been thoroughly revised, some have been discarded and some new ones added (e.g. equations of motion of an elastic fluid, black holes, gravitational waves).

The fractal geometry of nature. By Benoit B. Mandelbrot. W. H. Freeman & Co., San Francisco, CA, 1977. vii + 443 pp. 418 illustrations. \$32.50.

The author conceived and developed a new geometry, the geometry of fractal shapes, to describe the geometry of nature's shapes: clouds, mountains, coast lines, snowflakes, etc. This book follows and—according to the author—largely replaces his 1977 essay *Fractals: Form, Chance and Dimension* (see Quart. Appl. Math. vol. 35, July 1978, p. 128). Nearly every section has been recast, mainly by adding accounts of new developments and introducing much new art. Chapter headings: 1. Introduction. 2. Three classic fractals, tamed. 3. Galaxies and eddies. 4. Scaling fractals. 5. Nonscaling fractals. 6. Self-mapping fractals. 7. Randomness. 8. Stratified random fractals. 9. Fractional brown fractals. 10. Random tremas; texture. 11. Miscellany. 12. Of men and ideas.

Quantum mechanics in mathematics, chemistry and physics. Edited by Karl E. Gustafson and William P. Reinhardt. Plenum Press, New York and London, 1981. ix + 506 pp. \$59.50.

This volume grew from a Special Session in Mathematical Physics as part of a meeting of the American Mathematical Society in Boulder, Colorado, 27–29 March 1980. There are papers by thirty-seven mathematicians, physicists and chemists from the Federal Republic of Germany, India, Belgium, People's Republic of China, Switzerland, Iran, Mexico, German Democratic Republic, England and the United States.

K-theory of forms. By Anthony Bak. Princeton University Press and University of Tokyo Press, 1981. vii + 263 pp. \$8.50 paperback, \$20.00 cloth.

This is number 98 of Annals of Mathematics Studies. It contains a unified treatment of basic materials for the theories of quadratic, even hermitian and hermitian forms and their classical algebraic *K*-theories.

Random Fourier series with applications to harmonic analysis. By Michael B. Marcus and Gilles Pisier. Princeton University Press and University of Tokyo Press, Tokyo, 1981. v + 150 pp. \$7.00 paperback, \$17.50 cloth.

This is number 101 of Annals of Mathematics Studies. The authors obtain necessary and sufficient conditions for the almost surely uniform convergence of random Fourier series on locally compact Abelian groups and on compact non-Abelian groups. Many related results such as a central limit theorem are obtained. The methods developed are used to study questions in harmonic analysis which are not intrinsically random.

Seminar on differential geometry. Edited by Shing-Tung Yau. Princeton University, Princeton, NJ, and University of Tokyo Press, Tokyo, 1982. vii + 706 pp.

This is number 102 of Annals of Mathematics Studies. It contains thirty-four papers presented in the seminars of a 1979-80 Institute of Advanced Study and National Science Foundation sponsored special activities program in differential geometry, with special emphasis on partial differential equations. The papers on closed equations and minimal surfaces have been collected in a separate volume.

Introduction to multidimensional scaling: Theory, methods, and applications. By Susan S. Schiffman, M. Lance Reynolds and Forrest W. Young. Academic Press, New York, 1981. xiv + 413 pp. \$29.50.

Multidimensional scaling (MDS) is a powerful mathematical procedure which can systemize data by representing the similarities of objects spatially as in a map. This book covers the design, execution, and analysis of multidimensional scaling experiments and includes detailed descriptions and examples of six major MDS computer programs. Discussed are various options that allow the user, for example, to select the model—classical, replicated, or weighted—most appropriate to a particular data set, and to examine the effect of different measurement level assumptions. Later chapters show how to fit properties and preferences to derived stimulus spaces, how to interpret vector spaces of subject weights, and how to analyze rectangular data matrices. The book is organized into three sections. Part I, "Basic Concepts and Data Bank," describes MDS in nonmathematical terms, covers experimental designs and procedures, includes test data, and gives an overview of MDS models and programs. Part II, "Methods and Applications," gives examples for each of the MDS programs and compares the results. Methods for interpreting stimulus and subject spaces are described and evaluated. Part II concludes with a chapter on the use of MDS in product development. Part III gives detailed descriptions of the MDS models and how the programs work.

Stochastic methods of operations research. By J. Kohlas. Cambridge University Press, London, 1982. ix + 224 pp. \$34.50 hardcover, \$13.95 paperback.

This is a translation of the book originally published by B. G. Teubner in 1977, supplemented by a section on numerical computation of transient solutions to continuous parameter Markov chains and a chapter on semi-Markov models. The fundamental tools for the treatment of stochastic problems of operations research are renewal theory and Markov chain, outlined in chapters 2 and 3, respectively. Chapter 1 is a concise introduction to probability theory. Applications of renewal theory and Markov chains to queuing systems are treated in chapter 4 and to dynamic optimization in chapter 5. Numerical techniques, based on simulation and Monte Carlo, are given in chapter 6 and an important extension of both renewal processes and Markov chains is furnished in the theory of semi-Markov processes (chapter 7).

Mathematics for computing. By G. P. McKeown and V. J. Rayward-Smith. John Wiley & Sons, New York, 1982. ix + 428 pp. \$24.95.

This book contains basic, algorithmically-oriented mathematics for students interested in computing: algebra, calculus, probability.

Theory of laminar flames. By J. D. Buckmaster and G. S. S. Ludford. Cambridge University Press, London, 1982. xii + 266 pp. \$49.50.

This is a volume in the series Cambridge Monographs on Mechanics and Applied Mathematics. It deals with the structure, generation and stability of flames from a mathematical point of view, using an approach developed over the past ten years, to provide a unified theoretical description of fundamental flame phenomena. The entire discussion is connected by a singular perturbation procedure known as activation-energy asymptotics, based on the fact that the description of reacting systems characterized by Arrhenius kinetics can be simplified when the activation energy is large, corresponding to an extreme sensitivity to temperature. Chapter headings: 1. Governing equations of combustion. 2. The pre-mixed plane flame. 3. Perturbations: SVFs and NEFs. 4. Steady burning of a linear condensate. 5. Unsteady burning of a linear condensate. 6. Spherical diffusion flames. 7. Cylindrical and spherical premixed flames. 8. Multidimensional theory of premixed flames. 9. Burners. 10. Effects of shear and strain. 11. Stability. 12. Ignition and explosion.

Introduction to linear regression analysis. By Douglas C. Montgomery and Elizabeth A. Peck. John Wiley & Sons, New York, 1982. xiii + 504 pp. \$34.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. It is intended as a text for a basic course in linear regression analysis and oriented toward the analyst who uses computers for problem solution. Throughout the book, the output from major regression computer packages is displayed and related to concepts presented in the text. Chapter headings: 1. Introduction. 2. Simple linear regression and correlation. 3. Measures of model adequacy. 4. Multiple linear regression. 5. Polynomial regression models. 6. Indicator variables. 7. Variable selection and model building. 8. Multicollinearity. 9. Topics in the use of regression analysis. 10. Validation of regression models.

Stochasticity and partial order—Doubly stochastic maps and unitary mixing. By Peter M. Alberti and Armin Uhlmann. D. Reidel Publishing Co., Dordrecht, Boston, London, 1982. 124 pp. \$28.50.

This is a volume in the series Mathematics and its Applications. Its intention is to explain and to prove certain relations between stochasticity and partial order. Chapter headings: 1. Some classical results. 2. Order structures of matrices. 3. The order structure in the state space of C^* - and W^* -algebras. 4. The c -ideal. 5. The Σ -property. 6. The dual structure in W^* -algebras.

Computing in statistical science through APL. By Francis John Ascombe. Springer-Verlag, New York, Heidelberg, Berlin, 1981. xv + 426 pp. \$24.80.

The first aim of this book is to interest statisticians in using the programming language APL in their work—for statistical analysis of data, for numerical support of theoretical studies, for simulation of random processes. In Part A the language is described and illustrated with short examples of statistical calculations. Part B, presenting some more extended examples of statistical analysis of data, has also the further aim of suggesting the interplay of computing and theory that must in the future be typical of the development of statistical science. Among the examples treated are time series analysis (Yale enrollment), regression (public school expenditures), and contingency tables and Pearson-Plackett distributions. No previous knowledge of computing is expected of the reader, but familiarity with some statistical concepts and theory is assumed—different amounts in different parts of the book.