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ON STABILITY OF SHOCK WAVES IN RELATIVISTIC MAGNETOHYDRODYNAMICS

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Abstract. The structural stability of relativistic magnetohydrodynamic shock waves is studied. Stability results are obtained for the special case of fast parallel shock waves. It is proved that the instability and linear stability domains coincide with those of shock waves in relativistic gas dynamics. The domain of structural (nonlinear) stability, where the uniform Lopatinski condition is fulfilled for the stability problem, is found. It is shown that the structural stability domain is smaller than that of relativistic gas dynamic shock waves.

1. Introduction. The question of the stability of shock waves in relativistic magnetohydrodynamics (MHD) is of great importance in connection with various applications in astrophysics, cosmology, and plasma physics.

A first rigorous study of relativistic MHD shock waves was given in [21], [22], and [23], where the following are proved: the timelike character of the shock waves fronts; the main thermodynamic inequalities (compressibility conditions); some existence and uniqueness theorems for nontrivial solutions of the jump conditions satisfying the entropy inequality. Moreover, in these works, the relative location of the speeds of the shock waves with respect to the magnetosonic and Alvén speeds is found, and the classification of shock waves as fast and slow is given.

It is shown in [27] that the system of conservation laws of relativistic MHD can be rewritten in the form of a symmetric t-hyperbolic (in the sense of Friedrichs [12]) system. This fact allows us to obtain some results from [21], [22], [23] in a different (easier) way and to conclude the local (short-time) well-posedness of the Cauchy problem in a Sobolev space W_2^s ($s \geq 3$ for 3-D; see [30], [20], [17]).

The main aim of the present paper is to investigate the structural stability of shock waves in relativistic MHD. Here the term "structural stability" means that a surface of strong discontinuity preserves its structure under small perturbations. But, as will be noted below, the linear stability of a strong discontinuity with respect to small perturbations does not always guarantee the local existence of shock front solutions of a

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quasilinear system of conservation laws. In this connection, the concept of structural stability must be defined more exactly.

Following [2], [3], [4], [24], [25], [26] we will say that a strong discontinuity is *structurally stable* if the boundary conditions of the corresponding linear stability problem satisfy the *uniform Lopatinski condition* [18] (in [24], [25], [26] the term "uniform stability" is also used). Actually, the whole domain of parameters of the linear stability problem consists of the following subdomains:

I. The domain, where the Lopatinski condition is violated (*instability*);

II. The domain of fulfilment of the uniform Lopatinski condition (*structural stability*);

III. The domain, where only the Lopatinski condition holds, and the uniform Lopatinski condition is not fulfilled.

In domain II, a priori estimates *without loss of smoothness* (see, for example, [5] and Remark 7.1 of the present article) for solutions of the linear stability problem for gas dynamic shock waves are obtained (see other variants of such estimates in [24]), and the theorem of local existence and uniqueness of the classical solution in a Sobolev space W_2^s, where $s \geq 3$ as for the Cauchy problem (see [17]), to the quasilinear gas dynamics system behind the curvilinear shock wave is proved in [2], [3] (see also [4]) with the help of the dissipative energy integrals method [16]. In [4] it is discussed how these results can be generalized to the hyperbolic systems of a so-called acoustic type. Some general results are obtained also in [25], where the theorem on local existence of discontinuous shock fronts of a quasilinear system of conservation laws, which satisfies some structural conditions, in a Sobolev space W_2^s ($s \geq 10$ for 3-D) is proved for the case of shock waves (k-shocks, see [16]).

Note that the local existence and uniqueness theorems in [25], [4] do not cover all the stability problems for strong discontinuities in hyperbolic models of continuum mechanics. For example, the strong discontinuities for systems of conservation laws, which are supplemented by a set of stationary conservation laws (MHD is an example of such systems), need individual consideration. There are also evolutionary (see Sec. 4) strong discontinuities that are not k-shocks (for instance, rotational discontinuities in MHD). But, on the other hand, in domain II, an ill-posedness example of Hadamard type cannot be constructed for the stability problem and as well as for all close problems that are obtained by a perturbation of the system and the boundary conditions. So, with a certain degree of strictness we can say that domain II is the domain of structural—nonlinear—stability.

It should also be noted that the presence of an a priori estimate *with loss of smoothness* (it is the case of domain III) does not allow the main results obtained for the case of constant coefficients to be carried over to the quasilinear case. In such a case, we cannot judge the existence of a strong discontinuity (as a physical structure) on the linear level of investigation. For example, it is shown in [4] that for the linear stability problem for gas dynamic shock waves one can so perturb the system and the boundary conditions that an ill-posedness example of Hadamard type can be constructed in domain III for the perturbed stability problem (in fact, this perturbed problem is a variable coefficients stability problem with "frozen" coefficients, see [4]).

Thus, in view of the above reasoning, we concentrate the main attention in this paper to the finding of domain II, i.e., the domain of structural—nonlinear—stability of relativistic MHD shock waves.

By virtue of a big complication of relativistic MHD, we begin our investigation of structural stability of shock waves with the special case of fast parallel shock waves. Only the stability of this kind of fast shock waves, when the magnetic field is parallel to the normal to the shock wave front, is studied in the framework of this paper.

It is proved that the instability (domain I) and linear stability (domains II and III) domains for fast parallel shocks do not depend on the magnetic field, and, so, they coincide with the corresponding domains in relativistic gas dynamics [1], [29]. This result is analogous to that obtained in [13] for the usual MHD. On the other hand, it is shown that the magnetic field reduces the structural stability domain (domain II) in comparison with that of relativistic gas dynamic shocks [6].

Finally, we note that the results in [21], [22], [23], [27] are necessary but not sufficient for the 3-D stability of relativistic MHD shock waves. It should also be noted that the solving of the stability problem is the first and necessary step to investigate discontinuous flows in relativistic MHD. In particular, structural stability is a necessary condition to proving the convergence of the numerical solution to the weak solution of a hyperbolic system.

The paper is organized as follows. In Sec. 2 we write out the system of relativistic MHD equations and, in Sec. 3, the problem of their symmetrization and finding of the hyperbolicity domain is discussed. In Sec. 4 the stability problem for fast parallel shock waves is presented. Applying technical ideas of [13], we give in Sec. 5 an equivalent definition of the uniform Lopatinski condition for the case when, ahead of the plane shock front, all the characteristics of the system are arriving and, behind the plane shock front, only one characteristic is arriving, and the others are leaving. Section 6 is devoted to finding the instability domain for fast parallel shock waves, and in Sec. 7 conditions for structural stability are presented.

We will use the symbol * to denote matrix transposition. Repeated indices are considered to be summed, the Greek indices run from 0 to 3, and the Latin ones from 1 to 3, except where stated otherwise.

2. The equations of relativistic magnetohydrodynamics. Following [21], [22], [23], [27], the equations of relativistic MHD for an ideal medium are written as follows:

$$\nabla_\alpha(\rho u^\alpha) = 0, \tag{2.1}$$

$$\nabla_\alpha T^{\alpha\beta} = 0, \tag{2.2}$$

$$\nabla_\alpha(u^\alpha b^\beta - u^\beta b^\alpha) = 0, \tag{2.3}$$

where ∇_α is the covariant derivative with respect to the metric $g_{\alpha\beta} = \mathrm{diag}(1, -1, -1, -1)$ of the space-time; ρ is the rest frame density; u^α the unit 4-velocity, oriented towards the future; $T^{\alpha\beta}$ is the total energy momentum tensor with the components

$$T^{\alpha\beta} = \left(\rho h + \frac{1}{4\pi}B^2\right) u^\alpha u^\beta - \left(p + \frac{1}{8\pi}B^2\right) g_{\alpha\beta} - \frac{1}{4\pi}b^\alpha b^\beta;$$

p is the pressure; h the index of the fluid,

$$h = 1 + e_0 + pV, \qquad V = 1/\rho;$$

e_0 is the specific internal energy; b^α the magnetic field 4-vector, which satisfies

$$u^\alpha b_\alpha = 0;$$

$B^2 = -b^\alpha b_\alpha$ is strictly positive, and the speed of light is equal to unity.

Let (t, x^i) be inertial coordinates (we restrict ourselves to the case of special relativity). Then

$$u^\alpha = (u^0, u^i) = \Gamma(1, v^i), \qquad u_\alpha = \Gamma(1, -v^i),$$
$$b^\alpha = (b^0, b^i) = ((\mathbf{b}, \mathbf{v}), b^i) = \Gamma\left((\mathbf{v}, \mathbf{H}),\ \frac{H^i}{\Gamma^2} + v^i(\mathbf{v}, \mathbf{H})\right), \qquad b_\alpha = (b^0, -b^i),$$

where $\Gamma = (1 - |\mathbf{v}|^2)^{-1/2}$ is the Lorenz factor; $\mathbf{v} = (v^1, v^2, v^3)^*$ is the medium velocity vector-column, with $|\mathbf{v}|^2 = (\mathbf{v}, \mathbf{v}) = v^i v_i$; $\mathbf{H} = (H^1, H^2, H^3)^*$ is the magnetic field vector-column, with $|\mathbf{H}|^2 = (\mathbf{H}, \mathbf{H}) = H^i H_i$, $(\mathbf{v}, \mathbf{H}) = v^i H_i$; $\mathbf{b} = (b^1, b^2, b^3)^*$, $(\mathbf{b}, \mathbf{v}) = b^i v_i$. Furthermore,

$$B^2 = -b^\alpha b_\alpha = \frac{|\mathbf{H}|^2}{\Gamma^2} + (\mathbf{v}, \mathbf{H})^2 > 0,$$
$$u^\alpha u_\alpha = \Gamma^2(1 - |\mathbf{v}|^2) = 1, \qquad \mathbf{u} = (u^1, u^2, u^3)^* = \Gamma \mathbf{v}, \qquad \Gamma^2 = 1 + |\mathbf{u}|^2.$$

System (2.1)–(2.3) can now be rewritten in the form of the following system of conservation laws:

$$\rho_t + \operatorname{div}(\rho \mathbf{u}) = 0 \quad \text{(matter conservation)}, \tag{2.4}$$
$$(M^j)_t + \frac{\partial \Pi_{jk}}{\partial x^k} = 0 \quad \text{(momentum conservation)}, \tag{2.5}$$
$$\mathbf{H}_t - \operatorname{rot}(\mathbf{v} \times \mathbf{H}) = 0 \quad \text{(Maxwell equations)}, \tag{2.6}$$
$$(\mathcal{E}_0)_t + \operatorname{div} \boldsymbol{\mathcal{E}} = 0 \quad \text{(energy conservation)}. \tag{2.7}$$

Moreover, system (2.4)–(2.7) should be supplemented by the first equation from system (2.3):

$$\operatorname{div} \mathbf{H} = 0 \quad \text{(stationary conservation law)}, \tag{2.8}$$

which is, as a matter of fact, an additional requirement on the initial data for system (2.4)–(2.7). Here

$$M^j = \mathcal{P} v^j - \frac{(\mathbf{v}, \mathbf{H})}{4\pi} H^j, \qquad \mathcal{P} = \rho h \Gamma^2 + \frac{|\mathbf{H}|^2}{4\pi},$$
$$\Pi_{jk} = \mathcal{P} v^j v^k + \left(p + \frac{B^2}{4\pi}\right)\delta_{jk} - \frac{1}{4\pi}\left\{\frac{H^j H^k}{\Gamma^2} + (\mathbf{v}, \mathbf{H})(v^j H^k + H^j v^k)\right\},$$
$$\mathcal{E}_0 = \mathcal{P} - \left(p + \frac{B^2}{4\pi}\right), \qquad \boldsymbol{\mathcal{E}} = (\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3)^* = \mathcal{P}\mathbf{v} - \frac{(\mathbf{v}, \mathbf{H})}{4\pi}\mathbf{H}.$$

The proper temperature T of the fluid and its specific entropy s satisfy, as in classical gas dynamics, the Gibbs relation

$$T\, ds = de_0 + p\, dV. \tag{2.9}$$

Thus, with a state equation of medium

$$e_0 = e_0(\rho, s),$$

in view of (2.9), we have

$$T = (e_0)_s(\rho, s), \qquad p = -(e_0)_V(\rho, s) = \rho^2 (e_0)_\rho(\rho, s),$$

and we can consider system (2.4)–(2.7) as a system for deriving the components of the unknown variables vector

$$\mathbf{U} = \begin{pmatrix} p \\ s \\ \mathbf{u} \\ \mathbf{H} \end{pmatrix}.$$

Finally, system (2.4)–(2.7) implies the *additional conservation* law (see [21], [22], [23], [27])

$$(\rho\Gamma s)_t + \operatorname{div}(\rho s \mathbf{u}) = 0 \quad \text{(entropy conservation).} \tag{2.10}$$

Note that law (2.10) was used in [27] for symmetrization of the equations of relativistic MHD.

3. Symmetric form and hyperbolicity conditions. In view of the fact that system (2.4)–(2.7) is supplemented by the stationary conservation law (2.8), the system of relativistic MHD cannot be symmetrized with the help of the well-known symmetrization scheme first suggested in [14] (see also [11], [12], [10]). But this system can be symmetrized with the help of another scheme that was first used in [15] for the symmetrization of classical MHD (see also [28], [4], [8]). In fact, this was performed in [27], where the vector of canonic variables $\mathbf{Q}$ is found. In terms of this vector system, (2.4)–(2.7) is rewritten in the symmetric form

$$A^0 \mathbf{Q}_t + A^k \mathbf{Q}_{x^k} = 0, \tag{3.1}$$

where $A^\alpha = A^\alpha(\mathbf{Q})$ are symmetric matrices.

The components of the vector $\mathbf{Q}, q_j$, $j = 1, 2, \ldots, 8$, are found from the relations

$$d(\rho\Gamma s) = q_1 d(\rho\Gamma) + q_{i+1}\, dM^i + q_{i+4}\, dH^i + q_8\, d\mathcal{E}_0,$$

and $\mathbf{Q}$ has the form

$$\mathbf{Q} = \frac{1}{T} \begin{pmatrix} sT - h \\ -\mathbf{u} \\ -\frac{\mathbf{b}}{4\pi} \\ \Gamma \end{pmatrix}$$

(in [27] the components of $\mathbf{Q}$ are ordered in a different way and have the opposite sign).

Following [15], [8], let us introduce the so-called *productive functions*

$$L = L(\mathbf{Q}) = q_1 \rho\Gamma + q_{i+1} M^i + q_{i+4} H^i + q_8 \mathcal{E}_0 - \rho\Gamma s,$$

$$M^k = M^k(\mathbf{Q}) = q_1 \rho u^k + q_{i+1} \Pi_{ik} + q_{i+4}(v^k H^i - v^i H^k) + q_8 \mathcal{E}_k + R H^k - \rho s u^k,$$

where the function $R = R(\mathbf{Q})$ is determined from the relation

$$d(\rho s u^k) = q_1 d(\rho u^k) + q_{i+1}\, d\Pi_{ik} + q_{i+4} d(v^k H^i - v^i H^k) + q_8\, d\mathcal{E}_k + R\, dH^k.$$

As a result, we have

$$L = -\frac{\Gamma}{T}\left(p + \frac{B^2}{8\pi}\right), \qquad R = -\frac{(\mathbf{u}, \mathbf{H})}{4\pi T}, \qquad M^k = L v^k,$$

and, on smooth solutions of system (2.4)–(2.7), these can be rewritten as (see [8])

$$\frac{\partial}{\partial t} L_{q_j} + \operatorname{div}(\mathbf{M}_{q_j} - \mathbf{H} R_{q_j}) + R_{q_j} \operatorname{div} \mathbf{H} = 0, \quad j = 1, 2, \ldots, 8, \tag{3.2}$$

where $\mathbf{M} = (M^1, M^2, M^3)^*$. Finally, system (3.2) is written in the symmetric form (3.1) with

$$A^0 = (L_{q_i q_j}), \qquad A^k = (M^k_{q_i q_j} - H^k R_{q_i q_j}), \quad i, j = 1, 2, \ldots, 8.$$

We can find the explicit form of the symmetric matrices A^α using the following procedure (see [8], [9]). Let

$$\mathbf{L}_q = (L_{q_1}, \ldots, L_{q_8})^*, \qquad \mathbf{M}^k_q = (M^k_{q_1}, \ldots, M^k_{q_8})^*.$$

Then

$$d\mathbf{Q} = J\, d\mathbf{U}, \qquad d\mathbf{L}_q = I^0\, d\mathbf{U}, \qquad d\mathbf{M}^k_q = I^k\, d\mathbf{U}, \tag{3.3}$$

where J, I^α are quadratic matrices. In view of (3.3),

$$A^0 = I^0 J^{-1}, \qquad A^k = I^k J^{-1} - H^k I_R.$$

Here $I_R = (R_{q_i q_j})$, $i, j = 1, 2, \ldots, 8$.

Note that $\det J \neq 0$ on smooth solutions of system (2.4)–(2.7), and the equations of relativistic MHD can also be rewritten as the symmetric system

$$A_0 \mathbf{U}_t + A_k \mathbf{U}_{x^k} = 0 \tag{3.4}$$

in terms of the initial vector of unknowns $\mathbf{U}$. Here $A_0 = J^* I^0$ and $A_k = J^* I^k - H^k J^* I_R J$ are symmetric matrices.

Let us discuss now the hyperbolicity conditions for system (2.4)–(2.7). As is known, if the condition

$$A^0 > 0 \quad (\text{or } A^0 < 0)$$

holds, then system (3.1) is symmetric t-hyperbolic. It is proved in [27] that the last condition is fulfilled (to be exact, in our case it is the condition $A^0 < 0$) if, as in classical gas dynamics, the state equation $e_0 = e_0(\rho, s)$ satisfies the following *convexity conditions*:

$$(e_0)_V < 0, \quad (e_0)_s > 0, \quad (e_0)_{VV} > 0, \quad (e_0)_{VV}(e_0)_{ss} - (e_0)^2_{Vs} > 0 \tag{3.5}$$

(they are presented in [27] in a different way).

Furthermore, we will also assume *relativistic causality*, which means that the speed of sound is less than that of light (see also, for example, [27], [1]):

$$c_s^2 = \frac{c^2}{h} < 1, \tag{3.6}$$

where $c^2 = (\rho^2 (e_0)_\rho)_\rho$.

4. Shock waves in relativistic MHD: the stability problem. We consider piecewise smooth solutions to system (2.4)–(2.7) with smooth parts separated by the surface of strong discontinuity with the equation

$$\tilde{f}(t,\mathbf{x}) = f(t,\mathbf{x}') - x^1 = 0$$

($\mathbf{x} = (x^1, \mathbf{x}'), \mathbf{x}' = (x^2, x^3)$). From the usual reasons (see, for example, [19], [16], [26]) we conclude the following jump conditions on the surface of strong discontinuity in relativistic MHD:

$$f_t[\rho\Gamma] - [\rho u^1] + f_{x^2}[\rho u^2] + f_{x^3}[\rho u^3] = 0, \tag{4.1}$$

$$f_t[M^j] - [\Pi_{j1}] + f_{x^2}[\Pi_{j2}] + f_{x^3}[\Pi_{j3}] = 0, \tag{4.2}$$

$$[H^1] - f_{x^2}[H^2] - f_{x^3}[H^3] = 0, \tag{4.3}$$

$$f_t[H^2] - [v^1H^2 - H^1v^2] + f_{x^3}[v^3H^2 - H^3v^2] = 0, \tag{4.4}$$

$$f_t[H^3] - [v^1H^3 - H^1v^3] + f_{x^2}[v^2H^3 - H^2v^3] = 0, \tag{4.5}$$

$$f_t[\mathcal{E}_0] - [\mathcal{E}_1] + f_{x^2}[\mathcal{E}_2] + f_{x^3}[\mathcal{E}_3] = 0. \tag{4.6}$$

Here $[F] = F^+ - F^-$ for every regularly discontinuous function F, the subscripts $+, -$ denote the value of the function ahead ($\tilde{f} \to +0$) and behind ($\tilde{f} \to -0$) the discontinuity front. Below we will write F instead of F^+, and F_∞ instead of F^-.

Furthermore, according to [23] we will assume without loss of generality that the additional *compressibility conditions*

$$p > p_\infty, \quad \rho > \rho_\infty, \quad s > s_\infty, \quad h > h_\infty \tag{4.7}$$

hold for the case of shock waves, with $j \neq 0$, $[\rho] \neq 0$. Here

$$j = \rho\Gamma(v_N - D_N), \qquad v_N = (\mathbf{v}, \mathbf{N});$$

$\mathbf{N} = (1/|\nabla\tilde{f}|)(-1, f_{x_2}, f_{x_3})^*$ is the normal to the shock front, $D_N = -f_t/|\nabla\tilde{f}|$ is the speed of the shock front in the normal direction, $|\nabla\tilde{f}| = (1 + f_{x_2}^2 + f_{x_3}^2)^{1/2}$. Conditions (3.5), (3.6) are also supposed to be valid.

Let us consider a plane stepshock with the equation $x^1 = 0$ and the piecewise constant solution to system (2.4)–(2.7)

$$\mathbf{U}(t,\mathbf{x}) = \begin{cases} \widehat{\mathbf{U}}_\infty = (\hat{p}_\infty, \hat{s}_\infty, \hat{u}^1_\infty, \hat{u}^2_\infty, \hat{u}^3_\infty, \widehat{H}^1_\infty, \widehat{H}^2_\infty, \widehat{H}^3_\infty)^*, & x_1 < 0; \\ \widehat{\mathbf{U}} = (\hat{p}, \hat{s}, \hat{u}^1, \hat{u}^2, \hat{u}^3, \widehat{H}^1, \widehat{H}^2, \widehat{H}^3)^*, & x_1 > 0, \end{cases} \tag{4.8}$$

which satisfies the jump conditions (4.1)–(4.6) on the plane $x^1 = 0$ and conditions (3.5), (3.6), (4.7). Here $\hat{\rho}, \hat{\rho}_\infty, \hat{p}, \hat{p}_\infty, \dots, \hat{v}^k, \hat{v}^k_\infty$ are constants; $\widehat{\Gamma} = (1 - |\hat{\mathbf{v}}|^2)^{-1/2}, \widehat{\Gamma}_\infty = (1 - |\hat{\mathbf{v}}_\infty|^2)^{-1/2}$, etc.

We will assume the stepshock to be parallel, i.e.,

$$\widehat{H}^2 = \widehat{H}^2_\infty = \widehat{H}^3 = \widehat{H}^3_\infty = 0. \tag{4.9}$$

Then from system (4.1)–(4.6), the following can be obtained:

$$[\hat{v}^2] = [\hat{u}^2/\widehat{\Gamma}] = 0, \qquad [\hat{v}^3] = [\hat{u}^3/\widehat{\Gamma}] = 0.$$

Hence, as in relativistic gas dynamics [1], [6], we can choose a reference frame in which

$$\hat{u}^2 = \hat{u}^2_\infty = \hat{u}^3 = \hat{u}^3_\infty = 0. \tag{4.10}$$

Finally, from the jump conditions (4.1)–(4.6), we have the following relations on the stationary shock front $x^1 = 0$:

$$\hat{\rho}\hat{u}^1 = \hat{\rho}_\infty \hat{u}^1_\infty \quad (= \hat{j} \neq 0), \tag{4.11}$$

$$\hat{\rho}\hat{h}(\hat{u}^1)^2 + \hat{p} = \hat{\rho}_\infty \hat{h}_\infty (\hat{u}^1_\infty)^2 + \hat{p}_\infty, \tag{4.12}$$

$$\hat{h}\widehat{\Gamma} = \hat{h}_\infty \widehat{\Gamma}_\infty, \tag{4.13}$$

$$\widehat{H}^1 = \widehat{H}^1_\infty. \tag{4.14}$$

Equations (4.11)–(4.13) coincide with the stationary jump conditions in relativistic gas dynamics [6]. Without loss of generality, it is possible to suppose that $\widehat{H}^1 > 0$.

Linearizing system (2.4)–(2.7) with respect to solution (4.8) in the half-space $x^1 > 0$, and accounting for (4.9), (4.10), we obtain the following linear equations of relativistic MHD:

$$\begin{gathered}
\frac{\partial p}{\partial t} + a_0 \frac{\partial p}{\partial x^1} + a_k \frac{\partial u^k}{\partial x^k} = 0, \\
\frac{\partial s}{\partial t} + \hat{v}^1 \frac{\partial s}{\partial x^1} = 0, \\
\frac{\partial u^1}{\partial t} + b_0 \frac{\partial p}{\partial x^1} + b_k \frac{\partial u^k}{\partial x^k} = 0, \\
\frac{\partial u^j}{\partial t} + c_0 \frac{\partial p}{\partial x^j} + c_1 \frac{\partial u^j}{\partial x^1} + c_2 \left(\frac{1}{\widehat{\Gamma}^2} \frac{\partial H^j}{\partial x^1} - \frac{\partial H^1}{\partial x^j} \right) = 0, \quad j = 1, 2, \\
\frac{\partial H^1}{\partial t} + \hat{v}^1 \frac{\partial H^1}{\partial x^1} + \frac{\widehat{H}^1}{\widehat{\Gamma}} \left(\frac{\partial u^2}{\partial x^2} + \frac{\partial u^3}{\partial x^3} \right) = 0, \\
\frac{\partial H^j}{\partial t} + \hat{v}^1 \frac{\partial H^j}{\partial x^1} - \frac{\widehat{H}^1}{\widehat{\Gamma}} \frac{\partial u^j}{\partial x^1} = 0, \quad j = 1, 2,
\end{gathered}$$

which can be written in the form of the linear system

$$\mathbf{U}_t + B_k \mathbf{U}_{x^k} = 0, \quad x^1 > 0. \tag{4.15}$$

Here $\mathbf{U}$ is the vector of small perturbations with the components p, s, u^k, H^k; $B_k = B_k(\widehat{\mathbf{U}})$ constant matrices that can easily be found;

$$a_0 = \frac{\hat{v}^1(1 - \hat{c}_s^2)}{\widehat{\Delta}}, \quad \hat{c}_s^2 = \frac{\hat{c}^2}{\hat{h}}, \quad \widehat{\Delta} = 1 - (\hat{v}^1 \hat{c}_s)^2, \quad a_1 = \frac{\hat{\rho}\hat{h}\hat{c}_s^2}{\widehat{\Gamma}^3 \widehat{\Delta}}, \quad a_2 = a_3 = \frac{\hat{\rho}\hat{c}^2}{\widehat{\Gamma}\widehat{\Delta}},$$

$$b_0 = \frac{1}{\hat{\rho}\hat{h}\widehat{\Gamma}\widehat{\Delta}}, \quad \widehat{\Gamma}^2 = 1 + (\hat{u}^1)^2, \quad \hat{v}^1 = \frac{\hat{u}^1}{\widehat{\Gamma}}, \quad b_1 = a_1, \quad b_2 = b_3 = -\frac{\hat{v}^1 \hat{c}_s^2}{\widehat{\Delta}},$$

$$c_0 = \frac{1}{\hat{\rho}\hat{h}\widehat{\Gamma}(1 + \hat{q}^2)}, \quad \hat{q} = \frac{\widehat{H}^1}{\widehat{\Gamma}\sqrt{4\pi\hat{\rho}\hat{h}}}, \quad c_1 = \frac{\hat{v}^1(1 - \hat{q}^2)}{1 + \hat{q}^2}, \quad c_2 = \frac{\hat{q}^2}{(1 + \hat{q}^2)\sqrt{4\pi\hat{\rho}\hat{h}}};$$

the coefficients a_α, b_α of matrices B_k coincide with corresponding ones in relativistic gas dynamics [6]. Analogously we obtain, ahead of the plane stepshock, the linear system

$$\mathbf{U}_t + B_{k\infty}\mathbf{U}_{x^k} = 0, \quad x^1 < 0, \tag{4.16}$$

where $B_{k\infty} = B_k(\widehat{\mathbf{U}}_\infty)$.

Linearizing Eqs. (4.1)–(4.6) with respect to the piecewise constant solution (4.8), and accounting for (4.9)–(4.14), we obtain the boundary conditions

$$M\begin{pmatrix} \mathbf{U}|_{x^1\to-0} \\ \mathbf{U}|_{x^1\to+0} \\ F_t \\ F_{x^2} \\ F_{x^3} \end{pmatrix} = 0 \tag{4.17}$$

at $x^1 = 0$ $(t > 0, \mathbf{x}' \in R^2)$ for systems (4.15), (4.16). We do not yet give the concrete form of (4.17). Here M is a constant rectangular matrix of order 8×19, and $x^1 = F(t,\mathbf{x}')$ is a small displacement of the shock front. Thus, with the initial data

$$\mathbf{U}|_{t=0} = \mathbf{U}_0(\mathbf{x}), \quad \mathbf{x} \in R^3_\pm, \quad F|_{t=0} = F_0(\mathbf{x}'), \quad \mathbf{x}' \in R^2 \tag{4.18}$$

under $t = 0$ we have the linear stability problem (4.15)–(4.18) for parallel shock waves in relativistic MHD.

Before the investigation of well-posedness of the stability problem (4.15)–(4.18), one should be sure that the geometrical *Lax condition* [16] is fulfilled, i.e., in our case $8 = n^+(B_1) + n^-(B_{1\infty}) + 1$, where n^+ (n^-) is the number of positive (negative) eigenvalues of a matrix. Note that the Lax condition is a necessary condition for the well-posedness of the stability problem, i.e., as well as for the structural stability of strong discontinuities, which are said to be *evolutionary* if the Lax condition is fulfilled.

To find evolutionary conditions for parallel shock waves we seek the eigenvalues of the matrices $B_1, B_{1\infty}$. The matrix B_1 has the eigenvalues $\lambda_i = \lambda_i(\widehat{\mathbf{U}})$, $i = 1, 2, \ldots, 8$, where

$$\lambda_{1,2} = \hat{v}^1, \qquad \lambda_3 = \frac{\hat{v}^1 + \hat{c}_s}{1 + \hat{v}^1\hat{c}_s}, \qquad \lambda_4 = \frac{\hat{v}^1 - \hat{c}_s}{1 - \hat{v}^1\hat{c}_s},$$
$$\lambda_{5,6} = \frac{\hat{v}^1 + \hat{q}\sqrt{1 - (\hat{v}^1)^2 + \hat{q}^2}}{1 + \hat{q}^2}, \qquad \lambda_{7,8} = \frac{\hat{v}^1 - \hat{q}\sqrt{1 - (\hat{v}^1)^2 + \hat{q}^2}}{1 + \hat{q}^2}.$$

The eigenvalues of the matrix $B_{1\infty}$ are $\lambda_{i\infty} = \lambda_i(\widehat{\mathbf{U}}_\infty)$, $i = 1, 2, \ldots, 8$. Moreover, $\lambda_{1,\ldots,6} > 0$ if $\hat{c}_s < \hat{v}^1 < 1$, and $\lambda_{1,2,3,5,6} > 0$, $\lambda_4 < 0$ if $\hat{v}^1 < \hat{c}_s < 1$; $\lambda_{7,8} > 0$ if $\hat{q} < \hat{v}^1 < 1$, and $\lambda_{7,8} < 0$ if $\hat{v}^1 < \hat{q}$.

Therefore, one has evolutionary shocks when

$$\hat{q}_\infty < \hat{c}_{s\infty} < \hat{v}^1_\infty < 1, \qquad \hat{q} < \hat{v}^1 < \hat{c}_s < 1, \tag{4.19}$$

or when

$$\hat{c}_{s\infty} < \hat{v}^1_\infty < \hat{q}_\infty, \qquad \hat{v}^1 < \hat{c}_s < \hat{q}. \tag{4.20}$$

Following [23] we refer to types (4.19), (4.20) of shock waves as *fast* and *slow* ones respectively. It is easy to show that there are no more other types of evolutionary shocks. Note that evolutionary shock waves can also be selected according to the k-shocks criterion (see, for example, [16], [26]), where $k = 1$ for fast shocks, and $k = 3$ for slow ones (here we do not give detailed reasoning).

In the framework of the present paper we are interested in fast shocks. By virtue of the first inequalities from (4.19), all the eigenvalues of the matrix $B_{1\infty}$ are positive in this case. Consequently, system (4.16) does not need boundary conditions at $x^1 = 0$, and without loss of generality, we can presume that $\mathbf{U}(t, \mathbf{x}) \equiv 0$ under $x_1 < 0$. By virtue of the second inequalities from (4.19), which can be rewritten in the form

$$\frac{q^2}{1 + q^2 \hat{c}_s^2} < M^2 < 1,$$

system (4.15) needs seven boundary conditions, and one boundary condition is necessary to determine the function F. Here $M = \hat{v}^1/\hat{c}_s$ is the relativistic Mach number, $q = \widehat{H}^1/(\hat{c}\sqrt{4\pi\hat{\rho}}) = \hat{q}\widehat{\Gamma}\hat{h}^{1/2}/\hat{c}_s$.

Finally, applying the symmetric form (3.4), we obtain from the mixed problem (4.15)–(4.18) the following linear stability problem for fast parallel shock waves in relativistic MHD.

STABILITY PROBLEM. In the domain $\mathbf{x} \in R^3_+$, $t > 0$ we seek a solution to the system

$$A_0 \mathbf{U}_t + A_k \mathbf{U}_{x^k} = 0 \tag{4.21}$$

satisfying the boundary conditions

$$\begin{gathered} u^1 = d_1 p, \quad F_t = d_2 p, \quad u^k = d_3 F_{x^k}, \quad k = 2, 3, \\ s = d_4 p, \quad H^k = \frac{\widehat{H}^1}{\hat{u}^1} u^k, \quad k = 2, 3, \quad H_1 = 0 \end{gathered} \tag{4.22}$$

at $x^1 = 0$ $(t > 0, \mathbf{x}' \in R^2)$ and the initial data

$$\mathbf{U}(0, \mathbf{x}) = \mathbf{U}_0(\mathbf{x}), \quad \mathbf{x} \in R^3_+, \quad F(0, \mathbf{x}') = F_0(\mathbf{x}'), \quad \mathbf{x}' \in R^2 \tag{4.23}$$

for $t = 0$.

Here A_α symmetric matrices,

$$A_0 = \begin{pmatrix} \frac{\widehat{\Gamma}}{\hat{\rho}\hat{c}^2} & 0 & \hat{v}^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hat{v}^1 & 0 & \frac{\hat{h}\hat{\rho}}{\widehat{\Gamma}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \hat{\rho}\hat{h}\widehat{\Gamma} + \frac{(\widehat{H}^1)^2}{4\pi\widehat{\Gamma}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \hat{\rho}\hat{h}\widehat{\Gamma} + \frac{(\widehat{H}^1)^2}{4\pi\widehat{\Gamma}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\widehat{\Gamma}}{4\pi} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4\pi\widehat{\Gamma}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4\pi\widehat{\Gamma}} \end{pmatrix},$$

$$A_1 = \begin{pmatrix} \frac{\hat{u}^1}{\hat{\rho}\hat{c}^2} & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \hat{v}^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & \frac{\hat{h}\hat{\rho}\hat{v}^1}{\widehat{\Gamma}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \left(\hat{\rho}\hat{h}\widehat{\Gamma} - \frac{(\widehat{H}^1)^2}{4\pi\widehat{\Gamma}}\right)\hat{v}^1 & 0 & 0 & -\frac{\widehat{H}^1}{4\pi\widehat{\Gamma}^2} & 0 \\ 0 & 0 & 0 & 0 & \left(\hat{\rho}\hat{h}\widehat{\Gamma} - \frac{(\widehat{H}^1)^2}{4\pi\widehat{\Gamma}}\right)\hat{v}^1 & 0 & 0 & -\frac{\widehat{H}^1}{4\pi\widehat{\Gamma}^2} \\ 0 & 0 & 0 & 0 & 0 & \frac{\hat{u}^1}{4\pi} & 0 & 0 \\ 0 & 0 & 0 & -\frac{\widehat{H}^1}{4\pi\widehat{\Gamma}^2} & 0 & 0 & \frac{\hat{v}^1}{4\pi\widehat{\Gamma}} & 0 \\ 0 & 0 & 0 & 0 & -\frac{\widehat{H}^1}{4\pi\widehat{\Gamma}^2} & 0 & 0 & \frac{\hat{v}^1}{4\pi\widehat{\Gamma}} \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & \frac{\widehat{H}^1}{4\pi} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\widehat{H}^1}{4\pi} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$A_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\widehat{H}^1}{4\pi} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\widehat{H}^1}{4\pi} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix};$$

$$d_1 = -\frac{\widehat{\Gamma}(a - \hat{\beta}^2)}{\hat{\rho}\hat{h}\hat{v}^1 a}, \quad d_2 = \frac{\hat{v}^1_\infty \hat{\beta}^2}{\hat{\rho}\hat{h}\widehat{\Gamma}^2 a c_1 \hat{v}^1[\hat{v}^1]}, \quad d_3 = -\frac{\widehat{\Gamma}[\hat{v}^1](\hat{v}^1)^2}{(\hat{v}^1)^2 - \hat{q}^2},$$

$$d_4 = \frac{\hat{\beta}^2[\hat{v}^1]}{\hat{\rho}\widehat{T}\hat{v}^1 a c_1}, \quad a = 2\hat{c}_s^2 - \frac{\hat{v}^1[\hat{v}^1]}{c_1}\left(\hat{c}_s^2 + \frac{(e_0)_{VS}(\hat{\rho},\hat{s})}{\widehat{T}\hat{\rho}}\right), \quad \hat{\beta}^2 = \hat{c}_s^2 - (\hat{v}^1)^2,$$

$$c_1 = 1 - \hat{v}^1\hat{v}^1_\infty, \quad \widehat{T} = (e_0)_s(\hat{\rho},\hat{s}), \quad [\hat{v}^1] = \hat{v}^1 - \hat{v}^1_\infty < 0.$$

By virtue of (4.19), the matrix A_0 is positive definite.

While solving the stability problem (4.21)–(4.23), we also determine the function $F = F(t, \mathbf{x}')$. For this purpose, one of the boundary conditions (4.22) must be the equation for determination of the function F. On the other hand, excluding the function F from (4.22) we obtain the following boundary conditions at $x^1 = 0$ for system (4.22):

$$u^1 = d_1 p, \quad \frac{\partial u^k}{\partial t} = d_5 \frac{\partial p}{\partial x^k}, \quad H^k = \frac{\widehat{H}^1}{\hat{u}^1} u^k, \quad k = 2, 3, \quad s = d_4 p, \quad H_1 = 0, \qquad (4.24)$$

where $d_5 = d_2 d_3$.

5. Uniform Lopatinski condition. Applying technical ideas from [13], we give in this section an equivalent definition of the *uniform Lopatinski condition* for linear mixed problems in the form of the stability problem (4.21)–(4.23) when only one eigenvalue of the matrix $A_0^{-1}A_1$ (or B_1) is negative, and the others are positive.

Applying the Fourier-Laplace transform to system (4.21) and the boundary conditions (4.24), we obtain the following boundary-value problem for the system of ordinary differential equations:

$$-\frac{d\widehat{\mathbf{U}}}{dx^1} = \mathcal{M}(s,\boldsymbol{\omega})\widehat{\mathbf{U}}, \quad x^1 > 0, \tag{5.1}$$

$$\mathcal{M}_0(s,\boldsymbol{\omega})\widehat{\mathbf{U}} = 0, \quad x^1 = 0. \tag{5.2}$$

Here

$$\widehat{\mathbf{U}} = \widehat{\mathbf{U}}(x^1) = (2\pi)^{-3/2} \iiint\limits_{R^3} \exp(-st - i(\boldsymbol{\omega}, \mathbf{x}'))\mathbf{U}(t, x^1, \mathbf{x}')\, dt\, d\mathbf{x}'$$

is the Fourier-Laplace transform of the vector function $\mathbf{U}(t,\mathbf{x})$;

$$s = \eta + i\xi, \quad \eta > 0, \quad (\xi, \boldsymbol{\omega}) \in R^3, \quad \boldsymbol{\omega} = (\omega_2, \omega_3),$$

$$|\boldsymbol{\omega}|^2 = \omega_2^2 + \omega_3^2, \quad \mathcal{M} = \mathcal{M}(s,\boldsymbol{\omega}) = -A_1^{-1}(sA_0 + i\omega_2 A_2 + i\omega_3 A_3);$$

the matrix $\mathcal{M}_0$ has in our case the following form:

$$\mathcal{M}_0 = \mathcal{M}_0(s,\boldsymbol{\omega}) = \begin{pmatrix} d_1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ d_4 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \omega_2 d_5 & 0 & 0 & -s & 0 & 0 & 0 & 0 \\ \omega_3 d_5 & 0 & 0 & 0 & -s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{\widehat{H}^1}{\hat{u}^1} & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & \frac{\widehat{H}^1}{\hat{u}^1} & 0 & 0 & -1 \end{pmatrix}.$$

While applying the Fourier-Laplace transform, we assume as usual that

$$\mathbf{U}(t,\mathbf{x}) = 0 \quad \text{for } t \leq 0.$$

In view of (4.19), seven eigenvalues of the matrix B_1, as well as of the matrix $A_0^{-1}A_1$, are positive, and one eigenvalue is negative. From this fact it follows that for all $\boldsymbol{\omega} \in R^2$ and $\eta > 0$, seven eigenvalues λ of the matrix $\mathcal{M}$ lie in the left half-plane ($\operatorname{Re}\lambda < 0$), and one eigenvalue lies in the right half-plane ($\operatorname{Re}\lambda > 0$). Indeed, this property of the eigenvalues λ is valid for $\omega_{2,3} = 0$ and $\eta > 0$. On the other hand, since system (4.21) is symmetric t-hyperbolic and $\det A_1 \neq 0$, then the assumption $\operatorname{Re}\lambda = 0$ follows from $\eta = 0$. Hence, the location of the eigenvalues λ relative to the imaginary axis of the complex λ-plane is independent of $\boldsymbol{\omega}$ and, so, coincides with that for $\omega_{2,3} = 0$. Note also that the proved property of eigenvalues of the matrix $\mathcal{M}$ is a general fact for matrices in the form of $\mathcal{M}$ which is in particular proved (in other terms) in [13].

Thus, we reduce the matrix $\mathcal{M}$ to the form

$$\mathcal{M} = \Lambda \begin{pmatrix} b & 0 \\ 0 & Q \end{pmatrix} \Lambda^{-1},$$

where Λ is a nonsingular matrix; b is a number with $\operatorname{Re} b > 0$; all the eigenvalues λ of the matrix Q lie in the left half-plane ($\operatorname{Re}\lambda < 0$). We seek the *bounded* solution of problem (5.1), (5.2) in the form:

$$\widehat{\mathbf{U}} = \Lambda \begin{pmatrix} 0 \\ \exp(Qx^1)\mathbf{C} \end{pmatrix},$$

where $\mathbf{C}$ is a constant vector that is found from the system

$$\mathcal{M}_0 \Lambda \begin{pmatrix} 0 \\ \mathbf{C} \end{pmatrix} = \mathcal{L}(\eta, \xi, \boldsymbol{\omega})\mathbf{C} = 0.$$

If $\det \mathcal{L}(\eta, \xi, \boldsymbol{\omega}) = 0$ for some $\eta > 0$, $(\xi, \boldsymbol{\omega}) \in R^3$, then the sequence of vector functions

$$\mathbf{U}_k(t, \mathbf{x}) = \exp\{-\sqrt{k} + k(\eta t + i\xi t + i(\boldsymbol{\omega}, \mathbf{x}'))\}\Lambda(\eta, \xi, \boldsymbol{\omega}) \begin{pmatrix} 0 \\ \exp\{kQ(\eta, \xi, \boldsymbol{\omega})x^1\}\mathbf{C} \end{pmatrix}$$

($k = 1, 2, 3, \dots$), which are all the solutions of the mixed problem (4.21), (4.42) with special initial data, is the ill-posedness example of Hadamard type.

Thus, following [18] we say that the boundary conditions (4.24) satisfy the *Lopatinski condition* if $\det \mathcal{L}(\eta, \xi, \boldsymbol{\omega}) \neq 0$ for all $\eta > 0$, $(\xi, \boldsymbol{\omega}) \in R^3$. Moreover, the boundary conditions (4.24) satisfy the *uniform Lopatinski condition* if $\det \mathcal{L}(\eta, \xi, \boldsymbol{\omega}) \neq 0$ for all $\eta \geq 0$, $(\xi, \boldsymbol{\omega}) \in R^3$ $(\eta^2 + \xi^2 + |\boldsymbol{\omega}|^2 \neq 0)$.

On the other hand, according to [13] we can also present the solution of problem (5.1), (5.2) in the form

$$\widehat{\mathbf{U}}(x^1) = \frac{1}{2\pi i} \oint_C (sA_0 + \lambda A_1 + i\omega_2 A_2 + i\omega_3 A_3)^{-1} A_1 \widehat{\mathbf{U}}_0 \exp(\lambda x^1)\, d\lambda, \tag{5.3}$$

where C is a contour large enough to enclose all the singularities of the integrand; $\widehat{\mathbf{U}}_0$ is a constant vector satisfying the boundary conditions (5.2), i.e., $\mathcal{M}_0 \widehat{\mathbf{U}}_0 = \mathcal{M}_0 \widehat{\mathbf{U}}(0) = 0$. The singularities of the integrand are the eigenvalues of the matrix $\mathcal{M}$ and satisfy the equation

$$\det(sA_0 + \lambda A_1 + i\omega_2 A_2 + i\omega_3 A_3) = 0. \tag{5.4}$$

It follows from (5.3) that $\widehat{\mathbf{U}}(x^1)$ is a sum of residues at the poles of the integrand. As is noted above, there is one eigenvalue λ with $\operatorname{Re}\lambda > 0$, i.e., for this λ we have: $\exp(\lambda x^1) \to +\infty$ as $x^1 \to +\infty$. Hence the residue at this value of λ must be zero. As is shown in [13], this is the same as the statement that for given $\boldsymbol{\omega} \in R^2$ there exist complex numbers s and λ with

$$\operatorname{Re} s = \eta > 0, \qquad \operatorname{Re}\lambda > 0,$$

such that the homogeneous system

$$(sA_0 + \lambda A_1 + i\omega_2 A_2 + i\omega_3 A_3)\mathbf{X} = 0, \tag{5.5}$$

$$\mathbf{X}^* A_1 \widehat{\mathbf{U}}_0 = 0 \tag{5.6}$$

has a nonzero solution $\mathbf{X}$. We recall that these values of s, λ, and $\omega_{2,3}$ must satisfy (5.4).

Since λ with $\operatorname{Re}\lambda > 0$ is a simple eigenvalue, then we can choose seven linearly independent equations from system (5.5). Adding (5.6) to these equations we rewrite system (5.5), (5.6) in the form

$$G(\eta, \xi, \boldsymbol{\omega}, \lambda)\mathbf{X} = 0,$$

where $G = G(\eta, \xi, \boldsymbol{\omega}, \lambda)$ is a quadratic matrix. If $\det G = 0$, then the sequence of vector functions

$$\mathbf{U}_k(t, \mathbf{x}) = \exp\{-\sqrt{k} + k(\eta t + i\xi t + i(\boldsymbol{\omega}, \mathbf{x}'))\}\widehat{\mathbf{U}}(x^1)$$

($k = 1, 2, 3, \dots$) is the ill-posedness example of Hadamard type for the mixed problem (4.21), (4.24) with special initial data.

So, we can now formulate an equivalent definition of the uniform Lopatinski condition. The boundary conditions (4.24) satisfy the *Lopatinski condition* if $\det G(\eta, \xi, \boldsymbol{\omega}, \lambda) \neq 0$ for all $\eta > 0$, $(\xi, \boldsymbol{\omega}) \in R^3$ and under λ with $\operatorname{Re}\lambda > 0$, where λ is the solution of (5.4).

Note that the requirement $\eta = 0$ implies $\operatorname{Re}\lambda = 0$ (system (4.21) is symmetric). Let $\lambda = \lambda(\eta, \xi, \boldsymbol{\omega})$ with $\operatorname{Re}\lambda > 0$ for $\eta > 0$ be a solution of (5.4), and $\lambda_0 = \lambda(0, \xi, \boldsymbol{\omega})$. Thus, the boundary conditions (4.24) satisfy the *uniform Lopatinski condition* if $\det G(\eta, \xi, \boldsymbol{\omega}, \lambda) \neq 0$ for all $\eta \geq 0$, $(\xi, \boldsymbol{\omega}) \in R^3$ $(\eta^2 + \xi^2 + |\boldsymbol{\omega}|^2 \neq 0)$ and under λ with $\operatorname{Re}\lambda \geq 0$, where λ is the solution of (5.4), and $\lambda = \lambda_0$ for $\eta = 0$.

6. Instability domain for fast parallel relativistic MHD shock waves. In this section we find the domain where the Lopatinski condition for the stability problem (4.21)–(4.23) is violated. It is the domain of instability of fast parallel relativistic MHD shock waves.

For system (4.21), Eq. (5.4) can be rewritten explicitly as follows:

$$(\hat{q}^2(s^2 - \lambda^2 + |\boldsymbol{\omega}|^2) + \Omega^2)\left(\frac{\widehat{\Gamma}^2(1 - \hat{c}_s^2)}{\hat{c}_s^2}\Omega^2 + s^2 - \lambda^2\right) + |\boldsymbol{\omega}|^2\Omega^2 = 0, \tag{6.1}$$

where $\Omega = s + \hat{v}^1\lambda$. Since fast parallel relativistic MHD shock waves are evolutionary, then an ill-posedness example of Hadamard type in 1-D form $(\omega_{2,3} = 0)$ cannot be constructed for the stability problem. Thus, we can assume that $|\boldsymbol{\omega}| \neq 0$. Moreover, without loss of generality, we will presume that $|\boldsymbol{\omega}| = 1$.

Introducing the new values ν and μ by the formulae

$$\nu = \lambda/\Omega, \qquad \mu = -i/\Omega,$$

we present Eq. (6.1) in the form

$$\left(\hat{q}^2\left(1 - \hat{v}^1\nu - \frac{1}{\widehat{\Gamma}^2}\nu^2 - \mu^2\right) + 1\right)\left(\frac{\widehat{\Delta}\widehat{\Gamma}^2}{\hat{c}_s^2} - 2\hat{v}^1\nu - \frac{1}{\widehat{\Gamma}^2}\nu^2\right) - \mu^2 = 0. \tag{6.2}$$

Equation (6.2) has two roots $\mu = \mu_{1,2}$ $(\mu_2 = -\mu_1)$. We choose the root μ of (6.2) in the following form:

$$\mu = \mu_1 = -\frac{1}{z}\left\{\frac{(z - z_2)(z - z_3)(z + z_4)(z + z_5)}{(z - z_1)(z + z_6)}\right\}^{1/2}\left(\frac{\widehat{\Delta}(1 + \hat{q}^2)}{\widehat{\Delta}\hat{q}^2 + \hat{c}_s^2/\widehat{\Gamma}^2}\right)^{1/2}, \tag{6.3}$$

where

$$z = 1/\nu, \tag{6.4}$$

and

$$z_{1,6} = \frac{\hat{q}}{1+\hat{q}^2\widehat{\Delta}^2\widehat{\Gamma}^2/\hat{c}_s^2}\left(\sqrt{1-(\hat{v}^1)^2+\hat{q}^2/\hat{c}_2^2} \pm \hat{v}^1\hat{q}\right),$$

$$z_{2,4} = \frac{\hat{q}}{1+\hat{q}^2}\left(\sqrt{1-(\hat{v}^1)^2+\hat{q}^2} \pm \hat{v}^1\hat{q}\right), \qquad z_{3,5} = \frac{\hat{c}_s}{\widehat{\Gamma}^2(1\mp\hat{v}^1\hat{c}_s)}.$$

In view of (4.19), we have

$$z_1 < z_2 < \hat{q} < \hat{v}^1 < \hat{c}_s < z_3, \qquad z_{4,5,6} > 0. \tag{6.5}$$

Since

$$\eta > 0, \qquad \operatorname{Re}\lambda > 0, \tag{6.6}$$

then

$$\operatorname{Im}(1/\mu) > 0, \qquad \operatorname{Im}(\nu/\mu) > 0. \tag{6.7}$$

By virtue of (6.3)–(6.5), conditions (6.7) imply that the domain of z is the right half ($\operatorname{Re} z > 0$) of the z-plane with two segments of the real axis removed: the segment from z_1 to z_2 and the segment from z_3 to $+\infty$. Note that conditions (6.6) are not fulfilled for $\mu = \mu_2$.

From system (5.5) we obtain the relations

$$\begin{gathered} s_{(1)} = 0, \qquad u^1_{(1)} = \frac{1+\hat{v}^1\widehat{\Gamma}^2 z}{\hat{\rho}\hat{h}\widehat{\Gamma}z}p_{(1)}, \qquad H^1_{(1)} = \frac{\widehat{H}^1\widehat{\Delta}(z-z_3)(z-z_5)}{\hat{\rho}\hat{c}^2 z}p_{(1)}, \\ u^k_{(1)} = \frac{\widehat{\Delta}\widehat{\Gamma}(z-z_3)(z-z_5)\omega_k}{\hat{\rho}\hat{c}^2 z\mu}p_{(1)}, \qquad H^k_{(1)} = \frac{\widehat{H}^1\widehat{\Delta}(z-z_3)(z-z_5)\omega_k}{\hat{\rho}\hat{c}^2 z^2\mu}p_{(1)} \end{gathered} \tag{6.8}$$

($k = 2, 3$) for components of the vector $\mathbf{X} = (p_{(1)}, s_{(1)}, u^1_{(1)}, u^2_{(1)}, u^3_{(1)}, H^1_{(1)}, H^2_{(1)}, H^3_{(1)})^*$.

In view of (4.24), the components of the vector

$$\widehat{\mathbf{U}}_0 = (p_{(0)}, s_{(0)}, u^1_{(0)}, u^2_{(0)}, u^3_{(0)}, H^1_{(0)}, H^2_{(0)}, H^3_{(0)})^*$$

are connected by the following relations:

$$\begin{gathered} s_{(0)} = d_4 p_{(0)}, \qquad u^1_{(0)} = d_1 p_{(0)}, \qquad u^k_{(0)} = -d_5\frac{\mu\omega_k}{z-\hat{v}^1}p_{(0)}, \\ H^k_{(0)} = -\frac{\widehat{H}^1}{\hat{u}^1}d_5\frac{\mu\omega_k}{z-\hat{v}^1}p_{(0)}, \qquad k=2,3, \quad H^1_{(0)} = 0. \end{gathered}$$

Substituting these relations into (5.6) and applying (6.8), we obtain the following condition for existence of a nonzero solution $\mathbf{X}$ to system (5.5), (5.6):

$$h(z)p_{(0)} = 0, \tag{6.9}$$

where

$$h(z) = \frac{\hat{\beta}^2}{\hat{\rho}\hat{h}\widehat{\Gamma}az(z-\hat{v}^1)}g(z), \tag{6.10}$$

$$g(z) = \left(\frac{\hat{c}_s^2 - a\widehat{\Gamma}^2}{\hat{v}^1\hat{c}_s^2}z - 1\right)(z-\hat{v}^1) + \frac{\hat{v}_\infty^1\widehat{\Delta}\widehat{\Gamma}^2}{\hat{c}_s^2 c_1}(z-z_3)(z+z_5).$$

Since $p_{(0)}$ must not be equal to zero, then (6.9), (6.10) become the equality $g(z) = 0$, which can be rewritten as follows:

$$-\left(1 + \frac{\hat{v}^1[\hat{v}^1]}{\hat{\rho}\hat{e}_p\hat{c}_s^2} - \frac{\hat{v}^1\hat{v}_\infty^1}{\hat{c}_s^2}\right)z^2 + (\hat{v}^1)^2[\hat{v}^1]\left(1 + \frac{1}{\hat{\rho}\hat{e}_p\hat{c}_s^2}\right)z + \frac{\hat{v}^1[\hat{v}^1]}{\widehat{\Gamma}^2} = 0, \tag{6.11}$$

where

$$\hat{e}_p = \frac{\partial e}{\partial p}(\hat{p},\hat{p}) = -\frac{\widehat{T}}{(e_0)_{VS}(\hat{\rho},\hat{s})}, \qquad e = e(\rho,p), \quad e = e_0.$$

We recall that, in view of (4.7), (4.11), $[\hat{v}^1] = \hat{v}^1 - \hat{v}_\infty^1 < 0$.

Equation (6.11) does not depend on the magnetic field and coincides totally with a corresponding one in relativistic gas dynamics [1]. Thus, the domains of instability (domain I) and linear stability (domains II and III) coincide with those of shock waves in relativistic gas dynamics [1], [6].

Indeed, since $\hat{v}^1 < z_3$ (see (6.5)), then $g(\hat{v}^1) < 0$. If $g(z_3) > 0$, i.e.,

$$\mathcal{F} = 1 - (\hat{v}^1)^2 + \frac{\hat{v}^1}{\hat{c}_s} - \frac{(\hat{v}^1)^2\hat{v}_\infty^1}{\hat{c}_s} + \frac{\hat{v}^1[\hat{v}^1]}{\hat{\rho}\hat{e}_p\hat{c}_s^2} < 0, \tag{6.12}$$

then (6.11) admits a real solution $\tilde{z}$ such that $\hat{v}^1 < \tilde{z} < z_3$. So, in domain (6.12), the Lopatinski condition is violated, and fast parallel relativistic MHD shock waves are *unstable*. At the same time, it is easy to show (see [1]) that in the domain $\mathcal{F} \geq 0$, roots of Eq. (6.11) either are with negative real parts or lie to the right of z_3. In addition, the assumption of positive thermal coefficient ($\hat{e}_p > 0$) is made here as in [13], [1]. Note that the domains of linear stability and, as well, structural stability (domain II) are found in [6] without this assumption. But here we will keep it for simplicity of reasoning.

Thus, the Lopatinski condition is fulfilled in the domain $\mathcal{F} \geq 0$ (*linear stability*) and violated in the domain $\mathcal{F} < 0$ (*instability*). To separate the domain of fulfillment of the uniform Lopatinski condition (*structural stability*) it is necessary to find a subdomain of the domain $\mathcal{F} \geq 0$ where $g(z) \neq 0$ as well as for such z that correspond to the case $\eta = 0$, $\lambda = \lambda_0$ ($\operatorname{Re}\lambda_0 = 0$).

7. Structural stability of fast parallel relativistic MHD shock waves. At the beginning we consider for simplicity the case corresponding to relativistic gas dynamic shocks (the structural stability domain for this case is found in [6] in a different way). So, we set formally $\hat{q} = 0$. Then

$$\mu = -\frac{\sqrt{(z-z_3)(z+z_5)}}{z}\left(\frac{\widehat{\Gamma}\widehat{\Delta}^{1/2}}{\hat{c}_s}\right).$$

It is clear that only roots $z > z_3$ of Eq. (6.11) can correspond to the case $\eta = 0, \lambda = \lambda_0 = i\delta$, $\delta \in R$ (see Sec. 6). In the domain $z > z_3$ the function

$$\delta = \delta(z) = \left(\frac{\hat{c}_s}{\widehat{\Gamma}\widehat{\Delta}^{1/2}}\right) \frac{1}{\sqrt{(z - z_3)(z + z_5)}}$$

decreases, and the function

$$\xi = \xi(z) = \left(\frac{\hat{c}_s}{\widehat{\Gamma}\widehat{\Delta}^{1/2}}\right) \frac{z - \hat{v}^1}{\sqrt{(z - z_3)(z + z_5)}}$$

($s = i\xi$) decreases up to its minimum

$$z_* = \frac{\hat{c}_s^2}{\widehat{\Gamma}^2 \hat{v}^1 (1 - \hat{c}_s^2)}$$

($z_* > z_3$) and increases under $z > z_*$.

Solving (6.1) under $\hat{q} = 0$ we find

$$\lambda = \lambda_{1,2} = \frac{\hat{v}^1(1 - \hat{c}_s^2)s \pm \hat{c}_s(1 - (\hat{v}^1)^2)\sqrt{s^2 + \widehat{\Delta}\widehat{\Gamma}^2}}{\hat{\beta}^2}.$$

It is easy to see that $\lambda_0 = \lambda_1|_{\eta=0} = i\delta$. Then we have

$$\delta = \frac{\hat{v}^1(1 - \hat{c}_s^2)\xi + \hat{c}_s(1 - (\hat{v}^1)^2)\sqrt{\xi^2 - \widehat{\Delta}\widehat{\Gamma}^2}}{\hat{\beta}^2}. \tag{7.1}$$

The graph of the function $\xi = \xi(z)$ has two points of intersection $z = r_{1,2}$ ($z_3 < r_1 < z_* < r_2$) with the line $\xi = \hat{\xi} = \text{const}$ under $z > z_3, z \neq z_*$ (the constant $\hat{\xi}$ is supposed to lie in the range of values of the function $\xi = \xi(z)$ under $z > z_3$). One of these points of intersection corresponds to the case $\eta = 0, \lambda = \lambda_0 = i\delta$, where δ is determined by formula (7.1). In view of (7.1), $\delta'(\xi) > 0$. On the other hand, $\delta'(\xi) = \delta'(z)/\xi'(z)$. Since $\delta'(z) < 0$ under $z > z_3$, then the interval $z > z_*$, which contains the point of intersection $z = r_2$ ($\xi'(r_2) > 0$), determines a part of the domain of fulfillment of the uniform Lopatinski condition. The other part is determined by roots z of Eq. (6.11) with negative real parts. Both roots have such a property if the coefficient of z^2 in (6.11) is nonpositive, i.e.,

$$\mathcal{F} \geq \frac{\hat{v}^1}{\hat{c}_s^2}(1 - \hat{v}^1\hat{c}_s)(1 + \hat{v}^1_\infty \hat{c}_s). \tag{7.2}$$

At the same time, omitting detailed calculations, we find that Eq. (6.11) in the domain $\mathcal{F} \geq 0$ has a real root z lying to the right of z_* (the other root is negative) if

$$\frac{\hat{v}^1}{\hat{c}_s}\left(1 - \hat{v}^1\hat{v}^1_\infty - \frac{[\hat{v}^1]}{\hat{c}_s}\right) < \mathcal{F} < \frac{\hat{v}^1}{\hat{c}_s^2}(1 - \hat{v}^1\hat{c}_s)(1 + \hat{v}^1_\infty\hat{c}_s). \tag{7.3}$$

Thus, combining domains (7.2), (7.3) we present the domain of fulfillment of the uniform Lopatinski condition:

$$\mathcal{F} > \frac{\hat{v}^1}{\hat{c}_s}\left(1 - \hat{v}^1\hat{v}^1_\infty - \frac{[\hat{v}^1]}{\hat{c}_s}\right). \tag{7.4}$$

Requirement (7.4) on the state equation $e_0 = e_0(\rho, s)$, which is additional to (3.5), (3.6) and determines the domain of structural stability of relativistic gas dynamic shock waves, coincides totally with corresponding conditions in [6]. Finally, we conclude that inequalities (6.12), (7.4), and

$$0 \leq \mathcal{F} \leq \frac{\hat{v}^1}{\hat{c}_s}\left(1 - \hat{v}^1\hat{v}^1_\infty - \frac{[\hat{v}^1]}{\hat{c}_s}\right) \tag{7.5}$$

present domains I, II, III (see Sec. 1) for the relativistic gas dynamic case $\hat{q} = 0$.

Now we proceed to the more complicated case $\hat{q} > 0$. In the first place, it is clear that (7.2) determines a part of the whole domain II as well as for fast parallel relativistic MHD shock waves.

From (6.3) we have

$$\xi = \xi(z, \hat{q}) = \xi(z)\eta(z, \hat{q})$$

where the function $\xi(z)$ is described above, and

$$\eta = \eta(z, \hat{q}) = \left\{\frac{(z - z_1)(z + z_6)}{(z - z_2)(z + z_4)}\right\}^{1/2}.$$

It is not difficult to check that the function η (as a function of z) decreases on the interval $z > z_3$. Hence the function ξ under $z > z_3$ decreases up to its minimum $\tilde{z}_*$ ($\tilde{z}_* > z_*$) and increases under $z > \tilde{z}_*$.

In view of the continuous dependence of η on the parameter $\hat{q}$, the point of intersection of the graph of the function $\eta = \eta(z, \hat{q})$ with the line $\xi = \hat{\xi} =$ const, which corresponds to the case of no roots with $\eta = 0$, $\lambda = \lambda_0 = i\delta$, lies to the right of $\tilde{z}_*$ under sufficiently small $\hat{q}$ ($\tilde{z}_*$ is close to z_*) and cannot jump over the interval $z_3 < z < \tilde{z}_*$ while $\hat{q}$ increases up to $\hat{v}^1$.

Therefore, the domain determined by the condition $z > \tilde{z}_*$, where z is a positive root of (6.11), with domain (7.2) presents domain II of fulfillment of the uniform Lopatinski condition. Moreover, since $\tilde{z}_* > z_*$, the structural stability domain for fast parallel relativistic MHD shock waves is smaller than that of relativistic gas dynamic shocks (see (7.4)).

Now we obtain the exact requirement on the state equation $e_0 = e_0(\rho, s)$ which is determined by the condition $z > \tilde{z}_*$. To find $\tilde{z}_*$ we have to solve the equation $\xi'_z = 0$ ($z > z_3$) that is equivalent to the following, unfortunately, too complicated equation:

$$h(z) = \frac{z^4}{\widehat{\Gamma}^2}\left(\frac{z}{z_*} - 1\right) - \hat{q}^2 h_1(z) = 0, \tag{7.6}$$

where

$$h_1(z) = \frac{z}{\widehat{\Gamma}^2}(z - \hat{v}^1)\left(\hat{v}^1 + \frac{1}{\widehat{\Gamma}^2}\right)((z - \hat{v}^1)^2 - 1) + zh_2(z)(z - \widehat{\Gamma}^2\hat{v}^1 h_2(z)) + \hat{q}^2\widehat{\Gamma}^2 h_2^2(z),$$

$$h_2(z) = \frac{\widehat{\Delta}}{\hat{c}_s^2}(z - z_3)(z + z_5) = \frac{\widehat{\Delta}}{\hat{c}_s^2}z^2 - \frac{2\hat{v}^1}{\widehat{\Gamma}^2}z - \frac{1}{\widehat{\Gamma}^4}.$$

The coefficient of z^5 in (7.6), which has the form

$$\frac{1}{\widehat{\Gamma}^2 z_*} + \frac{\hat{q}^2\widehat{\Gamma}^2(1 - \hat{c}_s^2)}{\hat{c}_s^2}\left(\frac{\widehat{\Delta}}{\hat{c}_s^2} + \frac{1}{\widehat{\Gamma}^2}\right),$$

is positive. Equation (7.6) has one root lying to the right of $\tilde{z}_*$, and other roots are either less than z_3 ($z_3 < \tilde{z}_*$) or complex. Hence the polynomial $h(z)$ of order 5 is positive on the interval $z > \tilde{z}_*$ and negative under $z < \tilde{z}_*$. Consequently, the condition $\hat{z} > \tilde{z}_*$, where $\hat{z}$ is the positive root of (6.11), is equivalent to the inequality

$$h(\hat{z}) > 0. \tag{7.7}$$

Here

$$\hat{z} = \frac{1}{2}\left(-(\hat{v}^1)^2[\hat{v}^1]\left(1 + \frac{1}{\hat{\rho}\hat{e}_p\hat{c}_s^2}\right) + D^{1/2}\right) \Big/ \left(\frac{\hat{v}^1\hat{v}^1_\infty}{\hat{c}_s^2} - \frac{\hat{v}^1[\hat{v}^1]}{\hat{\rho}\hat{e}_p\hat{c}_s^2} - 1\right),$$

$$D = (\hat{v}^1)^4[\hat{v}^1]^2\left(1 + \frac{1}{\hat{\rho}\hat{e}_p\hat{c}_s^2}\right)^2 - \frac{\hat{v}^1[\hat{v}^1]}{\widehat{\Gamma}^2}\left(\frac{\hat{v}^1\hat{v}^1_\infty}{\hat{c}_s^2} - \frac{\hat{v}^1[\hat{v}^1]}{\hat{\rho}\hat{e}_p\hat{c}_s^2} - 1\right);$$

and it is required that the coefficient of z^2 in (6.11) be positive, i.e.,

$$0 \leq \mathcal{F} < \frac{\hat{v}^1}{\hat{c}_s^2}(1 - \hat{v}^1\hat{c}_s)(1 + \hat{v}^1_\infty\hat{c}_s). \tag{7.8}$$

So, the union of domains (7.2) and (7.7), (7.8) presents the domain of structural stability of fast parallel shock waves in relativistic MHD. Unfortunately, condition (7.7) is too complicated to be analyzed. We note only that, for example, for $\hat{q} \ll 1$ (it is the case of a weak magnetic field $q \ll 1$, i.e., $\widehat{H}^1 \ll \hat{c}\sqrt{4\pi\hat{\rho}}$) conditions (7.7), (7.8) are reduced to (7.3) and give with (7.2) inequality (7.4). On the other hand, we can always say that in the domain determined by inequality (7.2), which can be rewritten as

$$1 - \frac{\hat{v}^1\hat{v}^1_\infty}{\hat{c}_s^2} + \frac{\hat{v}^1[\hat{v}^1]}{\hat{\rho}\hat{e}_p\hat{c}_s^2} \geq 0,$$

fast parallel relativistic MHD shock waves are structurally stable.

REMARK 7.1. Using, as in [6], the method of dissipative energy integrals and applying ideas from [7], where the structural stability of fast shocks in the usual MHD in the case of a weak magnetic field is proved, it is possible to construct an expanded system of second order (a system for the vector $\mathbf{U}$ and its derivatives up to the second order, see [6]) with dissipative boundary conditions for system (4.21) in domain (7.4) for the asymptotic case of a weak magnetic field ($q \ll 1$). Writing the energy integral for this expanded system with dissipative boundary conditions we obtain in domain (7.4) and under $q \ll 1$ the following *a priori estimates without loss of smoothness* for the stability problem (4.21)–(4.23):

$$\|\mathbf{U}(t)\|_{W_2^2(R^3_+)} \leq K_1\|\mathbf{U}_0\|_{W_2^2(R^3_+)}, \quad 0 < t \leq \widetilde{T} < \infty,$$
$$\|F\|_{W_2^3((0,\widetilde{T})\times R^2)} \leq K_2,$$

where $K_{1,2} > 0$ are constants depending on $\widetilde{T}$.

REMARK 7.2. In the domain III, where $\mathcal{F} \geq 0$ and $h(\hat{z}) \leq 0$, the question of shock waves stability can be solved only for the quasilinear formulation of the stability problem, i.e., we have to consider the initial quasilinear system (2.4)–(2.7) and relations (4.1)–(4.6).

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