MIXED EMPIRICAL POINT RANDOM PROCESSES
IN COMPACT METRIC SPACES. II

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ABSTRACT. Models of finite simple mixed empirical ordered marked point processes in compact metric spaces are studied in the paper. The processes are constructed from simple samples drawn without replacement from a population. The notion of an ordered marked point process with independent and 1-dependent marks is introduced. Examples of ordered marked point processes with independent and 1-dependent marks are given.

INTRODUCTION

A model of a finite simple mixed empirical marked point process is studied in Section 4. We assume that the space of positions $X$ is equipped with a probability measure $P_x$, while the space of marks $K = [a, b] \subset R^1$ is equipped with a probability measure $P_k$. An arbitrary trajectory of an ordered marked point process is treated as a simple random sample drawn without replacement from a population $Y = X \times K$ and according to the probability measure $P_Y = P_x \otimes P_k$. We introduce the notion of ordered marked point processes with independent and 1-dependent marks in Section 5. Examples of ordered marked point processes with independent and 1-dependent marks are given in Section 6.

4. MIXED EMPIRICAL ORDERED MARKED POINT PROCESSES
IN COMPACT METRIC SPACES

We recall the definition of a trajectory introduced in Section 1\(^1\). According to this definition, a trajectory $E^*$ of a finite simple ordered marked point process $(E^*, X^*, P^*)$ in a bounded space

$$(Y = X \times K, \mathcal{A}_Y = \mathcal{A}_X \otimes \mathcal{A}_K, \mathcal{B}_Y = \mathcal{B}_X \otimes \mathcal{B}_K)$$

is a thinned set of the Cartesian product $Y = X \times K$. If $X$ is a compact metric space and the space of marks $K$ coincides with an interval $[a, b] \subset R^1$, then the trajectory $E^*$ consists of a finite number of points, namely

$$E^* = ([x_1; k_1], [x_2; k_2], \ldots, [x_n; k_n]).$$

The phase space $Y = X \times K$ can be considered to be a compact metric space if the distance between points $[x_1; k_1]$ and $[x_2; k_2]$ is defined by

$$\rho([x_1; k_1], [x_2; k_2]) = \rho_X(x_1, x_2) + |k_1 - k_2|.$$

\(^{1}\)Editorial Note: This paper is a continuation of [7], with successive numbering of sections and formulas.
In what follows we assume that $X$ is the population for the random variable $x$ equipped with the probability measure $P_x$ defined on the $\sigma$-algebra $\mathfrak{A}_X$. The space $K$ is regarded as the population for the random variable $k$ equipped with the probability measure $P_k$ defined on the $\sigma$-algebra $\mathfrak{A}_K$. Therefore $(X, \mathfrak{A}_X, P_x)$ and $(K, \mathfrak{A}_K, P_k)$ are the sampling spaces for the random variables $x$ and $k$.

We introduce the product of the probability measures $P_\gamma = P_x \otimes P_k$ on the $\sigma$-algebra of Borel sets $\mathfrak{A}_Y = \mathfrak{A}_X \otimes \mathfrak{A}_K$ in the phase space $Y = X \times K$. Then $(Y, \mathfrak{A}_Y, P_\gamma)$ can be viewed as the sampling probability space of the two-dimensional random variable $\gamma = [x; k]$. This means that the model of a mixed empirical ordered point process studied in Section 3 of [7] can be applied to the ordered marked point process $(E^*, x^*, P^*)$ in the ordered space $(Y = X \times K, \mathfrak{A}_Y, \mathfrak{B}_Y)$ if $X$ is a compact metric space and $K = [a, b]$.

Let $G_1$ and $G_2$ be two independent stochastic experiments corresponding to the probability spaces $(X, \mathfrak{A}_X, P_x)$ and $(K, \mathfrak{A}_K, P_k)$, respectively. Then $G = (G_1, G_2)$ is a “compound” stochastic experiment corresponding to the probability space $(Y, \mathfrak{A}_Y, P_\gamma)$. A number $n \in \mathbb{Z}_+$ is drawn randomly (according to the probability distribution determined by the generating sequence $\{p_n^*\}$). Any trajectory

$$E^* = ([x_1; k_1], [x_2; k_2], \ldots, [x_n; k_n])$$

of a size $n$ of the ordered marked point process is a result of $n$ independent “compound” stochastic experiments $G = (G_1, G_2)$. Each experiment $G$ is a random sampling without replacement of a marked pair $[x; k]$ from the phase space $Y = X \times K$, where the positions $x_i$ belong to the space $X$ (this corresponds to the experiment $G_1$), while the marks $k_i$ belong to the space $K$ (this corresponds to the experiment $G_2$). Then the trajectory $E^*$ can be viewed as a simple sample $(x_i \neq x_j, k_i \neq k_j, i \neq j)$ of a finite size $n$ drawn randomly and without replacement from the population $Y = X \times K$ for the two-dimensional random variable $\gamma = [x; k]$ with the joint probability measure $P_\gamma = P_x \otimes P_k$. Thus the random elements $[x_1; k_1], \ldots, [x_i; k_i], \ldots, [x_n; k_n]$ are independent and identically distributed according to the probability measure $P_\gamma$. Moreover the mark $k_i$ does not depend on the position $x_i$ for any marked pair $[x_i; k_i]$, $i = 1, \ldots, n$. The projection of the ordered marked point process $(E^*, x^*, P^*)$ to the space of positions $X$ is a simple mixed empirical ordered point process

$$(E, x, P) = \text{pr}_X(E^*, x^*, P^*)$$

in the bounded space of positions $(X, \mathfrak{A}_X, \mathfrak{B}_X)$ (see [1]) whose trajectories are simple samples

$$E = \text{pr}_X E^* = (x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n), \quad x_i \neq x_j, \ i \neq j,$$

of size $n$ drawn from the population $X$ for the random variable $x$ with probability measure $P_x$, while the projection of the ordered marked point process to the space of marks $K$ consists of trajectories $(k_1, \ldots, k_i, \ldots, k_j, \ldots, k_n)$, $k_i \neq k_j, i \neq j$, that can be viewed as samples of size $n$ drawn randomly and without replacement from the population $K = [a, b]$ for the random variable $k$ with probability measure $P_k$.

The process $(E^*, x^*, P^*)$ is called a strong simple mixed empirical ordered marked point process in the ordered space $(Y, \mathfrak{A}_Y, \mathfrak{B}_Y)$ (see Section 1).

The joint probability distribution $P_\gamma$ of an arbitrary marked pair, denoted also by $\gamma = [x; k]$ and belonging to the trajectory $E^*$ on the metric space $Y = X \times K$, can be expressed in terms of the probability measure $P^*$ of the ordered marked point process
$(\mathcal{E}^*, \mathcal{X}^*, P^*)$. Namely,

$$
P_{\mathcal{Y}}(B_Y) = P_{[x;k]}(B_X \times B_K) = P_{[x;k]}([x;k] \in B_X \times B_K)
$$

$$
= P^*\{E^* = ([x;k]): [x;k] \in B_X \times B_K \mid \mathcal{E}^*_1\}
$$

$$
= P^*\{E^* = ([x;k]): N^*(E^*, B_X \times B_K) = 1 \mid \mathcal{E}^*_1\}
$$

for all $B_X \in \mathcal{C}_X$ and $B_K \in \mathcal{C}_K$.

If one considers relation (8) for $B_K \equiv K$ and then for $B_X \equiv X$, then one obtains the marginal probability distributions $P_x(B_X)$ and $P_k(B_K)$ of the random variable (position) $x$ on the metric space $X$ and random variable (mark) $k$ on the interval $K = [a, b]$ in terms of the probability measure $P^*$. Indeed,

$$
P_x(B_X) = P_x\{x \in B_X\} = P_{[x;k]}(B_X \times K)
$$

$$
= P^*\{E^* = ([x;k]): N^*(E^*, B_X \times K) = 1 \mid \mathcal{E}^*_1\},
$$

$$
P_k(B_K) = P_k\{k \in B_K\} = P_{[x;k]}(X \times B_K)
$$

$$
= P^*\{E^* = ([x;k]): N^*(E^*, X \times B_K) = 1 \mid \mathcal{E}^*_1\}
$$

for all $B_X \in \mathcal{C}_X$ and $B_K \in \mathcal{C}_K$.

This means that $P_x(B_X)$ is a probability distribution of the position $x$ if its mark is $k \in K$, while $P_k(B_K)$ is the probability distribution of the mark $k$ if its position is $x \in X$.

Since $P_{\mathcal{Y}}$ is the product of the measures $P_x$ and $P_k$, the probability

$$
l = P_{\mathcal{Y}}(B_X \times B_K) = P_{[x;k]}\{[x;k] \in B_X \times B_K\} = P_{[x;k]}(B_X \times K) P_{[x;k]}(X \times B_K)
$$

$$
= P_x(B_X) P_k(B_K) = pr
$$

is well defined for an arbitrary rectangle $B_X \times B_K \in \mathcal{C}_{Y}$, where

$$
p = P_x(B_X) \quad \text{and} \quad r = P_k(B_K).
$$

If $n$ is fixed ($N^*(E^*, Y) = n$, $n \in \mathbb{Z}_+$), the conditional distribution of the counting measure $N^*(E^*, B_X \times B_K)$, $B_X \times B_K \in \mathcal{C}_Y$, of the ordered marked point process $(\mathcal{E}^*, \mathcal{X}^*, P^*)$ is the binomial $(n, l)$ distribution with parameter $l = P_{\mathcal{Y}}(B_X \times B_K)$, that is,

$$
P^*\{E^*: N^*(E^*, B_X \times B_K) = k \mid \mathcal{E}^*_n\} = \binom{n}{k} l^k (1 - l)^{n-k} = \binom{n}{k} (pr)^k (1 - pr)^{n-k},
$$

where $k = 0, 1, \ldots, n$.

Using (2), (8), and conditional distribution (9), we evaluate the distribution of the counting measure $N^*(E^*, B_X \times B_K)$:

$$
P^*\{E^*: N^*(E^*, B_X \times B_K) = k\} = \sum_{n=k}^{\infty} p^*_n \binom{n}{k} (pr)^k (1 - pr)^{n-k},
$$

where $k \in \mathbb{Z}_+$ and $\{p^*_n\}$ is a generating sequence of the distribution $P^*$.

5. Finite simple ordered marked point processes with independent
   and 1-dependent marks

Consider a finite simple ordered marked point process $(\mathcal{E}^*, \mathcal{X}^*, P^*)$ in the ordered space

$$(Y = X \times K, \mathfrak{A}_Y = \mathfrak{A}_X \otimes \mathfrak{A}_K, \mathfrak{B}_Y = \mathfrak{B}_X \otimes \mathfrak{B}_K),$$

where $X$ is a compact metric space of positions. The space of marks $K = [a, b] \subset \mathbb{R}^1$ is treated as the population of marks $k$ with an absolutely continuous distribution $P_k$. Looking at the trajectory $E^* = ([x_1; k_1], \ldots, [x_n; k_n])$ of an ordered marked point process,
one can see a difference between the sequence \( E = (x_1, \ldots, x_n) \) of positions and that of marks \((k_1, \ldots, k_n)\). Namely, if \( E \) is a realization of a finite simple ordered point process

\[
(E, X, P) = \text{pr}_X(E^*, X^*, P^*)
\]

in the ordered space of positions \((X, \mathfrak{A}_X, \mathfrak{B}_X)\) (see [1]), then \((k_1, \ldots, k_n)\) is a simple random sample of a random size \(n\) drawn from the population \(K\).

Given an arbitrary number \(n_0 \in \mathbb{N}\), vectors \((x_1, \ldots, x_{n_0}) = E \subset \mathfrak{E}_{n_0}\), and sets \(B_K^{(1)}, \ldots, B_K^{(n_0)} \subset \mathfrak{B}_K\), where \(B_K^{(j)}B_K^{(j)} = \emptyset\), \(i, j = 1, \ldots, n_0, i \neq j\), one can evaluate the conditional joint distribution

\[
P_{k_1\ldots k_{n_0}}(B_K^{(1)}, \ldots, B_K^{(n_0)} | \mathcal{E}_{n_0})
\]

of marks \(k_1, \ldots, k_{n_0}\) corresponding to given positions \((x_1, \ldots, x_{n_0}) = E\), namely

\[
P_{k_1\ldots k_{n_0}}(B_K^{(1)}, \ldots, B_K^{(n_0)} | \mathcal{E}_{n_0}) = P_{k_1\ldots k_{n_0}}(B_K^{(1)}, \ldots, B_K^{(n_0)} | (x_1, \ldots, x_{n_0}))
\]

\[
= P_{k_1\ldots k_{n_0}} \left\{ k_1 \in B_K^{(1)}, \ldots, k_{n_0} \in B_K^{(n_0)} | (x_1, \ldots, x_{n_0}) \right\}
\]

\[
= P^* \left\{ E^* : N^*(E^*, X \times B_K^{(1)}) = 1, \ldots, N^*(E^*, X \times B_K^{(n_0)}) = 1 | (x_1, \ldots, x_{n_0}) \right\}
\]

\[
= \mu \left( B_K^{(1)}, \ldots, B_K^{(n_0)} \mid x_1, \ldots, x_{n_0} \right),
\]

where \(\mu(B_K^{(1)}, \ldots, B_K^{(n_0)} | x_1, \ldots, x_{n_0})\) is a probability measure defined on the \(\sigma\)-algebra \(\mathfrak{A}_{K}^{(n_0)} = \mathfrak{A}_K \times \cdots \times \mathfrak{A}_K\) \((n_0\) times) and where \(x_1, \ldots, x_{n_0}\) are elements of the \(\sigma\)-algebra \(\mathfrak{A}_X\). Putting \(B_K^{(j)} = K\) in (10) for all \(j = 1, \ldots, n_0, j \neq i\), we get the conditional probability distribution of the mark \(k_i, i = 1, \ldots, n_0\):

\[
P_{k_i}(B_K^{(i)} \mid \mathcal{E}_{n_0}) = P_{k_i} \left\{ k_i \in B_K^{(i)} | (x_1, \ldots, x_{n_0}) \right\}
\]

\[
= P_{k_1\ldots k_{i-1}k_{i+1}\ldots k_{n_0}}(K, \ldots, K, B_K^{(i)}, K, \ldots, K | (x_1, \ldots, x_{n_0}))
\]

\[
= P^* \left\{ E^* : N^*(E^*, X \times B_K^{(i)}) = 1 | (x_1, \ldots, x_{n_0}) \right\} = \mu_i \left( B_K^{(i)} \mid x_1, \ldots, x_{n_0} \right),
\]

where \(\mu_i(B_K^{(i)} | x_1, \ldots, x_{n_0})\) is a probability measure defined on the \(\sigma\)-algebra \(\mathfrak{A}_K\) and where \(x_1, \ldots, x_{n_0}\) are elements of the \(\sigma\)-algebra \(\mathfrak{A}_X\). Given an arbitrary set \(B_K^{(i)} \in \mathfrak{B}_K\), the function \(\mu_i(B_K^{(i)} | x_1, \ldots, x_{n_0})\), \(x_1, \ldots, x_{n_0} \in X\), is measurable with respect to the \(\sigma\)-algebra \(\mathfrak{A}_X\) (see [2]).

If the neighboring positions

\[(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_{n_0})\]

have little influence on the mark \(k_i, i = 1, \ldots, n_0\), that is, if the mark \(k_i\) does not depend on the neighboring positions, then

\[
P^* \left\{ E^* : N^*(E^*, X \times B_K^{(i)}) = 1 | (x_1, \ldots, x_{n_0}) \right\}
\]

\[
= P^* \left\{ E^* : N^*(E^*, X \times B_K^{(i)}) = 1 | x_i \right\} = \mu_i \left( B_K^{(i)} \mid x_i \right),
\]

where

\[
\mu_i \left( B_K^{(i)} \mid x_i \right) = P_{k_i} \left( B_K^{(i)} \mid x_i \right) = P_{k_i} \left\{ k_i \in B_K^{(i)} \mid x_i \right\}
\]

is the probability measure of the transformation of the space \(X\) to the space \(K\) (see [2]), that is, \(\mu_i\) is the probability distribution of the mark \(k_i\) that depends on its position \(x_i\).
Also if the influence of positions \((x_1, \ldots, x_{n_0})\) on the mark \(k_i, i = 1, \ldots, n_0\), is rather inessential, that is, if the mark \(k_i\) does not depend on positions \((x_1, \ldots, x_{n_0})\), then
\[
\begin{align*}
P^* \left\{ E^* : N^* \left( E^*, X \times B_K^{(i)} \right) = 1 \mid (x_1, \ldots, x_{n_0}) \right\} &= P^* \left\{ E^* : N^* \left( E^*, X \times B_K^{(i)} \right) = 1 \right\} = \mu_i \left( B_K^{(i)} \right),
\end{align*}
\]
where
\[
\mu_i \left( B_K^{(i)} \right) = P_{k_i} \left( B_K^{(i)} \right) = P_{k_i} \left\{ k_i \in B_K^{(i)} \right\}
\]
is a probability distribution of the mark \(k_i, i = 1, \ldots, n_0\).

Some applications of marked point processes in stochastic geometry are based on the assumption that the marks are identically distributed random variables (see [4][5]). Note that this property is involved in the construction of a mixed empirical ordered marked point process (see Section 4). Moreover
\[
\mu_i \left( B_K^{(i)} \mid x_1, \ldots, x_{n_0} \right) = \mu \left( B_K^{(i)} \mid x_1, \ldots, x_{n_0} \right), \quad i = 1, \ldots, n_0,
\]
\[
\mu_i \left( B_K^{(i)} \mid x_i \right) = \mu \left( B_K^{(i)} \mid x_i \right), \quad i = 1, \ldots, n_0,
\]
\[
\mu_i \left( B_K^{(i)} \right) = \mu \left( B_K^{(i)} \right), \quad i = 1, \ldots, n_0.
\]

**Definition 14.** A finite simple ordered marked point process \((E^*, X^*, P^*)\) in a bounded space \((X \times K, \mathfrak{A}_X \otimes \mathfrak{A}_K, \mathfrak{B}_X \otimes \mathfrak{B}_K)\) is called a process with independent \(\mu\)-marks if
\[
B_K^{(1)}, \ldots, B_K^{(n_0)} \subset \mathfrak{B}_K
\]
for all \(n_0 \in \mathbb{N}\) where
\[
B_K^{(i)} B_K^{(j)} = \emptyset, \quad i, j = 1, \ldots, n_0, \quad i \neq j,
\]
and the marks \(k_1, \ldots, k_{n_0}\) of a random \(n_0\)-dimensional vector \(E^* = (x_i; k_i) : i = 1, \ldots, n_0\) are jointly independent random variables in the space \(K\), have the probability distribution \(\mu(B_K)\), and are such that \(k_1, \ldots, k_{n_0}\) do not depend on their positions \((x_1, \ldots, x_{n_0})\) in the space \(X\),
\[
\mu \left( B_K^{(1)}, \ldots, B_K^{(n_0)} \mid x_1, \ldots, x_{n_0} \right) = \prod_{i=1}^{n_0} \mu \left( B_K^{(i)} \right).
\]

**Definition 15.** A finite simple ordered marked point process \((E^*, X^*, P^*)\) in a bounded space \((X \times K, \mathfrak{A}_X \otimes \mathfrak{A}_K, \mathfrak{B}_X \otimes \mathfrak{B}_K)\) is called a process with 1-dependent \(\mu\)-marks if
\[
B_K^{(1)}, \ldots, B_K^{(n_0)} \subset \mathfrak{B}_K
\]
for all \(n_0 \in \mathbb{N}\) where
\[
B_K^{(i)} B_K^{(j)} = \emptyset, \quad i, j = 1, \ldots, n_0, \quad i \neq j,
\]
the marks \(k_1, \ldots, k_{n_0}\) of a random \(n_0\)-dimensional vector \(E^* = (x_i; k_i) : i = 1, \ldots, n_0\) are conditionally jointly independent random variables [3]:
\[
\mu \left( B_K^{(1)}, \ldots, B_K^{(n_0)} \mid x_1, \ldots, x_{n_0} \right) = \prod_{i=1}^{n_0} \mu_i \left( B_K^{(i)} \mid x_i \right) = \prod_{i=1}^{n_0} \mu \left( B_K^{(i)} \mid x_i \right)
\]
such that every mark \(k_i, i = 1, \ldots, n_0\), has conditional probability measure \(\mu_i \left( B_K^{(i)} \mid x_i \right)\) that depends on a single parameter, namely on its position \(x_i \in X\). Thus this conditional probability measure belongs to a one-parameter family of probability distributions \(\{\mu(B_K \mid x) : x \in X\}\):
\[
\mu_i \left( B_K^{(i)} \mid x_i \right) = \mu \left( B_K^{(i)} \mid x_i \right).
\]
We regard an ordered semispherical segment stochastic process defined on the unit two-dimensional Euclidean sphere as a result of the following random experiment. Let 

\[ B^{(1)}_K, \ldots, B^{(n_0)}_K \subseteq B_K \]

for all \( n_0 \in \mathbb{N} \) where

\[ B^{(i)}_K B^{(j)}_K = \emptyset, \quad i, j = 1, \ldots, n_0, \quad i \neq j, \]

the marks \( k_1, \ldots, k_{n_0} \) of a random \( n_0 \)-dimensional vector \( E^* = ([x_i; k_i]: i = 1, \ldots, n_0) \) are conditionally jointly independent random variables \([3]\),

\[ \mu\left(B^{(1)}_K, \ldots, B^{(n_0)}_K | x_1, \ldots, x_{n_0}\right) = \prod_{i=1}^{n_0} \mu_i\left(B^{(i)}_K | x_i\right), \]

and the conditional probability distribution \( \mu_i\left(B^{(i)}_K | x_i\right) \) of any mark \( k_i, i = 1, \ldots, n_0, \) in the space \( K \) depends only on its position \( x_i \in X \).

6. Examples of ordered marked point processes with independent and 1-dependent marks

Example 1 \([2]\). It is often the case in hydrology that atmospheric precipitates lead to the peak exceedances above the basic level of a river. Let \((x_1, \ldots, x_n)\) be the random moments of peak exceedances above the basic level in the interval \( X = [0, T] \subseteq \mathbb{R}_+^1 \). One can assume that the points \((x_1, \ldots, x_n)\) form a realization \( E \) of some ordered point process \((\mathcal{E}, \mathcal{X}, \mathcal{P})\) in the space of positions \( X \) and that \( n \) is an integer nonnegative random variable (the number of peak exceedances above the basic level in the interval \( X \)) that does not depend on \((x_1, \ldots, x_n)\). The height above the basic flow at the moment \( x_i \) is denoted by \( k_i, i = 1, \ldots, n \). We also assume that the flow drops below the base level between two successive exceedances. It is clear that \( k_i \) treated as a random variable (mark) assuming values in the space of marks \( K = (0, b) \) depends on the position \( x_i \) only, that is, \( k_i = k_i(x_i) \), and does not depend on other exceedances \( k_{i-1}, k_{i+1}, \ldots, k_n \). The pairs \((x_i; k_i)\) form a trajectory \( E^* = ([x_1; k_1], \ldots, [x_n; k_n]) \) of some ordered marked point process \((\mathcal{E}^*, \mathcal{X}^*, \mathcal{P}^*)\) in the phase space \( X \times K = [0, T] \times (0, b) \). If all marks \( k_i \) have the same conditional probability measure \( \mu_i\left(B^{(i)}_K | x_i\right) \) belonging to a one parameter family of probability distributions \( \mu_i\left(B^{(i)}_K | x_i\right) = \mu\left(B^{(i)}_K | x_i\right), \quad i = 1, 2, \ldots, \) \( \) then \((\mathcal{E}^*, \mathcal{X}^*, \mathcal{P}^*)\) is an ordered marked point process with 1-dependent \( 
mu \)-marks.

Example 2 (Mixed empirical Poisson semispherical segment process). Denote by \( \mathcal{A} \) an ordered semispherical segment stochastic process defined on the unit two-dimensional Euclidean sphere \( S^2 \) with trajectories

\[ E_A = (Q_1(u_1(\varphi_1, \theta_1), a_1), \ldots, Q_n(u_n(\varphi_n, \theta_n), a_n)\), \]

where \( u_i(\varphi_i, \theta_i) \) are the centers of segments, \((\varphi_i, \theta_i)\) are spherical coordinates of the centers, \( a_i \) are their angle diameters, \( a_i \in K = [0, A] \), and \( A << \pi, i = 1, \ldots, n \) (see \([3]\)). We regard an ordered semispherical segment stochastic process \( \mathcal{A} \) as an ordered marked point stochastic process \((E_A, \mathcal{X}_A, \mathcal{P}_A)\) with trajectories \( E_A = ([u_1; a_1], \ldots, [u_n; a_n]) \) in the ordered space \((S^2 \times K, \mathfrak{A}_{S^2} \otimes \mathfrak{A}_K, \mathfrak{B}_{S^2} \otimes \mathfrak{B}_K)\). We assume that every trajectory \( E_A \) is obtained as a result of the following random experiment. Let \( G_1 \) and \( G_2 \) be two independent random experiments corresponding to the probability spaces \((S^2, \mathfrak{A}_{S^2}, \mathcal{P}_u)\) and \((K, \mathfrak{A}_K, \mathcal{P}_u)\), respectively. Then \( G = (G_1, G_2) \) is a “compound” random experiment corresponding to the probability space \((S^2 \times K, \mathfrak{A}_{S^2} \otimes \mathfrak{A}_K, \mathcal{P}_u \otimes \mathcal{P}_u)\) (see Section 4).
A number \( n \in \mathbb{Z}_+ \) is drawn randomly according to the probability distribution generated by the Poisson sequence
\[
\left\{ p_n : p_n = \frac{\lambda^n}{n!} e^{-\lambda}, \lambda > 0 \right\}
\]
(see Section 3). Then every trajectory \( E^*_A \) of the ordered marked point process of size \( n \) is obtained as a result of \( n \) independent repetitions of the “compound” experiment \( G \) being the random sampling without replacement of a marked pair \([u_i; a_i]\) from the phase space \( S^2 \times K \): the positions \( u_i \) are drawn from the space \( S^2 \) (experiment \( G_1 \)), while the marks \( k_i \) are drawn from the space \( K \) (experiment \( G_2 \)). Thus the process \((E^*_A, X^*_A, P^*_A)\) is a finite strictly simple mixed empirical Poisson ordered marked point process with independent \( P^*_A \)-marks.

The stochastic process \( \mathcal{A} \) corresponding to the ordered marked point process
\[
(E^*_A, X^*_A, P^*_A)
\]
is called a mixed empirical Poisson stochastic process of segments.

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