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И
АНАЛИЗ

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Алгебра и анализ

“Наука”, С.-Петербургское отделение, 2022 г.

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We heartily congratulate Nikolai Kapitonovich Nikolski on his anniversary and wish him good spirits and creative longevity.

*Editors of the special issue of the journal
E. Abakumov, A. Baranov, A. Borichev,
S. Kislyakov, Yu. Tomilov*

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FOREWORD

On November 16, 2020 Nikolai Kapitonovich Nikolski turned 80 years old. For half a century, Nikolai Kapitonovich has been one of the leaders of the Leningrad–St. Petersburg community of analysts, who has made an invaluable contribution to the formation of the modern face of this scientific school.

These words are a paraphrase of what N. K. Nikolski himself said about another famous St. Petersburg mathematician, Viktor Petrovich Havin (see [1, p. 3]). One can read about the origin, shaping, and development of the mentioned community in the same paper [1] (for the most part, this information is personal memories of N. K.), however, the genre of that article and the team of authors ruled out the option to state in it that, except (naturally) the very early years, the school of analysis had two undoubted leaders, V. P. Havin and N. K. Nikolski.

A most important stage in the growth of this school, not mentioned (for the reasons indicated) in [1], is the organization by N. K. Nikolski in the mid-1970s of the Laboratory of mathematical analysis at LOMI–POMI. The author of these lines was accepted into it at about the same time, having finished graduate studies, and found himself in an unusually benevolent creative atmosphere of that wonderful team of like-minded people. It is appropriate to mention the role of N. K. in the development of research on mathematical analysis at the University of Bordeaux-1, with which N. K. has been associated since the early 1990s.

N. K.’s own scientific achievements and his contribution to the establishing of the school and the education of a number of famous mathematicians (including several winners of the Salem Prize) were awarded in 2010 with the prestigious Ampère Prize of the French Academy of Sciences.

Last not least, it should be said about the role of Nikolai Kapitonovich in the foundation of the journal “Algebra and Analysis”. N. K. was one of the leaders of the first editorial board (D. K. Faddeev was the editor-in-chief, and his deputies were A. N. Andrianov, N. K. Nikolski, and V. N. Popov). Moreover, the numerous fresh ideas of Nikolai Kapitonovich, as well as the enthusiasm and energy with which he implemented them, allowed the journal to quickly take its rightful place among other mathematical publications at that time.

The Editorial Board cordially congratulates Nikolai Kapitonovich on his anniversary and wishes him good health and long creative activity, dedicating this issue of the journal to him. The subject matter of the articles, as we hope, corresponds to the scientific interests of N. K. in different periods of his work, but, of course, cannot cover the entire field of his mathematical research.

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- [1] A. D. Baranov, S. V. Kislyakov, and N. K. Nikolski, *Victor Petrovich Havin, a life devoted to mathematics*. 50 years with Hardy spaces, Oper. Theory Adv. Appl., vol. 261, Birkhäuser/Springer, Cham, 2018, pp. 3–55. MR3792087

S. V. Kislyakov, Editor-in-chief



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