

**A CLASS OF INVERSE PROBLEMS
FOR VISCOELASTIC MATERIAL
WITH DOMINATING NEWTONIAN VISCOSITY**

BY

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Abstract. Memory kernels in linear stress-strain relations involving a Newtonian viscosity are identified by solving a class of inverse problems. The inverse problems are reduced to nonlinear Volterra integral equations of the first kind which in turn lead to corresponding Volterra equations of the second kind by differentiation. Applying the contraction principle with weighted norms we derive global (in time) existence, uniqueness and stability of the solution to the inverse problems under similar assumptions as for related inverse problems in heat flow.

1. Introduction. In the linear theory of viscoelasticity for general materials with memory the constitutive stress-strain relation involves an integral term over the past history of the material containing a time-dependent kernel (cf. [13]–[15]). For identifying such memory kernels *inverse problems* have been used starting with the papers by Grasselli, Kabanikhin, Lorenzi [4, 5] and continued, for instance, in Grasselli [2], Lorenzi [12] and in our papers [8, 9].

In this paper we continue the investigations in [8, 9] to inverse problems for memory kernels in stress-strain relations involving a Newtonian viscosity term (see [15, pp. 21, 28]). As in [8, 9], by means of Fourier’s method of eigenfunction expansion for the direct initial-boundary value problem, we reduce the inverse problem to a nonlinear Volterra integral equation of the first kind. Differentiation of it leads to a corresponding equation of the second kind. To this equation we apply the contraction principle in weighted norms obtaining global (in time) existence and uniqueness of the solution and stability estimates for it. This method has been applied to related problems first in Janno [7] and more generally in Bukhgeim [1]. In contrast to the “hyperbolic” cases dealt with

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in [8, 9] in the present “parabolic” case the needed assumptions are as weak as in the corresponding inverse problems for heat flow (cf. [10]).

The application of the contraction principle to the Volterra integral equation of the second kind gives a basis for the numerical computation of the solution by the iteration method. For simplicity, in the following we only deal with continuous memory kernels in the generic “main case”. Corresponding problems with weakly singular memory kernels can be dealt with as in [9] and [10] and additional cases to the main case as in [8, 9]. Further, as in [8]–[10] inverse problems with observation functionals containing the traction can be treated (see also Grasselli [3]).

2. Problem formulation. We deal with the problem of identifying the continuous kernel m appearing in the linear parabolic integrodifferential equation

$$\rho u_{tt}(t, x) = \eta \Delta u_t + \beta \Delta u - \int_0^t m(t - \tau) \Delta u(\tau, x) d\tau + \chi(t, x) \quad (1)$$

in $Q = D \times (0, T)$, where D is a bounded domain in \mathbb{R}^n with piecewise smooth boundary S , $\eta > 0$, $\beta \geq 0$ are given constants, $\rho > 0$ and χ are given continuous functions on Q , and Δ denotes the Laplacian.

In the case $n = 1$ Eq. (1) occurs for viscoelastic wave propagation in a material with memory governed by the stress-strain relation

$$\sigma(t, x) = \eta \varepsilon_t(t, x) + \beta \varepsilon(t, x) - \int_{-\infty}^t m(t - \tau) \varepsilon(\tau, x) d\tau \quad (2)$$

between the strain ε and the stress σ , where η is a Newtonian viscosity which we take as positive (for the case $\eta = 0$ see our papers [8, 9]) and m is the relaxation memory kernel (see [15], pp. 21, 28). Further, we have $\varepsilon = u_x$ with the displacement u and, as usual, we assume $u(x, t) \equiv 0$ for $t < 0$.

The solution u of Eq. (1) should meet the initial and boundary conditions

$$u(0, x) = \varphi(x), \quad u_t(0, x) = \psi(x) \quad \text{on } D, \quad (3)$$

$$u(t, x) = 0 \quad \text{or} \quad \partial_\nu u + \alpha u = 0 \quad \text{on} \quad \Sigma = S \times (0, T), \quad (4)$$

where ν is the outer normal to S and φ , ψ and $\alpha \geq 0$ are given continuous functions on D and S , respectively.

In the *inverse problem* we have to find m for $t \in [0, T]$ from Eqs. (1), (3), (4) and, in addition, from a condition of the form

$$\Psi[u](t) = h(t), \quad t \in [0, T], \quad (5)$$

where h is a given continuous and differentiable function on $[0, T]$ and Ψ a linear functional on $u(t, \cdot)$, for instance the values $u(t, x_0)$ of u in some fixed point $x_0 \in D$.

If we introduce the velocity $v = u_t$ in Eq. (1) we obtain the following equation for v (cp. [6] or [14, p. 135], for instance):

$$\rho v_t = \eta \Delta v - \int_0^t G(t - \tau) \Delta v(\tau, x) d\tau + \chi(t, x) \quad (6)$$

where the kernel G is given by

$$G(t) = \int_0^t m(s) ds - \beta \quad (7)$$

and the function χ by

$$\chi(t, x) = f(t, x) - \Delta\varphi(x)G(t). \quad (8)$$

So, in the case $\Delta\varphi \equiv 0$ we arrive at a corresponding inverse problem for the smooth kernel (7) in a heat conduction problem which has been treated in [11]. Therefore, we assume $\Delta\varphi \neq 0$.

In the following we consider the inverse problem for Eq. (1) as a particular case of the abstract problem

$$\ddot{u}(t) + \eta A \dot{u}(t) + \beta A u(t) - \int_0^t m(t - \tau) A u(\tau) d\tau = f(t) \quad (9)$$

in $[0, T]$ with the conditions $u(0) = \varphi$, $\dot{u}(0) = \psi$ and (5), where $u, f \in ([0, T] \rightarrow X)$ are abstract functions with values in a Hilbert space X , Ψ is a linear functional on X , and A is a linear symmetric operator in X possessing a complete orthonormal system of eigenelements v_n with nonnegative eigenvalues μ_n of finite multiplicity:

$$A v_n = \mu_n v_n, \quad 0 \leq \mu_1 \leq \mu_2 \leq \dots \quad (10)$$

In the case of Eq. (1), where $A = -(1/\rho)\Delta$ with (4), we will not specify the space X in advance and deal with a general functional Ψ not necessarily continuous; only the coefficients $\gamma_n = \Psi[v_n]$ are assumed as finite numbers.

3. Reduction to an integral equation of the first kind. The solution u of Eq. (9) is taken in the form of the Fourier expansion

$$u(t) = \sum_{n=1}^{\infty} B_n(t) v_n. \quad (11)$$

By Eq. (9) and $u(0) = \varphi$, $\dot{u}(0) = \psi$ the scalar coefficient functions B_n in (11) are the solutions of the initial-value problem

$$\ddot{B}_n(t) + \eta \mu_n \dot{B}_n(t) + \beta \mu_n B_n(t) = \mu_n (m * B_n)(t) + f_n(t), \quad t \in [0, T], \quad (12)$$

$$B_n(0) = \varphi_n, \quad \dot{B}_n(0) = \psi_n, \quad (13)$$

where $f_n(t)$, φ_n , ψ_n are the Fourier coefficients of $f(t)$, φ , ψ , respectively, and $*$ denotes convolution.

The problem (12), (13) is equivalent to the Volterra integral equation for B_n ,

$$B_n(t) - (L_n[m] * B_n)(t) = \Phi_n(t), \quad t \in [0, T], \quad (14)$$

where

$$L_n[m](t) = \mu_n (l_n * m)(t) \quad (15)$$

and Φ_n, l_n are the solutions of

$$\ddot{\Phi}_n + \eta\mu_n\dot{\Phi}_n + \beta\mu_n\Phi_n = f_n, \quad \Phi_n(0) = \varphi_n, \quad \dot{\Phi}_n(0) = \psi_n, \tag{16}$$

$$\ddot{l}_n + \eta\mu_n\dot{l}_n + \beta\mu_n l_n = 0, \quad l_n(0) = 0, \quad \dot{l}_n(0) = 1. \tag{17}$$

From (16), (17) we have

$$l_n(t) = \exp[-a_n t] \frac{\sinh b_n t}{b_n} \tag{18}$$

with

$$a_n = \eta\mu_n/2, \quad b_n = (\eta^2\mu_n^2/4 - \beta\mu_n)^{1/2}, \tag{19}$$

where $\text{Re } b_n \geq 0$, and

$$\Phi_n(t) = \varphi_n(\dot{l}_n(t) + \eta\mu_n l_n(t)) + \psi_n l_n(t) + (f_n * l_n)(t). \tag{20}$$

By (11) the additional condition (5) takes the form

$$\sum_{n=1}^{\infty} \gamma_n B_n(t) = h(t), \quad t \in [0, T], \tag{21}$$

provided the series in (21) is (pointwise) convergent. Using (14) with (15) the condition (21) is reduced to the nonlinear Volterra integral equation of the first kind

$$\int_0^t K[m](t-s)m(s) ds = g(t), \quad t \in [0, T], \tag{22}$$

where

$$K[m](t) = \sum_{n=1}^{\infty} \gamma_n \mu_n (l_n * B_n)(t), \tag{23}$$

$$g(t) = h(t) - \sum_{n=1}^{\infty} \gamma_n \Phi_n(t). \tag{24}$$

Defining the solution u of problem (9) with $u(0) = \varphi, \dot{u}(0) = \psi$ by Eq. (11), the integral equation (22) for m together with Eqs. (14) for the $B_n = B_n[m]$ are equivalent to the inverse problem.

4. Preparations. At first we prove the following estimates for the functions l_n in (18). For $t \geq 0$ and $k = 0, 1, \dots$ it follows that

$$\mu_n |l_n^{(k)}(t)| \leq C_k \mu_n^k, \quad n = 1, 2, \dots \tag{25}$$

with positive constants C_k . Namely, from (18) we have

$$\begin{aligned} \mu_n l_n^{(k)}(t) &= \mu_n \sum_{j=0}^k \binom{k}{j} (\exp[-a_n t])^{(k-j)} \left(\frac{\sinh b_n t}{b_n} \right)^{(j)} \\ &= (-1)^k (2/\eta) a_n^{k+1} e^{-a_n t} \frac{\sinh b_n t}{b_n} + S_k, \end{aligned} \tag{26}$$

where $S_0 = 0$ and

$$S_k = (\mu_n/2) \sum_{j=1}^k \binom{k}{j} (-a_n)^{k-j} b_n^{j-1} e^{-a_n t} (e^{b_n t} - (-1)^j e^{-b_n t}), \quad k \geq 1. \tag{27}$$

Further, since the function $\sinh y/y$ is increasing for $y \geq 0$ and $0 \leq b_n \leq a_n$ in the case of real b_n we can estimate

$$\left| a_n e^{-a_n t} \frac{\sinh b_n t}{b_n} \right| \leq a_n t e^{-a_n t} \frac{\sinh a_n t}{a_n t} \leq \frac{1}{2}. \tag{28}$$

For imaginary b_n we have $|\sinh b_n t/b_n t| \leq 1$ which together with $|a_n t e^{-a_n t}| \leq 1/e$ yields

$$\left| a_n e^{-a_n t} \frac{\sinh b_n t}{b_n} \right| \leq \frac{1}{e}. \tag{29}$$

Moreover, by (19)

$$|e^{-a_n t} e^{b_n t}|, |e^{-a_n t} e^{-b_n t}| \leq 1. \tag{30}$$

Using (28)–(30) in (26), (27) as well as $a_n = \eta\mu_n/2$ and the inequalities

$$|b_n| \leq \max\left(\frac{\eta\mu_n}{2}, (\beta\mu_n)^{1/2}\right), \quad \mu_n^{1/2} \leq \bar{\mu}^{-1/2} \mu_n$$

where $\bar{\mu}$ is the first non-vanishing eigenvalue, we obtain (25).

For proving existence and stability of the solution m to Eq. (22) we introduce the scale of norms

$$\|m\|_\sigma := \|e^{-\sigma t} m\|_{C[0,T]} = \max_{0 \leq t \leq T} (e^{-\sigma t} |m(t)|), \quad \sigma \geq 0$$

with $\|m\| := \|m\|_0$. It follows that

$$e^{-\sigma T} \|m\| \leq \|m\|_\sigma \leq \|m\|, \quad \sigma \geq 0. \tag{31}$$

Further, we denote the data set by $d = ((\varphi_n)_{n \geq 1}, (\psi_n)_{n \geq 1}, (f_n)_{n \geq 1})$. Then our main lemma becomes

LEMMA. Let $m, f_n \in C[0, T]$ and

$$\sum_{n=1}^\infty |\gamma_n| \mu_n^3 |\varphi_n| < \infty, \quad \sum_{n=1}^\infty |\gamma_n| \mu_n^2 |\psi_n| < \infty, \quad \sum_{n=1}^\infty |\gamma_n| \mu_n^2 \|f_n\| < \infty. \tag{32}$$

Then $K^{(3)}[m] \in C[0, T]$. Further, for kernels $K[m^\alpha, d^\alpha]$ corresponding to $m^\alpha, d^\alpha, \alpha = 1, 2$, respectively, the following inequality holds for $\sigma \geq 0$:

$$\begin{aligned} \|K^{(3)}[m^1, d^1] - K^{(3)}[m^2, d^2]\|_\sigma &\leq M(\|m^1\|_\sigma, \|m^2\|_\sigma, |d^1|_0) \\ &\times \{\|m^1 - m^2\|_\sigma + |d^1 - d^2|_0\}, \end{aligned} \tag{33}$$

where M is a continuous function nondecreasing in each of its arguments and

$$|d|_0 := \sum_{n=1}^\infty |\gamma_n| \mu_n^2 [\mu_n |\varphi_n| + |\psi_n| + \|f_n\|]. \tag{34}$$

Proof. First, observe that for $v_1, v_2 \in C[0, T]$ we have

$$\|v_1 * v_2\|_\sigma \leq T \|v_1\|_\sigma \|v_2\|_\sigma, \quad \sigma \geq 0. \tag{35}$$

Thus, by (15) and (25) the kernel $L_n[m]$ of Eq. (14) fulfills the estimation

$$\|L_n[m]\|_\sigma \leq c \|m\|_\sigma, \quad \sigma \geq 0, \tag{36}$$

with the positive constant $c = C_0 T$.

We write Eq. (14) in the form

$$e^{-\sigma t} B_n(t) - \int_0^t e^{-\sigma(t-s)} L_n[m](t-s) e^{-\sigma s} B_n(s) ds = e^{-\sigma t} \Phi_n(t)$$

and estimate its solution by means of Gronwall's lemma, obtaining the inequality

$$\|B_n\|_\sigma \leq \exp(T \|L_n[m]\|_\sigma) \|\Phi_n\|. \tag{37}$$

Further, the difference $\Delta_n = B_n[m^1, d^1] - B_n[m^2, d^2]$ of B_n for two m^α and d^α , $\alpha = 1, 2$, satisfies the equation

$$\begin{aligned} e^{-\sigma t} \Delta_n(t) - \int_0^t e^{-\sigma(t-s)} L_n[m^2](t-s) e^{-\sigma s} \Delta_n(s) ds \\ = \Phi_n^1(t) - \Phi_n^2(t) + (L_n[m^1 - m^2] * B_n[m^1, d^1])(t) \end{aligned}$$

where $\Phi_n^\alpha = \Phi_n[d^\alpha]$, $\alpha = 1, 2$. Again by Gronwall's lemma and (35) we get

$$\|\Delta_n\|_\sigma \leq \exp(T \|L_n[m^2]\|_\sigma) \times \{ \|\Phi_n^1 - \Phi_n^2\| + T \|L_n[m^1 - m^2]\|_\sigma \|B_n[m^1, d^1]\|_\sigma \}.$$

Using here (36) and (37), we derive the inequality

$$\|\Delta_n\|_\sigma \leq M_1 (\|m^1\|_\sigma, \|m^2\|_\sigma) \|\Phi_n^1\| \{ \|m^1 - m^2\|_\sigma + \|\Phi_n^1 - \Phi_n^2\| \} \tag{38}$$

with a continuous and nondecreasing function M_1 .

To obtain an analogous inequality for the derivative $\dot{\Delta}_n$ we observe that \dot{B}_n also satisfies Eq. (14) with the same kernel and the right-hand side $\hat{\Phi}_n = \dot{\Phi}_n + L_n[m]\varphi_n$. Therefore, estimating as above we get

$$\begin{aligned} \|\dot{\Delta}_n\|_\sigma \leq M_2 (\|m^1\|_\sigma, \|m^2\|_\sigma) (\|\dot{\Phi}_n^1\| + |\varphi_n^1|) \\ \times \{ \|m^1 - m^2\|_\sigma + \|\dot{\Phi}_n^1 - \dot{\Phi}_n^2\| + |\varphi_n^1 - \varphi_n^2| \} \end{aligned} \tag{39}$$

with a continuous and nondecreasing function M_2 again.

Now we can estimate the derivative $K^{(3)}$. In view of the relations $l_n(0) = 0$, $\dot{l}_n(0) = 1$, $\ddot{l}_n(0) = -\eta\mu_n$ in (17) from (23) we have

$$K^{(3)}[m] = \sum_{n=1}^\infty \gamma_n \mu_n (l_n^{(3)} * B_n + \dot{B}_n - \eta\mu_n B_n). \tag{40}$$

This on account of (25) and (35) implies

$$\|K^{(3)}[m^1, d^1] - K^{(3)}[m^2, d^2]\|_\sigma \leq c_1 \sum_{n=1}^\infty |\gamma_n| [(\mu_n^3 + \mu_n^2) \|\Delta_n\|_\sigma + \mu_n \|\dot{\Delta}_n\|_\sigma] \tag{41}$$

with a positive constant c_1 . For using here the inequalities (38) and (39) for Δ_n and $\dot{\Delta}_n$ we need estimations for Φ_n and $\dot{\Phi}_n$. Observing (25) from (20) we obtain

$$\mu_n \|\Phi_n^{(i)}\| \leq c_{2,i} \mu_n^i [\mu_n |\varphi_n| + |\psi_n| + \|f_n\|], \quad i = 0, 1, \tag{42}$$

with positive constants $c_{2,i}, i = 0, 1$. Now the inequalities (41), (38), (39), and (42) imply the asserted inequality (33) with (34).

Finally, due to $m, f_n \in C[0, T]$ from Eq. (14) we see that $B_n, \dot{B}_n \in C[0, T]$. Moreover, analogously as in the proof of (33) in view of the assumptions (32) we derive the uniform convergence of the series in (40). This implies $K^{(3)}[m] \in C[0, T]$. The Lemma is completely proved.

5. Reduction to an integral equation of the second kind. Under a regularity assumption we reduce Eq. (22) to an integral equation of the second kind by differentiating Eq. (22) three times. Precisely, we have

THEOREM 1. Let $h \in C^3[0, T], f_n \in C^1[0, T]$. Further, let the convergence conditions (32) and

$$\sum_{n=1}^{\infty} |\gamma_n| \|\dot{f}_n\| < \infty \tag{43}$$

be fulfilled. Moreover, the matching conditions

$$\begin{aligned} h(0) &= \sum_{n=1}^{\infty} \gamma_n \varphi_n, & \dot{h}(0) &= \sum_{n=1}^{\infty} \gamma_n \psi_n, \\ \ddot{h}(0) &= \sum_{n=1}^{\infty} \gamma_n f_n(0) - \eta \sum_{n=1}^{\infty} \gamma_n \mu_n \varphi_n - \beta \sum_{n=1}^{\infty} \gamma_n \mu_n \psi_n \end{aligned} \tag{44}$$

are assumed. Then

$$g \in C^3[0, T], \quad g(0) = \dot{g}(0) = \ddot{g}(0) = 0. \tag{45}$$

If, in addition,

$$a := \sum_{n=1}^{\infty} \gamma_n \mu_n \varphi_n \neq 0, \tag{46}$$

then Eq. (22) for $m \in C[0, T]$ is equivalent to the Volterra equation of the second kind

$$m(t) + \frac{1}{a} \int_0^t K^{(3)}[m](t-s)m(s) ds = \frac{1}{a} g^{(3)}(t), \quad t \in [0, T], \tag{47}$$

where $K^{(3)}[m] \in C[0, T]$.

Proof. By (24) we have for the derivatives of g

$$g^{(i)}(t) = h^{(i)}(t) - \sum_{n=1}^{\infty} \gamma_n \Phi_n^{(i)}(t), \quad i = 1, 2, 3, \tag{48}$$

where $h \in C^3[0, T]$ by assumption and $\Phi_n \in C^3[0, T]$ by (16) with the assumption $f_n \in C^1[0, T]$. Moreover, from (16)

$$\ddot{\Phi}_n = -\eta \mu_n \ddot{\Phi}_n - \beta \mu_n \dot{\Phi}_n + \dot{f}_n. \tag{49}$$

Using (42) from (16) and (49) we get the estimations

$$\begin{aligned} \|\ddot{\Phi}_n\| &\leq c_3(\mu_n + 1)(\mu_n|\varphi_n| + |\psi_n| + \|f_n\|), \\ \|\ddot{\Phi}_n\| &\leq c_4\mu_n(\mu_n + 1)(\mu_n|\varphi_n| + |\psi_n| + \|f_n\|) + \|\dot{f}_n\|. \end{aligned} \tag{50}$$

In view of the assumptions (32) and (43) the estimation (50) shows that $g \in C^3[0, T]$ by (48). Further by (16) and (48) at $t = 0$ for $i = 0, 1, 2$ and the matching conditions (44) the relations $g(0) = \dot{g}(0) = \ddot{g}(0) = 0$ follow. So, (45) is proved.

Differentiating the kernel (23) two times and observing that from (17) and (13) we have $l_n(0) = 0, \dot{l}_n(0) = 1$ and $B_n(0) = \varphi_n$ we see that

$$K[m](0) = \dot{K}[m](0) = 0 \quad \text{and} \quad \ddot{K}[m](0) = a, \tag{51}$$

where a is given by (46). Further by the Lemma we have $K^{(3)}[m] \in C[0, T]$ for $m \in C[0, T]$. Now, due to (45), (46), and (51) Eq. (22) is equivalent in $C[0, T]$ to Eq. (47) obtained by differentiating it three times. Theorem 1 is proved.

6. Existence, uniqueness and stability. Using the equivalence of Eqs. (47) and (22) we derive the existence, uniqueness and stability of the solution to the inverse problem. At first we prove existence and uniqueness.

THEOREM 2. Let the assumptions of Theorem 1 be fulfilled. Then Eq. (22) has a unique continuous solution in $[0, T]$. The solution can be calculated from the equivalent Eq. (47) by the method of successive approximation.

Proof. By Theorem 1 Eqs. (22) and (47) in $C[0, T]$ are equivalent. We write Eq. (47) in the form

$$m + L(G[m], m) = \chi \tag{52}$$

in the Banach space $E = C[0, T]$ equipped with the scale of weighted norms $\|m\|_\sigma$ where

$$G[m] = (1/a)K^{(3)}[m], \quad L(m_1, m_2) = m_1 * m_2 \tag{53}$$

and $\chi = (1/a)g^{(3)}$.

By Corollary 1 in [10], Eq. (52) has a unique solution for any $\chi \in E$ if G is an operator in E satisfying the condition

$$\|G[m_1] - G[m_2]\|_\sigma \leq M(\|m_1\|_\sigma, \|m_2\|_\sigma)\|m_1 - m_2\|_\sigma \tag{54}$$

for $\sigma \geq 0$ and for every $m_1, m_2 \in E$, where M is a continuous function from \mathbb{R}_+^2 into \mathbb{R}_+ increasing in each of its arguments, and L is a bilinear operator from $E \times E$ into E such that the inequalities

$$\|L(m_1, m_2)\|_\sigma \leq N_1\|m_1\|_\sigma\|m_2\|_\sigma, \quad \sigma \geq 0, \tag{55}$$

$$\|L(m_1, m_2)\|_\sigma \leq N_2(\sigma) \min(\|m_1\|_\sigma\|m_2\|_\sigma, \|m_1\| \|m_2\|_\sigma), \quad \sigma \geq 0 \tag{56}$$

are fulfilled for every $m_1, m_2 \in E$, where N_1 is a positive constant and N_2 is a continuous decreasing function from \mathbb{R}_+ into \mathbb{R}_+ vanishing for $\sigma \rightarrow \infty$.

The condition (54) is fulfilled for the operator G in (53) in view of the estimation (33) for $d^1 = d^2 = d$ in the Lemma. The inequality (55) with $N_1 = T$ follows from the basic estimation (35) of the convolution. Also we have the estimation

$$\|m_1 * m_2\|_\sigma \leq \|m_1\|_\sigma \|m_2\| \int_0^T e^{-\sigma s} ds = \frac{1}{\sigma} (1 - e^{-\sigma T}) \|m_1\|_\sigma \|m_2\|$$

implying (56).

The proof of the applied Corollary 1 in [10] uses the contraction principle in a scale of norms $\|m\|_\sigma$ for sufficiently large σ . Therefore the method of successive approximation can be used for calculation of the solution m to Eq. (47). Theorem 2 is proved.

Finally, by Lemma 4 in [10] using the estimation (33) for general d^α , $\alpha = 1, 2$, we can prove the following stability theorem (cp. Theorem 2 in [10]).

THEOREM 3. Let the assumptions of Theorem 1 be fulfilled for two data sets $D^\alpha = (d^\alpha, h_\alpha)$, $\alpha = 1, 2$, of the inverse problem. Then for the corresponding solutions m^α , $\alpha = 1, 2$, the stability estimate

$$\|m^1 - m^2\| \leq C(\|m^1\|, \|K^{(3)}[m^1, d^1]\|, |a^1|^{-1}, |a^2|^{-1}, |D^1|, |D^2|) |D^1 - D^2|$$

holds provided $|D^1 - D^2|$ is sufficiently small. Here C denotes a positive constant, a^i are the non-vanishing constants

$$a^i = \sum_{n=1}^{\infty} \gamma_n \mu_n \varphi_n^i, \quad i = 1, 2,$$

and the semi-norm $|D|$ is defined by

$$|D| := |d|_0 + \sum_{n=1}^{\infty} |\gamma_n| \|\dot{f}_n\| + \|h^{(3)}\|$$

with $|d|_0$ given by (34).

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