QUARTERLY OF APPLIED MATHEMATICS
VOLUME LXIV, NUMBER 1
MARCH 2006, PAGES 17–28
S 0033-569X(06)01003-8
Article electronically published on January 24, 2006

DIFFUSION OF CHEMICALLY REACTIVE SPECIES IN A POROUS MEDIUM

By

K. VAJRAVELU (*Dept. of Mathematics, University of Central Florida, Orlando, Florida 32816*),

J. R. CANNON (*Dept. of Mathematics, University of Central Florida, Orlando, Florida 32816*),

AND

D. ROLLINS (*Dept. of Mathematics, University of Central Florida, Orlando, Florida 32816*)

Abstract. Solutions for a class of nonlinear second-order differential equations, arising in diffusion of chemically reactive species of a Newtonian fluid immersed in a porous medium over an impervious stretching sheet, are obtained. Using the Schauder theory, existence and uniqueness results are established. Moreover, the exact analytical solutions (for some special cases) are obtained and are used to validate the numerical solutions. The results obtained for the diffusion characteristics reveal many interesting behaviors that warrant further study of the effects of reaction rate on the transfer of chemically reactive species.

1. Introduction. The transport of heat, mass and momentum in laminar boundary layers on moving extensible or stretching surfaces has practical applications in electrochemistry (see Chin [1] and Gorla [2]) and in polymer processing (see Griffith [3] and Erickson et al. [4]). The majority of the studies of these transport processes have so far been devoted to flows induced by surfaces moving with a constant velocity. However, Crane [5] considered the laminar boundary layer flow of a Newtonian fluid caused by a flat elastic sheet whose velocity varies linearly with the distance from a fixed point on the sheet. Recently, his analysis has been extended to fluids obeying non-Newtonian constitutive equations, that is, Rivlin-Ericksen fluids (see Siddappa and Khapate [6]), micropolar fluids (see Chiam [7]), second-order fluids (see Rajagopal et al. [8]), Walters' 'liquid B' (see Siddappa and Abel [9]), and power law fluids (see Andersson and Dandapat [10]).

The heat transfer problem associated with the Newtonian boundary layer flow past a stretching sheet has been studied by several authors (see references [11]–[17]). By taking advantage of the mathematical equivalence of the thermal boundary layer problem with the concentration analogue, results obtained for heat transfer characteristics can be

Received May 16, 2004.
2000 *Mathematics Subject Classification.* Primary 34B15, 76D03, 76S05, 76V05.

carried over directly to the case of mass transfer by replacing the Prandtl number by the Schmidt number. However, the presence of a chemical reaction term in the mass diffusion equation generally destroys the formal equivalence with the thermal energy problem and, moreover, prohibits the construction of the similarity solutions. Chambre and Young [18] considered diffusion of a reactive species into the fluid flow past a wedge-shaped body and concluded that a similarity solution exists only in the case of stagnation point flow studied earlier by Chambre [19]. Since numerical techniques for nonsimilar problems are significantly more time-consuming than solution schemes devised for sets or ordinary differential equations, Dural and Hines [20] recently proposed approximate methods like the Method of Weighted Residuals for many practical purposes.

Very recently, Anderson et al. [21] studied the transfer of a chemically reactive species in the laminar flow over a linearly stretching surface. The reactive component given off by the surface undergoes an isothermal and homogeneous one-stage reaction as it diffuses into the surrounding fluid. By taking advantage of an explicit analytical solution of the momentum boundary layer problem, they demonstrated that similarity can also be achieved for the concentration field.

The above investigators restrict their analyses to flow behavior in non-porous media. However, some metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid. The rate of cooling can be controlled and a final product of desired characteristics can be achieved if strips are drawn through porous media. In view of this, the study of visco-elastic fluid flow through porous media has gained importance in recent years (see Refs. [22]–[25]). It is well known that a variety of interaction forces (such as drag, virtual mass effect, Magnus effect, Basset effect, spin lift, shear lift, etc.) come into play due to the flow in a porous media (for details see Johnson et al. [26, 27]). Very recently, using the effect due to drag, Abel et al. [28] investigated the effect of variable viscosity on visco-elastic fluid flow and heat transfer in a porous medium over a non-isothermal impervious stretching sheet.

Motivated by these analyses, the present authors study the flow and mass transfer of a chemically reactive species of a Newtonian fluid in a porous medium over an impervious stretching sheet. Furthermore, they analyze the salient features of the flow and mass transfer characteristics by obtaining exact, analytical and numerical solutions with existence and uniqueness results for the resulting coupled nonlinear differential equations.

2. Flow analysis. Consider the flow of a Newtonian fluid past a flat sheet coinciding with the plane $y = 0$, the flow being confined to $y > 0$. Two equal and opposite forces are applied along the x-axis so that the wall is stretched, keeping the origin fixed. The steady two-dimensional boundary layer equations for this fluid in the usual notation are (see Andersson et al. [21] and Abel et al. [28])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k_0}u, \tag{2.2}$$

where $\nu = \mu/\rho$, and k_0 is the permeability of the porous medium.

The appropriate boundary conditions for the problem are

$$u = Bx, \quad v = 0 \quad \text{at} \quad y = 0, \quad B > 0,$$
$$u \to 0 \quad \text{as} \quad y \to \infty. \tag{2.3}$$

Eqs. (2.1) and (2.2) admit a self-similar solution of the form

$$u = Bxf'(\eta), \quad v = -(B\nu)^{1/2} f(\eta), \tag{2.4}$$
$$\eta = (B/\nu)^{1/2} y, \tag{2.5}$$

where the prime denotes differentiation with respect to η. Clearly, u and v defined above satisfy the continuity equation (2.1). Substituting Eqs. (2.4) and (2.5) in Eq. (2.2) gives

$$(f')^2 - ff'' = f''' - k_1 f', \tag{2.6}$$

where $k_1 = \nu/(k_0 B)$ is the porosity parameter. The boundary conditions (2.3) become

$$f' = 1, \quad f = 0 \quad \text{at} \quad \eta = 0,$$
$$f' \to 0 \quad \text{as} \quad \eta \to \infty. \tag{2.7}$$

The exact solution for the differential equation (2.6) satisfying conditions (2.7) is

$$f(\eta) = (1 - e^{-m\eta)}/m, \quad m = \sqrt{(1 + k_1)}\,. \tag{2.8}$$

This gives the velocity components

$$u = Bxe^{-m\eta},$$
$$v = -(B/v)^{1/2}(1 - e^{-m\eta})/m. \tag{2.9}$$

For $k_1 = 0$, $f(\eta) = 1 - e^{-\eta}$ is the unique solution of the problem (for details see Troy et al. [29] and McLeod and Rajagopal [30]). For $k_1 > -1$,

$$f(\eta) = (1 - e^{-m\eta})/m, \quad m = \sqrt{(1 + k_1)}. \tag{2.10}$$

Hence, in heat and mass transfer analyses we use this solution for the function f. From Eq. (2.10), we can say that f' is a decreasing function of the porosity parameter k_1.

3. Heat and mass transfer analyses. By knowing the mathematical equivalence of the concentration boundary layer problem with the thermal boundary layer analogue, results obtained for mass transfer characteristics can be carried directly to the heat transfer characteristics by replacing the Schmidt number with the Prandtl number. Hence, for brevity we do not present the heat transfer results here. For the flow problem discussed in section 2, the concentration field $c(x, y)$ is governed by the boundary layer diffusion equation (see Refs. [21, 27]), which reduces to

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2} - k_n c^n, \tag{3.1}$$

where c is the concentration of the species of the fluid, D is the diffusion coefficient of the diffusing species in the fluid and k_n denotes the reaction rate constant of the nth-order homogeneous and irreversible reaction. Since the concentration of the reactant is

maintained at a prescribed value c_w at the sheet and is assumed to vanish far away from the sheet, the relevant boundary conditions for the concentration Eq. (3.1) become

$$\begin{aligned} c(x,y) &= c_w \quad \text{at} \quad y = 0, \\ c(x,y) &\to 0 \quad \text{as} \quad y \to \infty. \end{aligned} \tag{3.2}$$

Knowing the mathematical nature of the physical problem the similarity solution can be obtained for the mass concentration. To that end, we introduce the transformation

$$c = c_w \phi(\eta), \tag{3.3}$$

where $\phi(\eta)$ is the dimensionless concentration field. The nonlinear partial differential equation (3.1) can now be transformed to

$$\phi'' + \alpha f \phi' = \alpha \beta \phi^n, \tag{3.4}$$

where a prime denotes differentiation with respect to η and $\alpha = S_c = \nu/D$ is the Schmidt number and $\beta = k_n c_w^{n-1}/B$ is the reaction rate parameter. The boundary conditions (3.2) become

$$\begin{aligned} \phi &= 1 \quad \text{at} \quad \eta = 0, \\ \phi &\to 0 \quad \text{as} \quad \eta \to \infty. \end{aligned} \tag{3.5}$$

Evidently, the concentration field ϕ is coupled to the velocity field through the dimensionless stream function f in the nonlinear mass transfer equation (3.4). However, in the special case of reactive species of order zero ($n = 0$), the nonlinear term in the right side of Eq. (3.4) becomes $\alpha\beta$, and hence the boundary value problem has no solution (since the condition at infinity can't be satisfied). Furthermore, in the special case of reactive species of first order ($n = 1$), the nonlinear term on the right-hand side of Eq. (3.4) becomes $\alpha\beta\phi(\eta)$ and the present concentration boundary value problem becomes formally equivalent with the analogous thermal boundary layer problem in which the Prandtl number replaces the Schmidt number. The analytical solution of Eq. (3.4) for the first-order reaction ($n = 1$) with respect to the boundary conditions (3.5) can be written in the form of Kummer's function:

$$\phi(\eta) = \frac{e^{-m(A+B)\eta} M(A+B, 1+2B, -(Sc/m^2)e^{-m\eta})}{M(A+B, 1+2B, (-Sc/m^2))},$$

where $A = Sc/(2m^2)$, $B = (4\beta m^2 Sc + Sc^2)^{1/2}/(2m^2)$ and M is the Kummer's function (see Ref. [32]). The exact analytical solution of the complete concentration equation (3.4) for the case when $\beta = 0$ is expressed in terms of the incomplete gamma function as noted in Andersson et al. [21]. In addition, if $Sc = m^2$, the exact solution is given by $\phi(\eta) = e(1 - e^{-e^{-m\eta}})/(e-1)$. It is worth mentioning that the exact analytical solution of the concentration equation for the general case $n > 1$ and $\beta = 1/(n-1)$, $Sc = m^2/(n-1)$, is given by $\phi(\eta) = e^{-m\eta/(n-1)}$. For practical purposes the parameters m, n, α are positive and β can be a real number. Due to the nonlinearity in equation (3.4), we cannot obtain an exact solution for all values of the parameters m, n, α and β. Hence in section 4 we shall prove the existence and uniqueness results; and in section 5 we shall present the numerical solution (through graphs) and the discussion of the results.

4. Existence and uniqueness results.

A. Existence and uniqueness for $\beta > 0$. We consider the two-point boundary problem of finding a solution ϕ of

$$\begin{aligned} \phi'' + \alpha f(\eta)\phi' = \alpha\beta\phi^n, \quad 0 < \eta < R, \\ \phi(0) = 1, \quad \phi(R) = 0, \end{aligned} \tag{4.1}$$

where α and β are known positive constants and $f(\eta)$ is a known continuous function defined on the interval $0 \le \eta \le R$.

Assuming the existence of a classical solution to (4.1), it is an elementary application of the maximum principle argument [33] to show that $0 \le \phi \le 1$ for n odd and $\phi \le 1$ for n even. In what follows we shall assume that $0 \le \phi \le 1$ and demonstrate the existence of such a classical solution.

We begin with the set S of continuous functions on the interval $0 \le \eta \le R$ with $\phi(0) = 1$ and $\phi(R) = 0$ and $0 \le \phi \le 1$. Clearly S is a convex subset of the Banach space of continuous functions on $0 \le \eta \le R$ subject to the norm $\|\phi\| = \sup_{0\le\eta\le R} |\phi(\eta)|$. We define the mapping $T : S \to S$ via $y = T\phi$ where y is the solution of the boundary value problem

$$\begin{cases} y'' + \alpha f(\eta)y' = \alpha\beta\phi^{n-1}y, \qquad 0 < x < R, \\ y(0) = 1, y(R) = 0. \end{cases} \tag{4.2}$$

From the literature on ordinary differential equations [34] we see that (4.2) has a unique classical solution $y = y(\eta)$. From the maximum principle [33] we see that $0 \le y \le 1$, which justifies our assertion that $T : S \to S$.

Considering the integrating factor $\exp\{\alpha \int_0^\eta f(\tau)d\tau\}$, we obtain the representation

$$y'(\eta) = y'(0)\exp\left\{-\alpha\int_0^\eta f(\tau)d\tau\right\} + \alpha\beta\int_0^\eta \exp\left\{-\alpha\int_\tau^\eta f(\xi)d\xi\right\}(\phi(\tau))^{n-1}y(\tau)d\tau \tag{4.3}$$

and

$$\begin{aligned} y(\eta) = 1 + y'(0)\int_0^\eta \exp\left\{-\alpha\int_0^\tau f(\xi)d\xi\right\}d\tau \\ + \alpha\beta\int_0^\eta\int_0^\tau \exp\left\{-\alpha\int_\zeta^\tau f(\xi)d\xi\right\}(\phi(\zeta))^{n-1}y(\zeta)d\zeta d\tau. \end{aligned} \tag{4.4}$$

It then follows from $y(R) = 0$ that

$$y'(0) = -\left[\frac{1 + \alpha\beta\int_0^R\int_0^\tau \exp\left\{-\alpha\int_\zeta^\tau f(\xi)d\xi\right\}(\phi(\zeta))^{n-1}y(\zeta)d\zeta d\tau}{\int_0^R \exp\left\{-\alpha\int_0^\tau f(\xi)d\xi\right\}d\tau}\right]. \tag{4.5}$$

Since $0 \le \phi \le 1$ and $0 \le y \le 1$, it follows from (4.5) that

$$-C_1 \le y'(0) < 0, \tag{4.6}$$

where

$$C_1 = \left(1 + \alpha\beta\frac{R^2}{2}\right)\left(\int_0^R \exp\left\{-\alpha\int_0^\tau f(\xi)d\xi\right\}d\tau\right)^{-1}. \tag{4.7}$$

From (4.3) and (4.6), we see that

$$|y'(\eta)| \leq C_2, \tag{4.8}$$

where

$$C_2 = C_1 + \alpha\beta R. \tag{4.9}$$

As

$$|y''(\eta)| \leq \alpha|f(\eta)|\,|y'(\eta)| + \alpha\beta|\phi^{n-1}(\eta)|\,|y(\eta)|, \tag{4.10}$$

it follows from $0 \leq \phi \leq 1$, $0 \leq y \leq 1$ and (4.8) that

$$|y''| \leq C_3, \tag{4.11}$$

where

$$C_3 = \alpha\|f\|C_2 + \alpha\beta. \tag{4.12}$$

Clearly, T maps S into a convex compact subset of S.

In order to apply the Schauder fixed point theorem, it remains to show that T is continuous. We consider two functions $\phi_1, \phi_2 \in S$ and their corresponding images y_1 and y_2 which satisfy

$$\begin{cases} y_i'' + \alpha f y_i' = \alpha\beta\phi_i^{n-1} y_i, & 0 < \eta < R, \\ y_i(0) = 1, y_i(R) = 0, & i = 1, 2. \end{cases} \tag{4.13}$$

Let $z = y_1 - y_2$. Then, z satisfies

$$\begin{cases} z'' + \alpha f z' = \alpha\beta\phi_1^{n-1} z + \alpha\beta(\phi_1^{n-1} - \phi_2^{n-1}) y_2, & 0 < \eta < R, \\ z(0) = 0, z(R) = 0. \end{cases} \tag{4.14}$$

First, we see that

$$\begin{aligned} z(\eta)^2 &= 2\int_0^\eta z(\tau)z'(\tau)d\tau \\ &\leq 2\int_0^R |z(\tau)|\,|z'(\tau)|d\tau \\ &\leq 2\left(\int_0^R z^2 d\tau\right)^{1/2}\left(\int_0^R (z'(\tau))^2 d\tau\right)^{1/2} \end{aligned} \tag{4.15}$$

and

$$\int_0^R (z(\eta))^2 d\eta \leq 2R\left(\int_0^R z^2 d\tau\right)^{1/2}\left(\int_0^R (z'(\tau))^2 d\tau\right)^{1/2}, \tag{4.16}$$

whence it follows that

$$\left(\int_0^R z^2 d\eta\right)^{1/2} \leq 2R\left(\int_0^R (z')^2 d\tau\right)^{1/2} \tag{4.17}$$

and

$$\|z\| \leq 2\sqrt{R}\left(\int_0^R (z')^2 d\tau\right)^{1/2}. \tag{4.18}$$

Next, we multiply the differential equation in (4.14) by $z\exp\left\{\alpha\int_0^\eta f(\tau)d\tau\right\}$, integrate from 0 to R, and integrate the term $\left(z'\exp\left\{\alpha\int_0^\eta f(\tau)d\tau\right\}\right)' z$ by parts to obtain

$$\alpha\beta\int_0^R \exp\left\{\alpha\int_0^\eta f(\tau)d\tau\right\}(\phi_1(\eta))^{n-1}(z(\eta))^2 d\eta + \int_0^R \exp\left\{\alpha\int_0^\eta f(\tau)d\tau\right\}(z'(\eta))^2 d\eta$$
$$= -\alpha\beta\int_0^R \exp\left\{\alpha\int_0^\eta f(\tau)d\tau\right\}(\phi_1^{n-1}-\phi_2^{n-1})y_2 z d\eta. \tag{4.19}$$

As $0\le\phi_1\le 1$ and $0\le y_2\le 1$, we see from (4.17) that

$$\int_0^R (z'(\eta))^2 d\eta \le \alpha\beta\exp\left\{2\alpha\int_0^R |f(\tau)|\alpha\tau\right\}\left(\int_0^R(\phi_1^{n-1}-\phi_2^{n-1})^2 d\eta\right)^{\frac{1}{2}}\left(\int_0^R z^2 d\eta\right)^{\frac{1}{2}}$$
$$\le C_4\|\phi_1-\phi_2\|\left(\int_0^R (z'(\eta))^2 d\eta\right)^{\frac{1}{2}}, \tag{4.20}$$

where

$$C_4 = 2(n-1)R\alpha\beta\exp\left\{2\alpha\int_0^R |f(\tau)|d\tau\right\}. \tag{4.21}$$

From (4.18) and (4.20) it follows that

$$\|y_1-y_2\| \le 2\sqrt{R}\, C_4\|\phi_1-\phi_2\|, \tag{4.22}$$

which demonstrates the continuity of the map T. From the Schauder fixed point theorem, it follows that there exists a fixed point $\phi = T\phi$ where $0\le\phi\le 1$. We summarize this result with the following statement.

THEOREM 1. For any positive parameters α and β and any continuous function $f = f(\eta)$ on $0\le\eta\le R$ there exists a classical solution $\phi=\phi(\eta)$ of the problem

$$\begin{cases}\phi'' + \alpha f(\eta)\phi' = \alpha\beta\phi^n, & 0<\eta<R,\\ \phi(0)=1, \phi(R)=0,\end{cases} \tag{4.23}$$

which satisfies $0<\phi(\eta)<1$ for $0<\eta<R$ and the bound $0\le\phi(\eta)\le 1$ for $0\le\eta\le R$.

Proof. See the analysis preceding the statement of the theorem.

For uniqueness, we have the following result.

THEOREM 2. There exists only one positive solution ϕ to problem (4.23).

Proof. As existence has been shown above, let $\phi_i, i=1,2$ denote two positive solutions to (4.23). For $z=\phi_1-\phi_2$, we have $z(0)=z(R)=0$ and

$$z'' + \alpha f z' = \alpha\beta(\phi_1^n-\phi_2^n)$$
$$= \alpha\beta g(\phi_1,\phi_2)z, \quad 0<\eta<R, \tag{4.24}$$

where

$$g(\phi_1,\phi_2) = \phi_1^{n-1}+\phi_1^{n-2}\phi_2+\cdots+\phi_1\phi_2^{n-2}+\phi_2^{n-1}>0 \quad \text{if} \quad \phi_1,\phi_2>0. \tag{4.25}$$

By the maximum principle, $z\equiv 0$, which implies $\phi_1\equiv\phi_2$.

B. Existence and uniqueness for $\beta < 0$

In this case we may rewrite (4.1) as

$$\begin{cases} y'' + \alpha f y' = -\alpha|\beta| y^n, & 0 < \eta < R, \\ y(0) = 1, y(R) = 0. \end{cases} \tag{4.26}$$

Utilizing the integrating factor $\exp\left\{\alpha \int_0^R f(\tau)d\tau\right\}$ and some elementary calculations, we obtain the representation

$$y(\eta) = 1 - E(\eta) + \alpha|\beta| L(R, y) \cdot E(\eta) - \alpha|\beta| L(\eta, y), \quad 0 \le \eta \le R, \tag{4.27}$$

where

$$E(\eta) = \int_0^\eta \exp\left\{-\alpha \int_0^\tau f(\xi)d\xi\right\} d\tau \Big/ \int_0^R \exp\left\{-\alpha \int_0^\tau f(\xi)d\xi\right\} d\tau \tag{4.28}$$

and

$$L(\eta, \phi) = \int_0^\eta \int_0^\tau \exp\left\{-\alpha \int_\mu^\tau f(\xi)d\xi\right\} (\phi(\mu))^n d\mu d\tau. \tag{4.29}$$

From the representation

$$y(\eta) = (1 - E(\eta))(1 - \alpha|\beta| L(R, y)) + \alpha|\beta|(L(R, y) - L(\eta, y)), \tag{4.30}$$

which follows from (4.27) via adding and subtracting $L(R, y)$, we see that

$$|y(\eta)| \le 1 + \alpha|\beta| R^2 \|y\|^n, \quad 0 \le \eta \le R. \tag{4.31}$$

Hence

$$\|y\| \le 1 + \alpha|\beta| R^2 \|y\|^n. \tag{4.32}$$

Given any positive constant $C > 1$ such that $\|y\| < C$, it is clear that if

$$0 < |\beta| < \frac{C - 1}{\alpha R^2 C^n}, \tag{4.33}$$

then (4.32) implies that

$$\|y\| < C. \tag{4.34}$$

The point of this exercise is that for

$$S = \left\{\phi \in C([0, R]) \middle| \|\phi\| < C, C > 1\right\}, \tag{4.35}$$

the map $y = T\phi$ defined by

$$(T\phi)(\eta) = 1 - E(\eta) + \alpha|\beta| L(R, \phi) \cdot E(\eta) - \alpha|\beta| L(\eta, \phi) \tag{4.36}$$

is well defined and for $0 < |\beta| < (C-1)/\alpha R^2 C^n$, the function T maps S into S. Estimates for $y', (T\phi)'$ and $y'' = (T\phi)''$ can be obtained from (4.26). Thus, T maps a convex subset S of the Banach space $C([0, R])$ with the supnorm $\|\phi\| = \sup_{0 \le \eta \le R} |\phi(\eta)|$ into a convex compact subset of S. The continuity of the map T follows from the estimate

$$\begin{aligned} |(T\phi_1)(\eta) - (T\phi_2)(\eta)| &\le \alpha|\beta| E(\eta) |L(R, \phi_1) - L(R, \phi_2)| + \alpha|\beta| \, |L(\eta, \phi_1) - L(R, \phi_2)| \\ &\le \alpha|\beta| R^2 n C^n \|\phi_1 - \phi_2\|, \quad 0 \le \eta \le R. \end{aligned} \tag{4.37}$$

An application of the Schauder fixed point theorem yields the following result.

THEOREM 3. For each constant $C > 1$ and for each $\beta < 0$ such that $0 \leq |\beta| < (C-1)/(\alpha R^2 C^n)$, there exists a classical solution $\phi(\eta), 0 \leq \eta \leq R$ of

$$\begin{cases} y'' + \alpha f y' = -\alpha|\beta| y^n, & 0 < \eta < R, \\ y(0) = 1, y(R) = 0, \end{cases} \tag{4.38}$$

where α is a positive constant and f is a known function defined on $0 \leq \eta \leq R$. Moreover, for each $\beta < 0$ so that $0 \leq |\beta| < \min\{(\alpha R^2 n C^n)^{-1}, (C-1)/(\alpha R^2 C^n)\} = (\alpha R^2 n C^n)^{-1}$, the solution ϕ is unique.

Proof. See the analysis preceding the statement of the theorem. For $0 \leq |\beta| < (\alpha R^2 n C^n)^{-1}$, it follows from (4.37) that T is a contraction which implies uniqueness of the fixed point.

Note. From Theorem 3, it is evident that there is no unique solution for all values of $\beta < 0$. That is, the parameters α, β, m and n need to satisfy an inequality $|\beta| > (\alpha R^2 C^n)^{-1}$. This fact was verified while computing the numerical solutions.

5. Numerical solution and discussion of the results. The boundary value problem

$$\phi'' + \alpha f(\eta)\phi' = \alpha\beta\phi^n, \quad \phi(\eta = 0) = 1, \quad \phi(\eta \to \infty) = 0 \tag{5.1}$$

is solved numerically for several sets of values of the parameters α, β, m and n. Some of the qualitatively interesting results are presented in Figures 1 and 2.

In Figures 1 and 2, we plotted the nondimensional concentration profiles $\phi(\eta)$ for several sets of values of the parameters α (the Schmidt number), β (the reaction rate parameter), n (the reaction order parameter) and $m = \sqrt{(1+k_1)}$ (the porosity-parameter). Here, an increase in m physically means an increase in porosity.

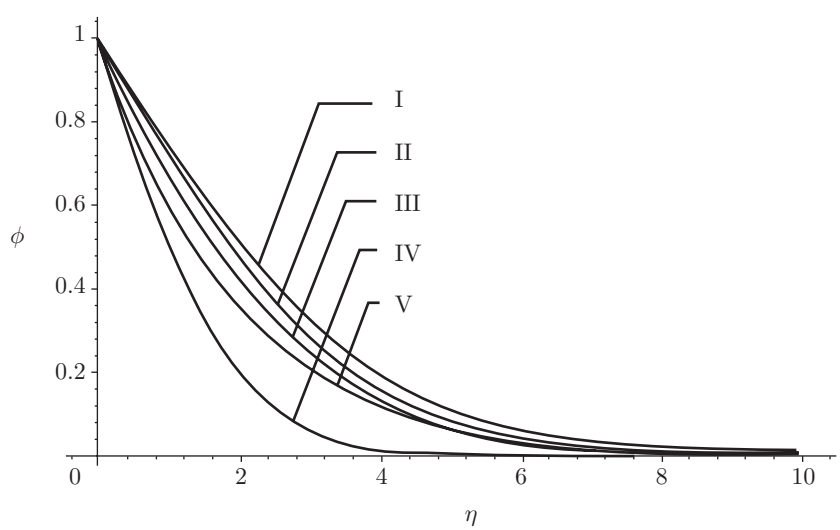

FIG. 1. ϕ versus η for $m = 0.2$

	I	II	III	IV	V
α	0.2	0.2	0.2	0.5	0.5
β	-0.2	-0.2	0.2	0.2	0.2
n	1	2	2	2	5

From Figure 1 it is evident that the fluid concentration decreases with an increase in the reaction order parameter n when the reaction rate parameter β is negative (see curves I and II). This is the opposite of the phenomenon when β is positive (see curves IV and V) — this is in conformity with the physical fact that the generative chemical reaction takes place when $\beta < 0$ and destructive chemical reaction takes place when $\beta > 0$. However, the effect of the Schmidt number α is to decrease ϕ considerably (see curves III and IV). Physically it means that the thickness of the concentration boundary layer decreases with Schmidt number. From Figures 1 and 2, it is evident that all the above said conclusions are qualitatively true for $m = 0.2$ and $m = 0.5$. However, the thickness of the concentration boundary layer increases with an increase in m. That is, an increase in porosity is to increase the fluid concentration in the boundary layer.

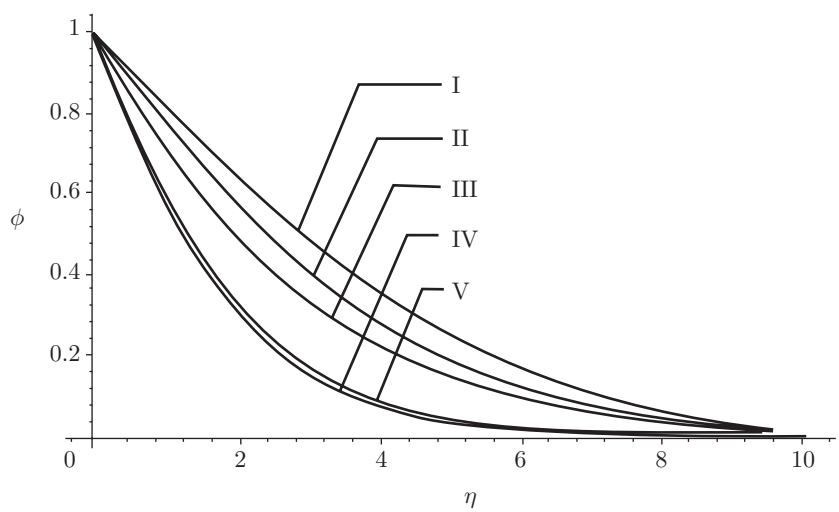

FIG. 2. ϕ versus η for $m = 0.5$; curves as in Figure 1

Acknowledgments. The authors thank the referee for constructive comments which led to a definite improvement in the paper.

REFERENCES

[1] D.-T. Chin, Mass transfer to a continuous moving sheet electrode, *J. Electrochem. Soc.* **122** (1975), 643.

[2] R. S. R. Gorla, Unsteady mass transfer in the boundary layer on a continuous moving sheet electrode, *J. Electrochem. Soc.* **125** (1978), 865.

[3] R. M. Griffith, Velocity, temperature, and concentration distributions during fibre spinning, *Ind. Engng. Chem. Fundam.* **3** (1964), 245.

[4] L. E. Erickson, L. T. Fan and V. G. Fox, Heat and mass transfer on a moving continuous flat plate with suction or injection, *Ind. Engng. Chem. Fundam.* **5** (1966), 19.
[5] L. J. Crane, Flow past a stretching plate, *Z. Agnew. Math. Phys.* **21** (1970), 645.
[6] B. Siddappa and B. S. Khapate, Rivlin-Ericksen fluid flow past a stretching plate, *Rev. Roum. Sci. Techn. Mec. Appl.* **21** (1976), 497.
[7] T. C. Chiam, Micropolar fluid flow over a stretching sheet, *Z. Angew. Math. Mech.* **62** (1982), 565.
[8] K. R. Rajagopal, T. Y. Na and A. S. Gupta, Flow of a viscoelastic fluid over a stretching sheet, *Rheol. Acta* **23** (1984), 213.
[9] B. Siddappa and S. Abel, Non-Newtonian flow past a stretching plate, *Z. Angew. Math. Phys.* **36** (1985), 890.
[10] H. I. Andersson and B. S. Dandapat, Flow of a power-law fluid over a stretching sheet, *Stability Appl. Anal. Continuous Media* **1** (1992), 339.
[11] J. Vleggaar, Laminar boundary-layer behaviour on continuous accelerating surfaces, *Chem. Engng. Sci.* **32** (1977), 1517.
[12] P. S. Gupta and A. S. Gupta, Heat and mass transfer on a stretching sheet with suction or blowing, *Can. J. Chem. Engng.* **55** (1977), 744.
[13] P. Carragher and L. J. Crane, Heat transfer on a continuous stretching sheet, *Z. Angew. Math. Mech.* **26** (1982), 564.
[14] L. J. Grubka and K. M. Bobba, Heat transfer characteristics of a continuous stretching surface with variable temperature, *ASMA J. Heat Transfer* **107** (1985), 148.
[15] D. R. Jeng, T. C. A. Chang and K. J. DeWitt, Momentum and heat transfer on a continuous moving surface, *ASMA J. Heat Transfer* **108** (1986), 532.
[16] C.-K. Chen and M.-I. Char, Heat transfer of a continuous stretching surface with suction or blowing, *J. Math. Anal. Appl.* **135** (1988), 568. MR0967227 (89i:80001)
[17] K. Vajravelu and D. Rollins, Heat transfer in a viscoelastic fluid over a stretching sheet, *J. Math. Anal. Appl.* **158** (1991), 241. MR1113413 (92b:76003)
[18] P. L. Chambre and J. D. Young, On the diffusion of a chemically reactive species in a laminar boundary layer flow, *Phys. Fluids* **1** (1958), 48.
[19] P. L. Chambre, On the ignition of a moving combustible gas stream, *J. Chem. Phys.* **25** (1956), 417.
[20] N. Dural and A. L. Hines, A comparison of approximate and exact solutions for homogeneous irreversible chemical reaction in the laminar boundary layer, *Chem. Engng. Commun.* **96** (1990), 1.
[21] H. I. Andersson, O. R. Hansen and B. Holmedal, Diffusion of a chemically reactive species from a stretching sheet, *Int. J. Heat Mass Transfer* **37** (1994), 659.
[22] S. Abel and P. H. Veena, Visco-elastic fluid flow and heat transfer in a porous medium over a stretching sheet, *Internat. J. Non-linear Mech.* **33** (1998), 531.
[23] R. K. Gupta and T. Sridhar, Visco-elastic effects in non-Newtonian flow through porous media, *Rheol. Acta* **24** (1985), 148.
[24] K. V. Prasad, M. S. Abel and A. Joshi, Oscillatory motion of a visco-elastic fluid over a stretching sheet in porous media, *J. Porous Media* **3** (2000), 61.
[25] K. V. Prasad, M. S. Abel and S. K. Khan, Momentum and heat transfer in visco-elastic fluid flow in porous medium over a non-isothermal stretching sheet, *Int. J. Num. Methods for Heat and Fluid Flow* **10** (2001), 784.
[26] G. Johnson, M. Massoudi and K. R. Rajagopal, Flow of a fluid infused with solid particles through a pipe, *Int. J. Engng. Sci.* **29** (1991), 649. MR1107195 (92c:76070)
[27] G. Johnson, M. Massoudi and K. R. Rajagopal, Flow of a fluid-solid mixture between flat plates, *Chemical Engng. Sci.* **46** (1991), 1713.
[28] M. S. Abel, S. K. Khan and K. V. Prasad, Study of visco-elastic fluid flow and heat transfer over a stretching sheet with variable viscosity, *Int. J. Nonlinear Mech.* **37** (2002), 81.
[29] W. C. Troy, E. A. Overman II, G. B. Ermentrout and J. P. Keener, Uniqueness of flow of a second-order fluid past a stretching sheet, *Quart. Appl. Math.* **44** (1987), 753. MR0872826 (87m:76009)
[30] J. B. McLeod and K. R. Rajagopal, On the uniqueness of flow of a Navier-Stokes fluid due to a stretching boundary, *Arch. Rational Mech. Anal.* **98** (1987), 385. MR0872753 (88c:35131)
[31] K. R. Rajagopal and L. Tao, *Mechanics of Mixtures*, World Scientific, River Edge, NJ, 1995. MR1370661 (97a:73001)

[32] M. Abramowitz and I. Stegun (Eds.), *Handbook of mathematical functions with formulas, graphs, and mathematical tables*, Dover, New York, 1965. MR0208797 (34:8606)
[33] M. H. Protter and H. F. Weinberger, *Maximum Principles in Differential Equations*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1967. MR0219861 (36:2935)
[34] P. Hartman, *Ordinary Differential Equations*, Second Edition, Birkhäuser, Boston-Basel-Stuttgart, 1982. MR0658490 (83e:34002)