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OPTIMAL FILTRATION IN SYSTEMS WITH NOISE MODELED BY A POLYNOMIAL OF FRACTIONAL BROWNIAN MOTION

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Abstract. The problem of optimal filtration in systems with noise modeled by a polynomial of fractional Brownian motion is partially solved by representing fractional Brownian motion in terms of standard Brownian motion.

1. Introduction

Systems governed by fractional Brownian motions are a generalization of those governed by standard Brownian motions.

We consider the real-valued processes X_t (signals) and Y_t (observations) represented by the following system of equations:

$$\begin{cases} X_t = \eta + \int_0^t a(s, X_s)\,ds + \int_0^t p(W_s)\,dW_s, & t \in [0,T], \\ Y_t = \xi + \int_0^t A(s, X_s)\,ds + \int_0^t B(s)\,dV_s, & t \in [0,T], \end{cases} \tag{1.1}$$

where $V = (V_t, t \in [0,T])$ and $W = (W_t, t \in [0,T])$ are fractional Brownian motions with Hurst parameters $H \in (\frac{1}{2}, 1)$ and $h \in (\frac{1}{2}, 1)$, respectively. The coefficients A and a are assumed to be continuous functions on $[0,T] \times \mathbb{R}$, p is a polynomial, $p(x) = \sum_{n=0}^{N-1} a_n x^n$, and B is a continuous function on $[0,T]$ that vanishes nowhere on $[0,T]$. Suppose the random initial conditions (η, ξ) do not depend on the fractional Brownian motions V and W, and the pair (X, ξ) has a given distribution $\mu_{(X,\xi)}$. Assume that the process Y is observed and one wants to estimate X. This is a classical problem of filtration of a signal X at time t from the process Y observed up to the time t. The conditional distribution $\pi_t(X)$ with respect to the σ-algebra $\mathcal{Y}_t = \sigma(\{Y_s, s \in [0,t]\})$ (called the optimal filter) is known to be the solution of this problem.

The filtration problem is considered in [1] and [2] for systems with noise modeled by a standard Brownian motion. The classical problem is extended in [3] to the case where the noise is a fractional Brownian motion. The filtration for linear systems with one-dimensional fractional Brownian motions is studied in [4]; the case of a multidimensional fractional Brownian motion is considered in [5]. All these papers treat the case where the noise X is of the form $\int_0^t f(s)\,dW_s$, and $f(t)$, $t \in [0,T]$, is a nonrandom function.

The present paper deals with the case where f is a particular random function, namely, a certain polynomial of fractional Brownian motion. We partially solve the problem of filtration by representing W^n for any $n \in \mathbb{N}$ in terms of a suitable Wiener process.

Section 1 gives the main properties of fractional Brownian motion that allow one to apply methods similar to the classical methods for solving the filtration problem. In

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Section 2 we construct the representation of W^n in terms of a suitable standard Brownian motion. A partial solution of the problem of optimal filtration is obtained in Section 3 for the system (1.1).

2. MAIN PART

2.1. **Fractional Brownian motion.** In what follows we assume that all random variables and stochastic processes are defined on a stochastic basis $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathsf{P})$ satisfying the standard conditions. The P-completion of the flow of σ-algebras generated by a stochastic process is treated as the natural flow of σ-algebras.

Let $T > 0$ be fixed. A stochastic process $V = (V_t, t \in [0,T])$ is called a normalized fractional Brownian motion with Hurst parameter $H \in (\frac{1}{2}, 1)$ if

(1) V is a Gaussian process with continuous paths and stationary increments;
(2) $V_0 = 0$, $\mathsf{E}\, V_t = 0$, $\mathsf{E}\, V_t^2 = t^{2H}$ for all $t \in [0,T]$.

The case of $H = \frac{1}{2}$ corresponds to standard Brownian motion. Since fractional Brownian motion is not a semimartingale, the classical theory of integration is not applicable for interpolation with respect to the fractional Brownian motion. Nevertheless, the integral with respect to fractional Brownian motion can be defined for a certain class of nonrandom functions. We now discuss this construction.

Let f and g be measurable functions on $[0,T]$. Put

$$\langle\langle f, g \rangle\rangle_H = H(2H-1) \int_0^T \int_0^T f(s) g(t) |s-t|^{2H-2} \, ds \, dt. \tag{2.1}$$

Then the space L_2^H of classes of equivalent measurable functions f on $[0,T]$ such that $\langle\langle f, f \rangle\rangle_H < \infty$ is a Hilbert space with scalar product $\langle\langle \cdot, \cdot \rangle\rangle_H$. The correspondence $\mathbb{1}[0,T] \longrightarrow V_T$ can be extended to the isometry between L_2^H and the Gaussian space generated by the random variables V_t, $t \in [0,T]$. The integral $\int_0^T f(s)\, dV_s$ is defined for $f \in L_H^2$ as the image of f under this isometry. For all $f, g \in L_H^2$, we have

$$\mathsf{E}\left[\left\{ \int_0^T f(s)\, dV_s \right\} \left\{ \int_0^T g(s)\, dV_s \right\} \right] = \langle\langle f, g \rangle\rangle_H, \tag{2.2}$$

where $\langle\langle f, g \rangle\rangle_H$ is defined by (2.1).

The process $\int_0^t f(s)\, dV_s$ is defined for $f \in L_H^2$ as follows:

$$\int_0^t f(s)\, dV_s = \int_0^T \mathbb{1}[0,t)(s) f(s)\, dV_s, \qquad t \in [0,T].$$

Since V is not a semimartingale, we construct an integral transformation V^* of the process V:

$$V_t^* = \int_0^t k_t^*(s)\, dV_s, \tag{2.3}$$

where k^* is a nonrandom kernel,

$$\begin{gathered} k_t^*(s) = k_H^{-1} s^{1/2-H} (t-s)^{1/2-H}, \qquad 0 < s < t, \\ k_H = 2H\Gamma(3/2-H)\Gamma(H+1/2). \end{gathered} \tag{2.4}$$

The process V^* is a martingale with covariance

$$\psi_H(t) = \langle V^* \rangle_t = \frac{\Gamma(3/2-H)}{2H\Gamma(3-2H)\Gamma(H+1/2)} t^{2-2H}. \tag{2.5}$$

Moreover, the flow of σ-algebras generated by the process V^* coincides (up to zero sets) with the flow of σ-algebras generated by the process V. The process V^* is called the fundamental martingale of the fractional Brownian motion V in [6].

Let

$$V_t^{**} = \frac{2H}{c_H} \int_0^t s^{H-1/2}\, dV_s^*,$$

$$c_H = \left(\frac{2H\Gamma(3/2-H)}{\Gamma(H+1/2)\Gamma(2-2H)} \right)^{1/2}.$$

It is worth mentioning that V_t^{**} is a standard Brownian motion such that

$$V_t = \int_0^t z(t,s)\, dV_s^{**}, \qquad t \in [0,T], \tag{2.6}$$

$$z(t,s) = \left(H - \frac{1}{2} \right) c_H s^{1/2-H} \int_s^t u^{H-1/2}(u-s)^{H-3/2}\, du, \qquad s \in [0,t]. \tag{2.7}$$

More details and facts about fractional Brownian motion can be found in [6].

2.2. **Representation of a power of fractional Brownian motion in terms of standard Brownian motion.** We show that V_t^n, $t \in [0,T]$, $n \in \mathbb{N}$, can be represented in the form

$$V_t^n = \int_0^t M_n(t,s)\, dV_s^{**} + \int_0^t K_n(t,s)\, ds, \tag{2.8}$$

where $M_n(t,s)$ is a random $\mathcal{F}_s$-measurable function, while $K_n(t,s)$ is a nonrandom function. We also obtain some recurrence equalities for $M_n(t,s)$ and $K_n(t,s)$.

If $n = 1$, we obtain from (2.6) that

$$V_t = \int_0^t M_1(t,s)\, dV_s^{**} + \int_0^t K_1(t,s)\, ds = \int_0^t z(t,s)\, dV_s^{**}.$$

Thus

$$M_1(t,s) = z(t,s), \qquad K_1(t,s) = 0, \qquad s \in [0,t],\ t \in [0,T]. \tag{2.9}$$

If $n = 2$, we apply the Itô formula for stochastic integrals with respect to the Wiener process and get

$$\begin{aligned} V_t^2 &= \int_0^t M_2(t,s)\, dV_s^{**} + \int_0^t K_2(t,s)\, ds = \left(\int_0^t z(t,s)\, dV_s^{**} \right)^2 \\ &= \int_0^t 2V_s z(t,s)\, dV_s^{**} + \int_0^t z^2(t,s)\, ds. \end{aligned}$$

Hence

$$M_2(t,s) = 2V_s z(t,s), \qquad K_2(t,s) = z^2(t,s), \qquad s \in [0,t],\ t \in [0,T]. \tag{2.10}$$

For an arbitrary $n \geq 3$, we have

$$V_t^n = \left(\int_0^t z(t,s)\, dV_s^{**} \right)^n = \int_0^t nV_s^{n-1} z(t,s)\, dV_s^{**} + \int_0^t \frac{n(n-1)}{2} V_s^{n-2} z^2(t,s)\, ds.$$

Using the equality

$$V_t^{n-2} = \int_0^t M_{n-2}(t,s)\, dV_s^{**} + \int_0^t K_{n-2}(t,s)\, ds$$

and changing the order of integration in the second term we obtain

$$\begin{aligned}
V_t^n &= \int_0^t M_n(t,s)\,dV_s^{**} + \int_0^t K_n(t,s)\,ds \\
&= \int_0^t nV_s^{n-1} z(t,s)\,dV_s^{**} \\
&\quad + \int_0^t \frac{n(n-1)}{2}\left(\int_0^s M_{n-2}(s,u)\,dV_u^{**} + \int_0^t K_{n-2}(s,u)\,du\right) z^2(t,s)\,ds \\
&= \int_0^t nV_s^{n-1} z(t,s)\,dV_s^{**} + \int_0^t \frac{n(n-1)}{2}\left(\int_s^t z^2(t,u) M_{n-2}(u,s)\,du\right) dV_s^{**} \\
&\quad + \int_0^t \frac{n(n-1)}{2}\int_s^t z^2(t,u) K_{n-2}(u,s)\,ds.
\end{aligned}$$

In fact, we obtained the recurrence equalities for $M_n(t,s)$ and $K_n(t,s)$, namely

$$\begin{gathered}
M_n(t,s) = nV_s^{n-1} z(t,s) + \frac{n(n-1)}{2}\left(\int_s^t z^2(t,u) M_{n-2}(u,s)\,du\right), \\
K_n(t,s) = \frac{n(n-1)}{2}\int_s^t z^2(t,u) K_{n-2}(u,s)\,du, \\
s\in[0,t], \qquad t\in[0,T].
\end{gathered} \tag{2.11}$$

The initial values are given by equalities (2.9) and (2.10).

It remains to prove that the integrals converge. In what follows we denote by C_i, $i\in\mathbb{N}$, constants whose precise values do not matter in the proof. First we estimate $z^2(t,s)$. Using equality (2.7) for $z^2(t,s)$ we get

$$\begin{aligned}
z^2(t,s) &= C_1 s^{1-2H}\left(\int_s^t u^{H-1/2}(u-s)^{H-3/2}\,ds\right)^2 \le C_2 s^{1-2H}\left((t-s)^{H-1/2}\right)^2 \\
&\le C s^{1-2H}.
\end{aligned}$$

Now we use induction to estimate $K_{2k}(t,s)$, $k\in\mathbb{N}$, with the help of the latter bound. Note that $K_{2k+1}(t,s)=0$, $k\in\mathbb{N}$. For $k=1$, we have

$$K_2(t,s) = z^2(t,s) \le C_2 s^{1-2H}.$$

Let $K_{2k}(t,s)\le C_{2k}s^{1-2H}$; then

$$K_{2(k+1)}(t,s) = \int_s^t z^2(t,u) K_{2k}(u,s)\,du \le C_{2k}C_2\int_s^t u^{1-2H}s^{1-2H}\,du \le C_{2(k+1)}s^{1-2H}.$$

Therefore all the integrals containing $K_n(t,s)$ are well defined.

The integral $\int_0^t M_n(t,s)\,dV_s^{**}$ is well defined for a fixed $n\in\mathbb{N}$ if the integral

$$\int_0^t \mathsf{E}\, M_n^2(t,s)\,ds$$

converges. This can be proved by induction.

For $k=1$, we use (2.9) to obtain $\mathsf{E}\, M_1^2(t,s)\le C_1 s^{1-2H}$ and

$$\int_0^t \mathsf{E}\, M_1^2(t,s)\,ds \le \int_0^t C_1 s^{1-2H}\,ds \le \tilde{C}_1 t^{2-2H} < \infty.$$

For $k=2$, we use (2.10) to prove that $\mathsf{E}\, M_2^2(t,s)\le C_2\,\mathsf{E}\,|V_s|^2 s^{1-2H}$ and

$$\int_0^t \mathsf{E}\, M_2^2(t,s)\,ds \le \int_0^t C_2\,\mathsf{E}\,|V_s|^2 s^{1-2H}\,ds \le \tilde{C}_2 t^{2-2H}t^{2H} < \infty.$$

For some k, let

$$\mathsf{E}\, M_k^2(t,s) \le C_k s^{1-2H} \left(1 + \mathsf{E}\, V_s^2 + \dots + \mathsf{E}\, V_s^{2(k-1)}\right)$$

and

$$\int_0^t \mathsf{E}\, M_k^2(t,s)\, ds \le \tilde{C}_k t^{2-2H} \left(1 + \mathsf{E}\, V_t^2 + \dots + \mathsf{E}\, V_t^{2(k-1)}\right) < \infty.$$

Then using (2.11) we get

$$\begin{aligned} \mathsf{E}\, M_{k+1}^2(t,s) &\le C'_{k+1}\, \mathsf{E} \left(V_s^k z(t,s) + \left(\int_s^t z^2(t,u) M_{k-1}(u,s)\, du \right) \right)^2 \\ &\le 2C'_{k+1} \left(\mathsf{E} \left(V_s^k z(t,s)\right)^2 + \mathsf{E} \left(\int_s^t u^{1-2H} M_{k-1}(u,s)\, du \right)^2 \right) \\ &\le C_{k+1} s^{1-2H} \left(1 + \mathsf{E}\, V_s^2 + \dots + \mathsf{E}\, V_s^{2k}\right), \end{aligned}$$

whence

$$\int_0^t \mathsf{E}\, M_{k+1}^2(t,s)\, ds \le \tilde{C}_{k+1} t^{2-2H} \left(1 + \mathsf{E}\, V_t^2 + \dots + \mathsf{E}\, V_t^{2k}\right) < \infty.$$

This proves the existence of all previous integrals.

2.3. **Solution of the problem of optimal filtration.** The problem of filtration is considered in [3] for the case where the noise is a fractional Brownian motion. It is shown in [3] that a method similar to the classical method where the noise is a standard Brownian motion can be used to establish equations for the optimal filtration $\pi_t(X) = \mathsf{E}[X_t \mid \mathcal{Y}_t]$, $\mathcal{Y}_t = \sigma\{Y_s, s \in [0,t]\}$. Following this method it is proved in [3] that the fundamental martingale for fractional Brownian motion exists and has some nice properties. Then it is shown in [3] that

$$Z_t = \int_0^t k_t^*(s) B^{-1}(s)\, dY_s \tag{2.12}$$

is a P-semimartingale. It is also proved in [3] that the σ-algebras

$$\mathcal{F}^{\xi,Z} = \sigma\left\{\xi; Z_s, s \in [0,t]\right\}$$

and $\mathcal{Y}_t$ coincide for all $t \in [0,T]$ up to P-zero sets.

The following result provides the equations for the optimal filter of some semimartingale.

Theorem 2.1 ([3, Theorem 2])**.** *Let $\xi = (\xi_t, t \in [0,T])$ be a $((\mathcal{F}_t), \mathsf{P})$-semimartingale such that*

$$\xi_t = \xi_0 + \int_0^t \beta_s\, ds + m_t, \qquad t \in [0,T],$$

where

$$\mathsf{E}\left[\xi_0^2\right] < \infty, \qquad \mathsf{E}\left[\int_0^T \beta_t^2\, dt\right] < \infty,$$

and let $m = (m_t, t \in [0,T])$ be a square integrable $((\mathcal{F}_t), \mathsf{P})$-martingale such that

$$\langle m, V^* \rangle_t = \int_0^t \lambda_s\, d\psi_H(s), \qquad t \in [0,T].$$

Then the process $\pi(\xi) = \pi_t(\xi)$, $t \in [0,T]$, *satisfies the following stochastic differential equation:*

$$\pi_t(\xi) = \pi(\xi_0) + \int_0^t \pi_s(\beta)\,ds + \int_0^t \big[\pi_s(\lambda) + \pi_s(\xi q(X)) - \pi_s(\xi)\pi_s(q(X))\big]\,dv_s, \quad t \in [0,T], \tag{2.13}$$

where

$$q_t(X) = \frac{d}{d\psi_H(t)} \int_0^t k_t^*(s) B^{-1}(s) a(s, X_s)\,ds, \qquad t \in [0,T], \tag{2.14}$$

and ψ_H *is defined by* (2.5). *The process* v_t *satisfies*

$$v_t = Z_t - \int_0^t \pi_s(q(X))\,d\psi_H(s). \tag{2.15}$$

The process v plays the same role as the innovation process does in the classical model. Namely, the process v is a continuous Gaussian $(\mathcal{Y}_t)$-martingale with covariance ψ_H. Moreover, any square integrable $(\mathcal{Y}_t)$-martingale M_t, $M_0 = 0$, with respect to the measure P is represented as

$$M_t = \int_0^t P_s\,dv_s, \qquad t \in [0,T],$$

where $P = (P_t, t \in [0,T])$ is an $(\mathcal{Y}_t)$-adapted process, $\mathsf{E}[\int_0^T P_t^2\,d\psi_H(t)] < \infty$.

We assume that the fractional Brownian motions V and W are dependent in the sense that there exists the quadratic characteristic

$$\langle W^{**}, V^* \rangle_t = \int_0^t \lambda(s)\,d\psi_H(s), \qquad t \in [0,T],$$

and the function $\lambda(t)$, $t \in [0,T]$, is known. We seek a solution that is the optimal filter for the functional $\phi(X_t)$, where $\phi \in C^2[0,T]$. First we use representation (2.8):

$$X_t = \eta + \int_0^t a(s, X_s)\,ds + \sum_{n=1}^N b_n \int_0^t M_n(t,s)\,dW_s^{**} + \sum_{n=1}^N b_n \int_0^t K_n(t,s)\,ds. \tag{2.16}$$

Let

$$\tilde{a}(t, s, X_s) = a(s, X_s) + \sum_{n=1}^N b_n K_n(t,s)$$

and

$$\tilde{M}(t,s) = \sum_{n=1}^N b_n M_n(t,s).$$

Then

$$X_t = \eta + \int_0^t \tilde{a}(t, s, X_s)\,ds + \int_0^t \tilde{M}(t,s)\,dW_s^{**}. \tag{2.17}$$

Consider the family of semimartingales X_s^t, $s \in [0,t]$, defined by

$$X_s^t = \eta + \int_0^s \tilde{a}(t, u, X_u)\,du + \int_0^s \tilde{M}(t,u)\,dW_u^{**}, \qquad s \in [0,t]. \tag{2.18}$$

It is clear that $X_t^t = X_t$. Using the Itô formula we prove that $\phi(X_s^t)$, $s \in [0,t]$, is a semimartingale such that

$$\phi(X_s^t) = \phi(\eta) + \int_0^s \mathrm{Ł}_u\left(\phi(X_u^t)\right)\,du + \int_0^s \phi'(X_u^t)\tilde{M}(t,u)\,dW_u^{**}, \qquad s \in [0,t], \tag{2.19}$$

where

$$\text{Ł}_u(\phi(\cdot)) = \tilde{a}(t,u,X_u)\phi'(\cdot) + \frac{1}{2}\phi''(\cdot)\left(\tilde{M}(t,u)\right)^2.$$

Applying Theorem 2.1 to the semimartingale $\phi(X^t)$ we get the equation for the optimal filter $\pi_s(\phi(X^t))$:

$$\begin{aligned}\pi_s(\phi(X^t)) = \pi(\phi(\eta)) &+ \int_0^s \pi_u\left(\text{Ł}_u(\phi(X^t))\right)du \\ &+ \int_0^s \Big[\pi_u\left(\phi'(X^t)\tilde{M}(t,u)\lambda(u)\right) \\ &\qquad + \pi_u(\phi(X^t)q) - \pi_u(\phi(X^t))\pi_u(q)\Big]\,dv_u.\end{aligned} \tag{2.20}$$

Setting $s=t$ we obtain the equation for the filter $\pi_t(\phi(X))$:

$$\begin{aligned}\pi_s(\phi(X)) &= \pi(\phi(\eta)) + \int_0^t \pi_s\left(\text{Ł}_s\big(\phi(X^t)\big)\right)ds \\ &+ \int_0^t \Big[\pi_s\big(\phi'(X^t)\tilde{M}(t,s)\lambda(s)\big) + \pi_s\big(\phi(X^t)q\big) - \pi_s\big(\phi(X^t)\big)\pi_s(q)\Big]\,dv_s,\end{aligned} \tag{2.21}$$

since $X_t^t = X_t$.

Remark 2.2. From equation (2.21) we see that the complete solution of the problem (1.1) requires an extra equation for $\pi_t(q)$.

Remark 2.3. Applying the results obtained in Section 2.2, one can solve the filtration problem for the case where $p = p(t, W_t)$ is a polynomial of two variables t and W_t. Indeed, in this case the noise $\int_0^t p(s,W_s)\,dW_s$ is a sum of finitely many terms $\int_0^t a_{n,k}s^nW_s^k\,dW_s$. We show that the latter integral can be represented in the form (2.8), that is,

$$\int_0^t s^nW_s^k\,dW_s = \int_0^t \hat{M}_{nk}(t,s)\,dW_s^{**} + \int_0^t \hat{K}_{nk}(t,s)\,ds.$$

Integrating by parts we get

$$\int_0^t a_{nk}s^nW_s^k\,dW_s = \frac{a_{nk}}{k+1}t^nW_t^{k+1} - \frac{na_{nk}}{k+1}\int_0^t s^{n-1}W_s^{k+1}\,ds.$$

It follows from (2.8) that

$$\begin{aligned}\int_0^t s^nW_s^k\,dW_s &= \frac{1}{k+1}t^n\left(\int_0^t M_{k+1}(t,s)\,dW_s^{**} + \int_0^t K_{k+1}(t,s)\,ds\right) \\ &\quad - \frac{n}{k+1}\int_0^t s^{n-1}\left(\int_0^s M_{k+1}(s,u)\,dW_u^{**} + \int_0^s K_{k+1}(s,u)\,du\right)ds \\ &= \int_0^t \frac{1}{k+1}t^nM_{k+1}(t,s)\,dW_s^{**} \\ &\quad + \int_0^t \frac{1}{k+1}t^nK_{k+1}(t,s)\,ds \\ &\quad - \int_0^t \frac{n}{k+1}\left(\int_0^s u^nM_{k+1}(u,s)\,du\right)dW_s^{**} \\ &\quad - \int_0^t \frac{n}{k+1}\left(\int_0^s u^nK_{k+1}(u,s)\,du\right)ds.\end{aligned}$$

Thus

$$\hat{M}_{nk}(t,s) = \frac{1}{k+1} t^n M_{k+1}(t,s) - \frac{n}{k+1}\left(\int_0^s u^n M_{k+1}(u,s)\,du\right),$$
$$\hat{K}_{nk}(t,s) = \frac{1}{k+1} t^n K_{k+1}(t,s) - \frac{n}{k+1}\left(\int_0^s u^n K_{k+1}(u,s)\,du\right).$$

The latter results allow one to apply the method described in Section 2.3 to solve the filtration problem.

3. CONCLUDING REMARKS

We considered the filtration problem for systems governed by fractional Brownian motions in the case where the noise is a polynomial. Using the representation of this polynomial in terms of integrals with respect to a suitable Brownian motions we established an equation for the optimal filter of the signal X with respect to the σ-algebra generated by the observations Y.

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