MIXED EMPIRICAL POINT RANDOM PROCESSES
IN COMPACT METRIC SPACES. II

YU. I. PETUNIN AND M. G. SENEUKO

Abstract. Models of finite simple mixed empirical ordered marked point processes in compact metric spaces are studied in the paper. The processes are constructed from simple samples drawn without replacement from a population. The notion of an ordered marked point process with independent and 1-dependent marks is introduced. Examples of ordered marked point processes with independent and 1-dependent marks are given.

Introduction
A model of a finite simple mixed empirical marked point process is studied in Section 4. We assume that the space of positions $X$ is equipped with a probability measure $P_x$, while the space of marks $K = [a, b] \subset \mathbb{R}^1$ is equipped with a probability measure $P_k$. An arbitrary trajectory of an ordered marked point process is treated as a simple random sample drawn without replacement from a population $Y = X \times K$ and according to the probability measure $P_Y = P_x \otimes P_k$. We introduce the notion of ordered marked point processes with independent and 1-dependent marks in Section 5. Examples of ordered marked point processes with independent and 1-dependent marks are given in Section 6.

4. Mixed empirical ordered marked point processes
in compact metric spaces

We recall the definition of a trajectory introduced in Section 1. According to this definition, a trajectory $E^*$ of a finite simple ordered marked point process $(E^*, X^*, P^*)$ in a bounded space

$$(Y = X \times K, \mathcal{A}_Y = \mathcal{A}_X \otimes \mathcal{A}_K, \mathcal{B}_Y = \mathcal{B}_X \otimes \mathcal{B}_K)$$

is a thinned set of the Cartesian product $Y = X \times K$. If $X$ is a compact metric space and the space of marks $K$ coincides with an interval $[a, b] \subset \mathbb{R}^1$, then the trajectory $E^*$ consists of a finite number of points, namely

$$E^* = ([x_1; k_1], [x_2; k_2], \ldots, [x_n; k_n]).$$

The phase space $Y = X \times K$ can be considered to be a compact metric space if the distance between points $[x_1; k_1]$ and $[x_2; k_2]$ is defined by

$$\rho([x_1; k_1], [x_2; k_2]) = \rho_X(x_1, x_2) + |k_1 - k_2|.$$
In what follows we assume that \( X \) is the population for the random variable \( x \) equipped with the probability measure \( P_x \) defined on the \( \sigma \)-algebra \( \mathfrak{A}_X \). The space \( K \) is regarded as the population for the random variable \( k \) equipped with the probability measure \( P_k \) defined on the \( \sigma \)-algebra \( \mathfrak{A}_K \). Therefore \((X, \mathfrak{A}_X, P_x)\) and \((K, \mathfrak{A}_K, P_k)\) are the sampling spaces for the random variables \( x \) and \( k \).

We introduce the product of the probability measures \( P_\mathfrak{Y} = P_x \otimes P_k \) on the \( \sigma \)-algebra of Borel sets \( \mathfrak{A}_Y = \mathfrak{A}_X \otimes \mathfrak{A}_K \) in the phase space \( Y = X \times K \). Then \((Y, \mathfrak{A}_Y, P_\mathfrak{Y})\) can be viewed as the sampling probability space of the two-dimensional random variable \( \mathfrak{y} = [x; k] \). This means that the model of a mixed empirical ordered point process studied in Section 3 of [7] can be applied to the ordered marked point process \((\mathcal{E}^*, \mathfrak{X}^*, P^*)\) in the ordered space \((Y = X \times K, \mathfrak{A}_Y, \mathfrak{B}_Y)\) if \( X \) is a compact metric space and \( K = [a, b] \).

Let \( G_1 \) and \( G_2 \) be two independent stochastic experiments corresponding to the probability spaces \((X, \mathfrak{A}_X, P_x)\) and \((K, \mathfrak{A}_K, P_k)\), respectively. Then \( \mathcal{G} = (G_1, G_2) \) is a “compound” stochastic experiment corresponding to the probability space \((Y, \mathfrak{A}_Y, P_\mathfrak{Y})\). A number \( n \in \mathbb{Z}_+ \) is drawn randomly (according to the probability distribution determined by the generating sequence \( \{p_n\} \)). Any trajectory

\[
E^* = ([x_1; k_1], \ldots, [x_i; k_i], \ldots, [x_j; k_j], \ldots, [x_n; k_n])
\]

of a size \( n \) of the ordered marked point process is a result of \( n \) independent “compound” stochastic experiments \( \mathcal{G} = (G_1, G_2) \). Each experiment \( G_i \) is a random sampling without replacement of a marked pair \([x; k]\) from the phase space \( Y = X \times K \), where the positions \( x_i \) belong to the space \( X \) (this corresponds to the experiment \( G_1 \)), while the marks \( k_i \) belong to the space \( K \) (this corresponds to the experiment \( G_2 \)). Then the trajectory \( E^* \) can be viewed as a simple sample \((x_i \neq x_j, k_i \neq k_j, i \neq j)\) of a finite size \( n \) drawn randomly and without replacement from the population \( Y = X \times K \) for the two-dimensional random variable \( \mathfrak{y} = [x; k] \) with the joint probability measure \( P_\mathfrak{Y} = P_x \otimes P_k \).

Thus the random elements

\[
[x_1; k_1], \ldots, [x_i; k_i], \ldots, [x_n; k_n]
\]

are independent and identically distributed according to the probability measure \( P_\mathfrak{Y} \). Moreover the mark \( k_i \) does not depend on the position \( x_i \) for any marked pair \([x_i; k_i]\), \( i = 1, \ldots, n \). The projection of the ordered marked point process \((\mathcal{E}^*, \mathfrak{X}^*, P^*)\) to the space of positions \( X \) is a simple mixed empirical ordered point process

\[
(\mathcal{E}, \mathfrak{X}, P) = \text{pr}_X((\mathcal{E}^*, \mathfrak{X}^*, P^*))
\]

in the bounded space of positions \((X, \mathfrak{A}_X, \mathfrak{B}_X)\) (see 1) whose trajectories are simple samples

\[
E = \text{pr}_X E^* = (x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n), \quad x_i \neq x_j, \ i \neq j,
\]

of size \( n \) drawn from the population \( X \) for the random variable \( x \) with probability measure \( P_x \), while the projection of the ordered marked point process to the space of marks \( K \) consists of trajectories \((k_1, \ldots, k_i, \ldots, k_j, \ldots, k_n)\), \( k_i \neq k_j, \ i \neq j \), that can be viewed as samples of size \( n \) drawn randomly and without replacement from the population \( K = [a, b] \) for the random variable \( k \) with probability measure \( P_k \).

The process \((\mathcal{E}^*, \mathfrak{X}^*, P^*)\) is called a strong simple mixed empirical ordered point process in the ordered space \((Y, \mathfrak{A}_Y, \mathfrak{B}_Y)\) (see Section 1).

The joint probability distribution \( P_\mathfrak{Y} \) of an arbitrary marked pair, denoted also by \( \mathfrak{y} = [x; k] \) and belonging to the trajectory \( E^* \) on the metric space \( Y = X \times K \), can be expressed in terms of the probability measure \( P^* \) of the ordered marked point process

\[
(\mathcal{E}, \mathfrak{X}, P) = \text{pr}_X((\mathcal{E}^*, \mathfrak{X}^*, P^*))
\]
\((\mathcal{E}^*, \mathcal{X}^*, P^*)\). Namely,

\[
P^*_{B_Y}(B_Y) = P_{[x;k]}(B_X \times B_K) = P_{[x;k]}([x; k] \in B_X \times B_K)
\]

(8)

\[
= P^*\{E^* = ([x; k]): [x; k] \in B_X \times B_K | \mathcal{E}^*_1\}
\]

\[
= P^*\{E^* = ([x; k]): N^*(E^*, B_X \times B_K) = 1 | \mathcal{E}^*_1\}
\]

for all \(B_X \in \mathfrak{C}_X\) and \(B_K \in \mathfrak{C}_K\).

If one considers relation (8) for \(B_K \equiv K\) and then for \(B_X \equiv X\), then one obtains the marginal probability distributions \(P_x(B_X)\) and \(P_k(B_K)\) of the random variable (position) \(x\) on the metric space \(X\) and random variable (mark) \(k\) on the interval \(K = [a, b]\) in terms of the probability measure \(P^*\). Indeed,

\[
P_x(B_X) = P_x\{x \in B_X\} = P_{[x;k]}(B_X \times K)
\]

\[
= P^*\{E^* = ([x; k]): N^*(E^*, B_X \times K) = 1 | \mathcal{E}^*_1\},
\]

\[
P_k(B_K) = P_k\{k \in B_K\} = P_{[x;k]}(X \times B_K)
\]

\[
= P^*\{E^* = ([x; k]): N^*(E^*, X \times B_K) = 1 | \mathcal{E}^*_1\}
\]

for all \(B_X \in \mathfrak{C}_X\) and \(B_K \in \mathfrak{C}_K\).

This means that \(P_x(B_X)\) is a probability distribution of the position \(x\) if its mark is \(k \in K\), while \(P_k(B_K)\) is the probability distribution of the mark \(k\) if its position is \(x \in X\).

Since \(P_{\mathfrak{M}}\) is the product of the measures \(P_x\) and \(P_k\), the probability

\[
l = P^*_{\mathfrak{M}}(B_X \times B_K) = P_{[x;k]}\{[x; k] \in B_X \times B_K\} = P_{[x;k]}(B_X \times K)P_{[x;k]}(X \times B_K)
\]

\[
= P_x(B_X)P_k(B_K) = pr
\]

is well defined for an arbitrary rectangle \(B_X \times B_K \in \mathfrak{C}_Y\), where

\[
p = P_x(B_X) \quad \text{and} \quad r = P_k(B_K).
\]

If \(n\) is fixed \((N^*(E^*, Y) = n, n \in Z^+)\), the conditional distribution of the counting measure \(N^*(E^*, B_X \times B_K)\), \(B_X \times B_K \in \mathfrak{C}_Y\), of the ordered marked point process \((\mathcal{E}^*, \mathcal{X}^*, P^*)\) is the binomial \(B(n, l)\) distribution with parameter \(l = P^*_{\mathfrak{M}}(B_X \times B_K)\), that is,

\[
P^*\{E^*: N^*(E^*, B_X \times B_K) = k | \mathcal{E}^*_n\} = \binom{n}{k}l^k(1-l)^{n-k} = \binom{n}{k}(pr)^k(1-pr)^{n-k},
\]

where \(k = 0, 1, \ldots, n\).

Using (2), (8), and conditional distribution (9), we evaluate the distribution of the counting measure \(N^*(E^*, B_X \times B_K)\):

\[
P^*\{E^*: N^*(E^*, B_X \times B_K) = k\} = \sum_{n=k}^{\infty} p_n^* \binom{n}{k}(pr)^k(1-pr)^{n-k},
\]

where \(k \in Z^+\) and \(\{p_n^*\}\) is a generating sequence of the distribution \(P^*\).

5. Finite simple ordered marked point processes with independent and 1-dependent marks

Consider a finite simple ordered marked point process \((\mathcal{E}^*, \mathcal{X}^*, P^*)\) in the ordered space

\[
(Y = X \times K, \mathfrak{A}_Y = \mathfrak{A}_X \otimes \mathfrak{A}_K, \mathfrak{B}_Y = \mathfrak{B}_X \otimes \mathfrak{B}_K),
\]

where \(X\) is a compact metric space of positions. The space of marks \(K = [a, b] \subset R^1\) is treated as the population of marks \(k\) with an absolutely continuous distribution \(P_k\).

Looking at the trajectory \(E^* = ([x_1; k_1], \ldots, [x_n; k_n])\) of an ordered marked point process,
one can see a difference between the sequence \( E = (x_1, \ldots, x_n) \) of positions and that of marks \((k_1, \ldots, k_n)\). Namely, if \( E \) is a realization of a finite simple ordered point process

\[
(E, \mathcal{X}, P) = \Pr_X (E^*, \mathcal{X}^*, P^*)
\]

in the ordered space of positions \((X, \mathfrak{A}_X, \mathfrak{B}_X)\) (see [3]), then \((k_1, \ldots, k_n)\) is a simple random sample of a random size \( n \) drawn from the population \( K \).

Given an arbitrary number \( n_0 \in \mathbb{N} \), vectors \((x_1, \ldots, x_{n_0}) = E \subset \mathcal{E}_{n_0}\), and sets \( B_K^{(1)}, \ldots, B_K^{(n_0)} \subset \mathfrak{B}_K \), where \( B_K^{(i)} = \varnothing \), \( i, j = 1, \ldots, n_0 \), \( i \neq j \), one can evaluate the conditional joint distribution

\[
P_{k_1 \ldots k_{n_0}} (B_K^{(1)}, \ldots, B_K^{(n_0)} | \mathcal{E}_{n_0})
\]

of marks \( k_1, \ldots, k_{n_0} \) corresponding to given positions \((x_1, \ldots, x_{n_0}) = E\), namely

\[
P_{k_1 \ldots k_{n_0}} (B_K^{(1)}, \ldots, B_K^{(n_0)} | \mathcal{E}_{n_0}) = P_{k_1 \ldots k_{n_0}} \left( B_K^{(1)}, \ldots, B_K^{(n_0)} | (x_1, \ldots, x_{n_0}) \right)
\]

\[
= P_{k_1 \ldots k_{n_0}} \left\{ k_1 \in B_K^{(1)}, \ldots, k_{n_0} \in B_K^{(n_0)} | (x_1, \ldots, x_{n_0}) \right\}
\]

\[
= P^* \left\{ E^*: N^*(E^*, X \times B_K^{(i)}) = 1, \ldots,
\right. \]

\[
N^*(E^*, X \times B_K^{(n_0)}) = 1 | (x_1, \ldots, x_{n_0}) \}
\]

\[
= \mu \left( B_K^{(1)}, \ldots, B_K^{(n_0)} | x_1, \ldots, x_{n_0} \right),
\]

where \( \mu (B_K^{(1)}, \ldots, B_K^{(n_0)} | x_1, \ldots, x_{n_0}) \) is a probability measure defined on the \( \sigma \)-algebra \( \mathfrak{A}_K^{(n_0)} = \mathfrak{A}_K \times \cdots \times \mathfrak{A}_K \) \( n_0 \) times and where \( x_1, \ldots, x_{n_0} \) are elements of the \( \sigma \)-algebra \( \mathfrak{A}_X \). Putting \( B_K^{(j)} \equiv K \) in (10) for all \( j = 1, \ldots, n_0 \), \( j \neq i \), we get the conditional probability distribution of the mark \( k_i, i = 1, \ldots, n_0 \):

\[
P_{k_i} \left( B_K^{(i)} | \mathcal{E}_{n_0} \right) = P_{k_i} \left\{ k_i \in B_K^{(i)} | (x_1, \ldots, x_{n_0}) \right\}
\]

\[
= P_{k_1 \ldots k_{i-1}k_{i+1}\ldots k_{n_0}} \left( K, \ldots, K, B_K^{(i)}, K, \ldots, K | (x_1, \ldots, x_{n_0}) \right)
\]

\[
= P^* \left\{ E^*: N^*(E^*, X \times B_K^{(i)}) = 1 | (x_1, \ldots, x_{n_0}) \right\} = \mu_i \left( B_K^{(i)} | x_1, \ldots, x_{n_0} \right),
\]

where \( \mu_i (B_K^{(i)} | x_1, \ldots, x_{n_0}) \) is a probability measure defined on the \( \sigma \)-algebra \( \mathfrak{A}_K \) and where \( x_1, \ldots, x_{n_0} \) are elements of the \( \sigma \)-algebra \( \mathfrak{A}_X \). Given an arbitrary set \( B_K^{(i)} \in \mathfrak{B}_K \), the function \( \mu_i (B_K^{(i)} | x_1, \ldots, x_{n_0}) \), \( x_1, \ldots, x_{n_0} \in X\), is measurable with respect to the \( \sigma \)-algebra \( \mathfrak{A}_X \) (see [2]).

If the neighboring positions

\[
(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_{n_0})
\]

have little influence on the mark \( k_i, i = 1, \ldots, n_0 \), that is, if the mark \( k_i \) does not depend on the neighboring positions, then

\[
P^* \left\{ E^*: N^*(E^*, X \times B_K^{(i)}) = 1 | (x_1, \ldots, x_{n_0}) \right\}
\]

\[
= P^* \left\{ E^*: N^*(E^*, X \times B_K^{(i)}) = 1 | x_i \right\} = \mu_i \left( B_K^{(i)} | x_i \right),
\]

where

\[
\mu_i \left( B_K^{(i)} | x_i \right) = P_{k_i} \left( B_K^{(i)} | x_i \right) = P_{k_i} \left\{ k_i \in B_K^{(i)} | x_i \right\}
\]

is the probability measure of the transformation of the space \( X \) to the space \( K \) (see [2]), that is, \( \mu_i \) is the probability distribution of the mark \( k_i \) that depends on its position \( x_i \).
Also if the influence of positions \((x_1, \ldots, x_{n_0})\) on the mark \(k_i, i = 1, \ldots, n_0\), is rather inessential, that is, if the mark \(k_i\) does not depend on positions \((x_1, \ldots, x_{n_0})\), then
\[
P^* \{ E^* : N^* (E^*, X \times B_K^{(i)}) = 1 | (x_1, \ldots, x_{n_0}) \} = P^* \{ E^* : N^* (E^*, X \times B_K^{(i)}) = 1 \} = \mu_i (B_K^{(i)}),
\]
where
\[
\mu_i (B_K^{(i)} | x_1, \ldots, x_{n_0}) = \mu (B_K^{(i)} | x_1, \ldots, x_{n_0}), \quad i = 1, \ldots, n_0,
\]
\[
\mu_i (B_K^{(i)} | x_i) = \mu (B_K^{(i)} | x_i), \quad i = 1, \ldots, n_0,
\]
\[
\mu_i (B_K^{(i)}) = \mu (B_K^{(i)}), \quad i = 1, \ldots, n_0.
\]

**Definition 14.** A finite simple ordered marked point process \((E^*, X^*, P^*)\) in a bounded space \((X \times K, \mathfrak{A}_X \otimes \mathfrak{A}_K, \mathfrak{B}_X \otimes \mathfrak{B}_K)\) is called a process with independent \(\mu\)-marks if
\[
B_K^{(1)}, \ldots, B_K^{(n_0)} \subset \mathfrak{B}_K
\]
for all \(n_0 \in \mathbb{N}\) where
\[
B_K^{(i)} B_K^{(j)} = \emptyset, \quad i, j = 1, \ldots, n_0, \ i \neq j,
\]
and the marks \(k_1, \ldots, k_{n_0}\) of a random \(n_0\)-dimensional vector \(E^* = ([x_i; k_i]: i = 1, \ldots, n_0)\) are jointly independent random variables in the space \(K\), have the probability distribution \(\mu(B_K)\), and are such that \(k_1, \ldots, k_{n_0}\) do not depend on their positions \((x_1, \ldots, x_{n_0})\) in the space \(X\),
\[
\mu (B_K^{(1)}, \ldots, B_K^{(n_0)} | x_1, \ldots, x_{n_0}) = \prod_{i=1}^{n_0} \mu (B_K^{(i)}).
\]

**Definition 15.** A finite simple ordered marked point process \((E^*, X^*, P^*)\) in a bounded space \((X \times K, \mathfrak{A}_X \otimes \mathfrak{A}_K, \mathfrak{B}_X \otimes \mathfrak{B}_K)\) is called a process with 1-dependent \(\mu\)-marks if
\[
B_K^{(1)}, \ldots, B_K^{(n_0)} \subset \mathfrak{B}_K
\]
for all \(n_0 \in \mathbb{N}\) where
\[
B_K^{(i)} B_K^{(j)} = \emptyset, \quad i, j = 1, \ldots, n_0, \ i \neq j,
\]
the marks \(k_1, \ldots, k_{n_0}\) of a random \(n_0\)-dimensional vector \(E^* = ([x_i; k_i]: i = 1, \ldots, n_0)\) are conditionally jointly independent random variables \([3]:\)
\[
\mu (B_K^{(1)}, \ldots, B_K^{(n_0)} | x_1, \ldots, x_{n_0}) = \prod_{i=1}^{n_0} \mu_i (B_K^{(i)} | x_i) = \prod_{i=1}^{n_0} \mu (B_K^{(i)} | x_i)
\]
such that every mark \(k_i, i = 1, \ldots, n_0\), has conditional probability measure \(\mu_i (B_K^{(i)} | x_i)\) that depends on a single parameter, namely on its position \(x_i \in X\). Thus this conditional probability measure belongs to a one-parameter family of probability distributions \(\{\mu(B_K | x): x \in X\}\):
\[
\mu_i (B_K^{(i)} | x_i) = \mu (B_K^{(i)} | x_i).
\]
Definition 16. A finite simple ordered marked point process \((\mathcal{E}^*, \mathcal{X}^*, P^*)\) in a bounded space \((X \times K, \mathcal{A}_X \otimes \mathcal{A}_K, \mathcal{B}_X \otimes \mathcal{B}_K)\) is called a process with 1-dependent marks if

\[
B^{(1)}_K, \ldots, B^{(n_0)}_K \subset B_K
\]

for all \(n_0 \in \mathbb{N}\) where

\[
B^{(i)}_K B^{(j)}_K = \emptyset, \quad i, j = 1, \ldots, n_0, \ i \neq j,
\]

the marks \(k_1, \ldots, k_{n_0}\) of a random \(n_0\)-dimensional vector \(E^* = ([x_i; k_i]: i = 1, \ldots, n_0)\) are conditionally jointly independent random variables \([3]\),

\[
\mu\left(B^{(1)}_K, \ldots, B^{(n_0)}_K | x_1, \ldots, x_{n_0}\right) = \prod_{i=1}^{n_0} \mu_i\left(B^{(i)}_K | x_i\right),
\]

and the conditional probability distribution \(\mu_i(B^{(i)}_K | x_i)\) of any mark \(k_i, i = 1, \ldots, n_0\), in the space \(K\) depends only on its position \(x_i \in X\).

6. Examples of ordered marked point processes with independent and 1-dependent marks

Example 1 ([2]). It is often the case in hydrology that atmospheric precipitates lead to the peak exceedances above the basic level of a river. Let \((x_1, \ldots, x_n)\) be the random moments of peak exceedances above the basic level in the interval \(X = [0, T] \subset \mathbb{R}^+_1\). One can assume that the points \((x_1, \ldots, x_n)\) form a realization \(E\) of some ordered point process \((\mathcal{E}, \mathcal{X}, P)\) in the space of positions \(X\) and that \(n\) is an integer nonnegative random variable (the number of peak exceedances above the basic level in the interval \(X\)) that does not depend on \((x_1, \ldots, x_n)\). The height above the basic flow at the moment \(x_i\) is denoted by \(k_i, i = 1, \ldots, n\). We also assume that the flow drops below the base level between two successive exceedances. It is clear that \(k_i\) treated as a random variable (mark) assuming values in the space of marks \(K = [0, b]\) depends on the position \(x_i\) only, that is, \(k_i = k_i(x_i)\), and does not depend on other exceedances \(k_{i-1}, \ldots, k_{i+1}\). The pairs \([x_i; k_i]\) form a trajectory \(E^* = ([x_i; k_i], \ldots, [x_n; k_n])\) of some ordered marked point process \((\mathcal{E}^*, \mathcal{X}^*, P^*)\) in the phase space \(X \times K = [0, T] \times [0, b]\). If all marks \(k_i\) have the same conditional probability measure \(\mu_i((B^{(i)}_K | x_i)\) belonging to a one parameter family of probability distributions \(\mu_i(B^{(i)}_K | x_i) = \mu(B^{(i)}_K | x_i), i = 1, 2, \ldots,\) then \((\mathcal{E}^*, \mathcal{X}^*, P^*)\) is an ordered marked point process with 1-dependent \(\mu\)-marks.

Example 2 (Mixed empirical Poisson semispherical segment process). Denote by \(\mathcal{A}\) an ordered semispherical segment stochastic process defined on the unit two-dimensional Euclidean sphere \(S^2\) with trajectories

\[
E_A = (Q_1(u_1(\varphi_1, \theta_1), a_1), \ldots, Q_n(u_n(\varphi_n, \theta_n), a_n)),
\]

where \(u_i(\varphi_i, \theta_i)\) are the centers of segments, \((\varphi_i, \theta_i)\) are spherical coordinates of the centers, \(a_i\) are their angle diameters, \(a_i \in K = [0, A]\), and \(A \ll \pi, i = 1, \ldots, n\) (see \([3]\)). We regard an ordered semispherical segment stochastic process \(\mathcal{A}\) as an ordered marked point stochastic process \((\mathcal{E}^*_A, \mathcal{X}^*_A, P^*_A)\) with trajectories \(E^*_A = ([u_1; a_1], \ldots, [u_n; a_n])\) in the ordered space \((S^2 \times K, \mathcal{A}_{S^2} \otimes \mathcal{A}_K, \mathcal{B}_{S^2} \otimes \mathcal{B}_K)\). We assume that every trajectory \(E^*_A\) is obtained as a result of the following random experiment. Let \(G_1\) and \(G_2\) be two independent random experiments corresponding to the probability spaces \((S^2, \mathcal{A}_{S^2}, P_u)\) and \((K, \mathcal{A}_K, P_u)\), respectively. Then \(G = (G_1, G_2)\) is a “compound” random experiment corresponding to the probability space \((S^2 \times K, \mathcal{A}_{S^2} \otimes \mathcal{A}_K, P_u \otimes P_u)\) (see Section 4).
A number \( n \in \mathbb{Z}_+ \) is drawn randomly according to the probability distribution generated by the Poisson sequence
\[
\left\{ p_n : p_n = \frac{\lambda^n}{n!} e^{-\lambda}, \lambda > 0 \right\}
\]
(see Section 3). Then every trajectory \( E^*_A \) of the ordered marked point process of size \( n \) is obtained as a result of \( n \) independent repetitions of the “compound” experiment \( G \) being the random sampling without replacement of a marked pair \([u_i; a_i]\) from the phase space \( S^2 \times K \): the positions \( u_i \) are drawn from the space \( S^2 \) (experiment \( G_1 \)), while the marks \( k_i \) are drawn from the space \( K \) (experiment \( G_2 \)). Thus the process \((E^*_A, X^*_A, P^*_A)\) is a finite strictly simple mixed empirical Poisson ordered marked point process with independent \( P_a \)-marks.

The stochastic process \( A \) corresponding to the ordered marked point process
\[
(E^*_A, X^*_A, P^*_A)
\]
is called a mixed empirical Poisson stochastic process of segments.

**Bibliography**


Faculty for Cybernetics, National Taras Shevchenko University, Academician Glushkov Avenue, 6, Kyiv 03127, Ukraine

E-mail address: vm214@dcp.kiev.ua

Department of Higher Mathematics, Kyiv National University for Economy, Peremogy Avenue, 54/1, Kyiv, 03057, Ukraine

E-mail address: semejko@ukr.net

Received 13/APR/2005

Translated by N. SEMENOV