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АЛГЕБРА И АНАЛИЗ

ТОМ 33 № 2 2021

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The Editorial Board of “Algebra i analiz” congratulates Vasiliĭ Mikhaĭlovich Babich, an outstanding scientist, the leader of the Leningrad–St. Petersburg school of diffraction and wave propagation theory, on his anniversary.

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FOREWORD

This issue is dedicated to the ninetieth anniversary of Vasiliĭ Mikhaĭlovich Babich, an outstanding scientist, the leader of the Leningrad–St. Petersburg school of diffraction and wave propagation theory.

Vasiliĭ Mikhaĭlovich was born on June 13, 1930 in Leningrad in a family of teachers. During the war he had to survive the siedge winter, evacuation across the Ladoga lake, staying in an orphanage. Upon his return to Leningrad in 1947, he enrolled at the Mathematics and Mechanics Faculty of Leningrad State University. His teachers were G. I. Petrashen′ and S. G. Mikhlin. After graduation from the university, Vasiliĭ Mikhaĭlovich started his career in 1954. He has been teaching at the Faculty of Mathematics and Mechanics and at the Faculty of Physics of Leningrad State University (St. Petersburg State University), and since 1967 he has been the Head of the Laboratory of Mathematical Problems in Geophysics at the Leningrad (St. Petersburg) Department of Steklov Mathematical Institute, RAS.

Vasiliĭ Mikhaĭlovich is the author of more than 160 papers and six monographs.[1]

His results on wave propagation theory and asymptotic methods in the theory of differential equations are world famous. Vasiliĭ Mikhaĭlovich is one of the founders of the ray method in elastic wave theory. A large series of his papers is devoted to the justification of the high-frequency asymptotics of the solution for the problem of diffraction on a smooth convex body. His works on Rayleigh surface waves are widely known; by the way, in them, among other things, the emergence of the geometric phase (currently known as the Berry phase) was observed. Another series of his works is devoted to the construction of localized solutions for linear equations of mathematical physics; his theory of quasimodes is especially famous in this range of ideas. Vasiliĭ Mikhaĭlovich is a creator of the method of summation of Gaussian beams for asymptotic calculation of wave fields. He owns profound results on cone and wedge diffraction. In recent years, Vasiliĭ Mikhaĭlovich has continued to work actively, in particular, on the theory of interference waves and applications of the Hadamard *Ansatz*.

[1] [1] V. M. Babich and V. S. Buldyrev, *Asymptotic methods in short-wavelength diffraction theory*, Nauka, Moscow, 1972; English transl., Alpha Science, Oxford, 2009. MR0426630

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[6] V. M. Babich and A. P. Kiselev, *Elastic waves*: *high frequency theory*, BHV, S.-Peterburg, 2014; English transl., CRC Press, Taylor&Chapman, Boca Raton–London–New York, 2018. MR3822378

Vasiliĭ Mikhaĭlovich's scientific activity is inseparably connected with teaching and education of students. Of these, more than 25 became Candidates of Sciences (Ph.D) and 6 obtained Doctor of Science degrees. In 2008 he was awarded the SPbSU Prize "For Pedagogical Excellence". For many years Vasiliĭ Mikhaĭlovich leads the St. Petersburg Seminar on wave propagation theory, which he created together with V. S. Buldyrev at the end of the 1950s, and is the chairman of the organizing committee of the annual international conference "Days on Diffraction" held since 1968.

Vasiliĭ Mikhaĭlovich Babich has received many awards and prizes: the series of works on ray methods in seismic wave calculations (jointly with a group of colleagues) was awarded by the State Prize of the USSR in 1982; works on asymptotic methods of diffraction theory were awarded by the V. A. Fock Prize of RAS in 1998; in 2010 Vasiliĭ Mikhaĭlovich was awarded by the honorary title "Honored Scientist of the Russian Federation". In 2014, Vasiliĭ Mikhaĭlovich won the "Life dedicated to mathematics" prize of the "Dinasty" Foundation.

A detailed account of Vasiliĭ Mikhaĭlovich Babich's activity can be found in the articles in Uspekhi Mat. Nauk **57** (2002), no. 3, 181–187; **66** (2011), no. 1, 205–207 (English transl., Russ. Math. Surveys **57** (2003), no. 3, 627–635, **66** (2011), no. 1, 205–208 MR1918875, MR2841696), and by the paper: A. P. Kiselev and V. P. Smyshlyaev, *V. M. Babich, on the occasion of his seventith birthday*, Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI) **275** (2001), 9–16; English transl., J. Math. Sci. (N.Y.) **117** (2003), no. 2, 3891–3894. MR1854496

Friends, colleagues and students greatly appreciate communication with Vasiliĭ Mikhaĭlovich, who is distinguished by his responsive and respectful attitude towards people. A big and close-knit family — his wife, three children, four grandchildren and three great-grandchildren — became his happiness and great success in life.

The authors of the issue warmly congratulate Vasiliĭ Mikhaĭlovich on his anniversary and wish him health and further success.

A. P. Kiselev and T. A. Suslina

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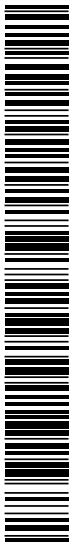
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