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PROPERTIES OF THE OPTIMAL STOPPING DOMAIN IN THE LÉVY MODEL

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Abstract. The optimal exercise problem is considered for an American type contingent claim in a Lévy financial market model. Sufficient conditions are proposed under which the stopping domain is non-empty and has the threshold structure.

1. Introduction

We consider the Lévy model of a (B, S) financial market with two assets, namely the model consists of a riskless bond and a risky asset. The price of the riskless asset is defined by

$$B_t = e^{qt}, \qquad t \geq 0,$$

where $q \geq 0$ is the riskless interest rate. The evolution of the price of the riskless asset is modeled by the process

$$S_t = S_0 e^{X_t}, \qquad t \geq 0,$$

where $X = \{X_t, t \geq 0\}$ is a stochastic process with independent increments and with an initial value $X_0 = 0$. The process $X = \{X_t, t \geq 0\}$ is defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathsf{P})$ equipped with a filtration $\{\mathcal{F}_t\}$ that satisfies the usual assumptions; we also assume that $\mathcal{F}_0 = \{\varnothing, \Omega\}$.

An American contingent claim with a payoff function $f\colon \mathbf{R} \to \mathbf{R}$ and exercise time T guarantees the reward $f(S_t)$ when exercised at time $t \in [0, T]$. The optimal exercise problem is to maximize the expected payoff $\mathsf{E}\left(f(S_\tau)e^{-q\tau}\right)$ in the class M_T of all $\{\mathcal{F}_t\}$-Markov moments τ assuming values in $[0, T]$. It is natural to reformulate this problem in terms of the process X, namely if $g(z) = f(e^z)$, then the optimal exercise problem is

$$\mathsf{E}\left(g(X_\tau + x)e^{-q\tau}\right) \to \max.$$

Introduce the value function

$$V(T, x) = \sup_{\tau \in M_T} \mathsf{E}\left(g(X_\tau + x)e^{-q\tau}\right).$$

A stopping time $\tau^* \in M_T$ such that

$$V(T, x) = \mathsf{E}\left(g(X^*_\tau + x)e^{-q\tau^*}\right)$$

is called an optimal stopping time or optimal exercise time for the underlying contingent claim. It is well known (see, for example, [6]) that, for the stopping time problem, the minimal optimal stopping time in the case of Markov processes is of the form

$$\tau^* = \inf\{t \geq 0\colon (t, X_t) \in G\},$$

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where the set

$$G = \{(t,x) \in [0,T] \times \mathbf{R} \mid V(T-t,x) = g(x)\}$$

is called the optimal stopping domain for the underlying contingent claim.

We introduce the t-section of the set G, namely

$$G_t = \{x \in \mathbf{R} \mid V(T-t,x) = g(x)\}.$$

When solving the optimal exercise problem and numerically constructing the optimal domain, it is important to know that the domain is non-empty and that it has the threshold structure, that is,

$$G_t = [c(t), \infty).$$

Several papers are devoted to the study of stopping domains being of a threshold structure. For example, [6] deals with the problems of the non-emptiness and threshold structure of the stopping domain for an American option written for several assets. The behavior of the price in an exponential Lévy model is studied in [4] for an American put option with dividends. The paper [4] deals with the boundary of the stopping domain. The threshold structure for the stopping set is established in [3] in the reselling problem for an European call option. The non-emptiness and threshold structure is studied in [7] for the diffusion model of the financial market. It was proved in [2] that the stopping domain is non-empty and that the stopping domain has the threshold structure in the Lévy model for a contingent claim with a differentiable payoff function g. In the current paper, we obtain similar results for a wider class of payoff functions.

2. Auxiliary results

Let X_t be a Lévy process with a finite Lévy measure ν. Then

$$X_t = at + bW_t + \int_0^t \int_{\mathbf{R}} y\, \mu(ds, dy),$$

where W is a Wiener process and where μ a Poisson measure of jumps such that $\mathsf{E}(\mu(dt,dx)) = \nu(dx)\, dt$. In what follows we assume that the Lévy measure of the process X_t satisfies the condition

$$\int_{|x| \geqslant 1} e^{p|x|}\, \nu(dx) < \infty$$

for all $p \geqslant 0$.

Denote by $C_b^2(\mathbf{R})$ the set of all bounded twice continuously differentiable functions whose derivatives are bounded. The generator of the process X_t is defined for a function $g \in C_b^2(\mathbf{R})$ by

$$Ag(x) = ag'(x) + \frac{b^2}{2} g''(x) + \int_{\mathbf{R}} (g(x+y) - g(x))\, \nu(dy).$$

We introduce the following operators:

$$A_q g(x) = ag'(x) + \frac{b^2}{2} g''(x) + \int_{\mathbf{R}} (g(x+y) - g(x))\, \nu(dy) - qg(x).$$

The action of the operators $A_q g$ for functions $g \in C(\mathbf{R})$ is understood in the generalized sense. Denote by $D(\mathbf{R})$ the set of all main functions, that is, the set of infinitely differentiable functions with bounded supports.

Assumption 2.1. (i) *For some $c > 0$, $\alpha > 0$, and all $x \in \mathbf{R}$,*

$$|g(x)| \leqslant c(1 + |x|^\alpha);$$

(ii) *$g \in C^1(\mathbf{R})$;*

(iii) *there exists* $\beta \in \mathbf{R}$ *such that*

$$g(x) > x^{\beta}$$

for sufficiently large $x > 0$ (*the case* $\beta < 0$ *is also possible*);

(iv) $\lim_{x\to+\infty} \sup_{|a|\leqslant \ln x} g(x+a)/g(x) = 1$.

In what follows we need the following result.

Theorem 2.1 ([2]). *Let a payoff function* $g \in C(\mathbf{R})$ *satisfy condition (i) of Assumption 2.1. Then the stopping domain in the Lévy model is empty if and only if* $A_q g$ *is a non-zero non-negative measure in* $\mathbf{R}$.

3. Main result

3.1. Non-emptiness of the stopping domain.

Theorem 3.1. *The stopping domain for a Lévy process with a finite Lévy measure and without Brownian component is non-empty if the payoff function satisfies Assumption 2.1.*

Proof. Assume the converse; that is, the stopping domain is empty despite all the assumptions of Theorem 3.1 hold. Then Theorem 2.1 implies that

$$\langle A_q g(x), \theta(x)\rangle \geqslant 0 \tag{1}$$

for all $\theta \in D(\mathbf{R})$ such that $\theta \geqslant 0$.

By assumption of the theorem, $b = 0$ and the Lévy measure ν is finite and symmetric. Then A_q can be rewritten as follows:

$$\begin{aligned} A_q g(x) &= a g'(x) + \int_{-\infty}^{\infty} (g(x+y) - g(x))\, \nu(dy) - q g(x) \\ &= a g'(x) - \frac{q}{2} g(x) + \mathsf{E}\left[g(x+\xi) - \left(1 + \frac{q}{2\nu(R)} g(x)\right)\right], \end{aligned}$$

where ξ is a random variable that has the same distribution as the jumps of the process X_t. It is proved in [2] that there exists a number x_0 such that

$$\mathsf{E}\left[g(x+\xi) - \left(1 + \frac{q}{2\nu(\mathbf{R})} g(x)\right)\right] < 0 \tag{2}$$

for all $x > x_0$. Put

$$\tilde{A}_q g(x) = a g'(x) - \frac{q}{2} g(x),$$

$$\tilde{\tilde{A}}_q g(x) = \mathsf{E}\left[g(x+\xi) - \left(1 + \frac{q}{2\nu(\mathbf{R})} g(x)\right)\right].$$

Inequality (2) implies that

$$\left\langle \tilde{\tilde{A}}_q g\ , \theta \right\rangle \leqslant 0 \tag{3}$$

for all $\theta \in D(\mathbf{R})$ such that $\theta \geqslant 0$ and $\operatorname{supp} \theta \subset [x_0, \infty)$. For such a number θ,

$$\left\langle \tilde{A}_q g\ , \theta \right\rangle \geqslant 0 \tag{4}$$

in view of (1). Consider

$$\begin{aligned} \left\langle \tilde{A}_q g, \theta \right\rangle = \left\langle g, (\tilde{A}_q)^* \theta \right\rangle &= \int_{\mathbf{R}} g(x) \left(-a\theta'(x) - \frac{q}{2}\theta(x)\right) dx \\ &= -\int_{\mathbf{R}} g(x) \left(a\theta'(x) + \frac{q}{2}\theta(x)\right) dx. \end{aligned}$$

Making the change $\theta(x) = e^{\beta x}\phi(x)$ in the latter equality we get

$$\begin{aligned}\left\langle \tilde{A}_q g, \theta \right\rangle &= -\int_{\mathbf{R}} g(x)\left(a\theta'(x) + \frac{q}{2}\theta(x)\right)\,dx \\ &= -\int_{\mathbf{R}} g(x)e^{\beta x}\left(a\phi'(x) + \left(a\beta + \frac{q}{2}\right)\phi(x)\right)\,dx.\end{aligned}$$

According to (4),

$$\int_{\mathbf{R}} g(x)e^{\beta x}\left(a\phi'(x) + \left(a\beta + \frac{q}{2}\right)\phi(x)\right)\,dx \leqslant 0$$

for all $\phi \in D(\mathbf{R})$ such that $\operatorname{supp}\phi \subset [x_0, \infty)$.

Put $\beta = -q/(2a)$. Then the latter inequality is equivalent to

$$\int_{\mathbf{R}} g(x)e^{-\frac{q}{2a}x}(a\phi'(x))\,dx \leqslant 0 \qquad \forall \phi \in D(\mathbf{R}),\ \operatorname{supp}\phi \subset [x_0, \infty).$$

Now we consider the cases of $a > 0$ and $a < 0$ separately.

1) Case of $a > 0$. Then

$$\left\langle g(x)e^{-\frac{q}{2a}x}, \phi'(x)\right\rangle \leqslant 0 \Leftrightarrow \left\langle \left(g(x)e^{-\frac{q}{2a}x}\right)', \phi(x)\right\rangle \geqslant 0,$$

whence we conclude that the function $g(x)\exp\{-qx/(2a)\}$ is increasing with respect to x. Thus

$$g(x) \geqslant ce^{-\frac{q}{2a}x}, \qquad c > 0,$$

for sufficiently large x which contradicts condition (i) of Assumption 2.1.

2) Case of $a < 0$. Then

$$\left\langle g(x)e^{-\frac{q}{2a}x}, \phi'(x)\right\rangle \geqslant 0 \Leftrightarrow \left\langle \left(g(x)e^{-\frac{q}{2a}x}\right)', \phi(x)\right\rangle \leqslant 0,$$

whence we conclude that the function $g(x)\exp\{-qx/(2a)\}$ is decreasing with respect to x. Thus

$$g(x) \leqslant ce^{-\frac{q}{2a}x}, \qquad c > 0,$$

for sufficiently large x which contradicts condition (ii) of Assumption 2.1.

Therefore we obtain a contradiction in both cases, and this proves that the stopping domain is non-empty. □

Note that a similar result is obtained in [2] for $g \in C^1(\mathbf{R})$.

Let a Lévy process have a Brownian component. Then we can prove that the stopping domain is non-empty for the call option in the Black–Scholes models with the payoff function $g(x) = (K - e^x)^+$.

Theorem 3.2. *Let*

$$g(x) = (K - e^x)^+.$$

Then the stopping domain is non-empty.

Proof. For $x < \ln K$, the function $g(x)$ is twice continuously differentiable. Thus $A_q g(x)$ is defined in the classical sense and

$$A_q g(x) = ag'(x) + \frac{b^2}{2}g''(x) + \int_{\mathbf{R}} (g(x+y) - g(x))\,\nu(dy) - qg(x).$$

Then

$$ag'(x) + \frac{b^2}{2}g''(x) \to 0, \qquad -qg(x) \to -qK < 0$$

as $x \to -\infty$. Since $g(x+y) - g(x) \to 0$ as $x \to -\infty$, we get $|g(x+y) - g(x)| \leqslant K$, and since $\nu(\mathbf{R}) < \infty$, the dominated convergence theorem implies that

$$\int_{\mathbf{R}} (g(x+y) - g(x))\, \nu(dy) \to 0.$$

Hence there exists a number $x_0 < \ln K$ such that $A^q g(x) < 0$ for all $x < x_0$, and this proves that the stopping domain is non-empty by Theorem 2.1. □

3.2. **The threshold structure.** Consider the optimal exercise problem in an infinite time interval (in other words, we consider the problem of the optimal exercise of an American contingent claim). As before, we introduce the value function

$$V(x) = \sup_{\tau \in M} \mathsf{E}\left(g(X_\tau + x)e^{-q\tau}\right),$$

where the supremum on the right hand side is evaluated with respect to the set of all $\{\mathcal{F}_t\}$-stopping times. If

$$\mathsf{E}\left(\sup_{t \geq 0} g(X_t)e^{-qt}\right) < \infty,$$

the value function is well defined, that is, $V(x) < \infty$. The optimal stopping time in this case is given by

$$\tau^* = \inf\{t \geq 0 \colon X_t \in G_0\},$$

which is the first entrance time to the stopping domain

$$G_0 = \{x \in \mathbf{R} \colon V(x) = g(x)\}.$$

Below we list the assumptions imposed on the function g.

Assumption 3.1. *(i) Let $D = \{a_1, \dots, a_n\}$, where $n \in \mathbf{N}$ and $a_1, \dots, a_n$ are positive real numbers such that $a_1 < a_2 < \dots < a_n$. Assume that the function g is continuous in $\mathbf{R}$ and such that the derivatives g' and g'' exist and are continuous in $\mathbf{R} \setminus D$ and that the limits*

$$g'(a_i\pm) := \lim_{x \to a_i\pm} g'(x)$$

and

$$g''(a_i\pm) := \lim_{x \to a_i\pm} g''(x)$$

exist and are finite.

(ii) There exist two numbers $C > 0$ and $p > 0$ such that

$$|g(x)| + |g'(x)| + |g''(x)| \leqslant C(1 + |x|^p)$$

for all $x \in \mathbf{R}$.

According to Assumption 3.1, the operator $A_q g$ can be rewritten as follows:

$$\begin{aligned} A_q g(x) = {} & \left[a g'(x) + \frac{b^2}{2} g''(x)\right] \mathbf{1}_{\{x \notin D\}} + \int_{\mathbf{R}} (g(x+y) - g(x))\, \nu(dy) \\ & + \sum_{i=1}^{N} \Big(a\big(g'(a_i+) - g'(a_i-)\big) + \frac{b^2}{2}\big(g''(a_i+) - g''(a_i-)\big)\Big)\delta_{a_i} - qg(x), \end{aligned}$$

where δ_{a_i} is the Dirac measure concentrated at the point a_i.

We need several auxiliary results to prove the main result of this section.

Lemma 3.1. *Let g satisfy Assumption 3.1. Then*

$$\mathsf{E}\left(\sup_{t \geqslant 0} g(X_t)e^{-qt}\right) < \infty.$$

Proof. Without loss of generality we assume that $p \geqslant 1$. It is proved in [1] that

$$\mathsf{E}\left[\sup_{t\in[0,T]} |X_t|^p\right] < C_1(T \wedge T^p) \leqslant C_1 T^p$$

for all $T \geqslant 1$. Thus

$$\begin{aligned}\mathsf{E}\left[\sup_{t\geqslant 0} g(X_t)e^{-qt}\right] &\leqslant \mathsf{E}\left[\sum_{n=1}^{\infty} \sup_{t\in[n-1,n]} g(X_t)e^{-qt}\right] \leqslant \sum_{n=1}^{\infty} e^{-q(n-1)}\,\mathsf{E}\left[\sup_{t\in[0,n]} |g(X_t)|\right] \\ &\leqslant C\sum_{n=1}^{\infty} e^{-q(n-1)}\,\mathsf{E}\left[\sup_{t\in[0,n]} (1+|X_t|^p)\right] \\ &\leqslant CC_1\sum_{n=1}^{\infty} e^{-q(n-1)}(1+n^p) < \infty. \end{aligned}$$

□

Lemma 3.2. *Let g satisfy Assumption 3.1. We further assume that $g'(x_0+) > g'(x_0-)$ for some x_0. Then $V(x_0) > g(x_0)$.*

Proof. By Tailor's formula,

$$g(x) = g(x_0) + (x-x_0)^+ g'(x_0+) - (x-x_0)^- g'(x_0-) + (x-x_0)\varepsilon(x-x_0),$$

where $\lim_{y\to 0}\varepsilon(y) = 0$.

Let the random event H_t mean that the process X does not have jumps in $[0,t]$ and let $\overline{H_t} = \Omega \setminus H_t$. Then $X_s = as + bW_s$, $s \in [0,t]$, on the event H_t. Therefore one can apply the Itô–Tanaka formula

$$g(X_t + x_0) = g(x_0) + \big(g'(x_0+) - g'(x_0-)\big)\frac{b^2}{2}L_t^0 + \int_0^t \theta_s\,dX_s + X_t\varepsilon(X_t),$$

where L_t^0 is the local time of the process X at zero in the interval $[0,t]$,

$$\theta_s = g'(x_0+)\mathbf{1}_{\{X_s>0\}} - g'(x_0-)\mathbf{1}_{\{X_s<0\}}.$$

This implies that

$$\begin{aligned}&\mathsf{E}(g(X_t+x_0)) \\ &\quad = \mathsf{E}\left[\left(g(x_0) + \big(g'(x_0+) - g'(x_0-)\big)\frac{b^2}{2}L_t^0 + \int_0^t \theta_s\,dX_s + X_t\varepsilon(X_t)\right)\mathbf{1}_{H_t}\right] \\ &\qquad + \mathsf{E}\left[g(X_t+x_0)\mathbf{1}_{\overline{H_t}}\right]. \end{aligned} \tag{5}$$

Now we estimate the terms on the right hand side of the latter equality. By the Itô–Tanaka formula,

$$\begin{aligned}\mathsf{E}\left[L_t^0\mathbf{1}_{H_t}\right] &= \mathsf{E}\left[|X_t| - \left(\int_0^t \operatorname{sign} X_s\,dX_s\right)\mathbf{1}_{H_t}\right] \\ &= \mathsf{E}\left[|X_t| - \left(\int_0^t \operatorname{sign} X_s(a\,ds + b\,dW_s)\right)\mathbf{1}_{H_t}\right]. \end{aligned}$$

Then

$$\begin{aligned}\mathsf{E}\left[\int_0^t \operatorname{sign} X_s\,dW_s\mathbf{1}_{H_t}\right] &= \mathsf{E}\left[\mathsf{E}\left[\int_0^t \operatorname{sign} X_s\,dW_s\mathbf{1}_{H_t}\,\middle|\,\mu\right]\right] \\ &= \mathsf{E}\left[\mathsf{E}\left[\int_0^t \operatorname{sign} X_s\,dW_s\,\middle|\,\mu\right]\mathbf{1}_{H_t}\right] \\ &= \mathsf{E}\left[\mathsf{E}\left[\int_0^t \mathsf{E}[\operatorname{sign} X_s \mid \mu]\,dW_s\right]\mathbf{1}_{H_t}\right] = 0, \end{aligned}$$

where we used the independence of W and μ. Hence

$$\begin{aligned}\mathsf{E}\left[L_t^0 \mathbf{1}_{H_t}\right] &= \mathsf{E}\left[\left(|X_t| - \int_0^t a \operatorname{sign} X_s\, ds\right)\mathbf{1}_{H_t}\right] \\ &\geq \mathsf{E}[|at+bW_t|] - \mathsf{E}\left[|at+bW_t|\,\mathbf{1}_{\overline{H_t}}\right] - a\,\mathsf{E}\left[\int_0^t \operatorname{sign} X_s ds \mathbf{1}_{H_t}\right] \\ &\geqslant \mathsf{E}[|at+bW_t|] - \mathsf{E}\big[|at+bW_t|^2\big]^{1/2}\mathsf{P}(\overline{H_t})^{1/2} - at \geqslant b\sqrt{t} + o(\sqrt{t}), \qquad t \to 0.\end{aligned}$$

Hence we used the Cauchy–Bunyakovskiĭ inequality above together with the following property of Lévy's processes: $\mathsf{P}(\overline{H_t}) = O(t)$ as $t \to 0$. Further,

$$\mathsf{E}\left[\int_0^t \theta_s\, dX_s \mathbf{1}_{H_t}\right] = \mathsf{E}\left[\int_0^t \theta_s (a\, ds + b\, dW_s)\mathbf{1}_{H_t}\right].$$

As above, we obtain

$$\mathsf{E}\left[\int_0^t \theta_s\, dW_s \mathbf{1}_{H_t}\right] = 0.$$

On the other hand,

$$\left|\mathsf{E}\left[\int_0^t \theta_s\, ds \mathbf{1}_{H_t}\right]\right| \leqslant |a|\,\mathsf{E}\left[\int_0^t \big|\theta_s\big|\, ds\right] \leqslant |a|\,\mathsf{E}\int_0^t C\, ds = Cat = o(\sqrt{t}).$$

The middle term in (5) is estimated in view of the independence of W and μ and by using the property that $\varepsilon(x) = o(x)$ as $x \to 0$:

$$\begin{aligned}|\mathsf{E}[X_t\varepsilon(X_t)\mathbf{1}_{H_t}]| &= |\mathsf{E}[(at+bW_t)\varepsilon(at+bW_t)\mathbf{1}_{H_t}]| \\ &= |\,\mathsf{E}[(at+bW_t)\varepsilon(at+bW_t)]\,\mathsf{E}[\mathbf{1}_{H_t}]| \leqslant \big|\mathsf{E}[(at+bW_t)\varepsilon(at+bW_t)]\big| \\ &= o(\sqrt{t}), \qquad t \to 0.\end{aligned}$$

Finally we estimate the last term in (5) by using the Hölder inequality:

$$\begin{aligned}\mathsf{E}[g(X_t+x_0)\mathbf{1}_{\bar{H}_t}] &\leqslant \Big(\mathsf{E}\,|g(X_t+x_0)|^4\Big)^{1/4}\mathsf{P}(\overline{H_t})^{3/4} \\ &\leqslant C\Big(\mathsf{E}\,[1+|X_t+x_0|^p]^4\Big)^{1/4}\mathsf{P}(\overline{H_t})^{3/4} \leqslant \big(C_1 + C_2(p)t^{4p}\big)^{1/4} O\big(t^{3/4}\big) \\ &= o(\sqrt{t}), \qquad t \to 0.\end{aligned}$$

Now we get $\mathsf{E}(g(X_t+x_0)) = g(x_0) + b\sqrt{t} + o(\sqrt{t})$ as $t \to 0$. Since $e^{-qt} = 1 + O(t)$ as $t \to 0$, thus

$$\mathsf{E}\left(e^{-qt} g(X_t+x_0)\right) > g(x_0),$$

for sufficiently small t. This is what had to be proved. □

Lemma 3.3. *Assume that $x_0 \in G_0$ and that the payoff function g satisfies Assumption 3.1. If $A_q g$ is a non-positive measure on the open interval (x_0, ∞), then G_0 contains the interval (x_0, ∞).*

Proof. The proof is analogous to that given in [7, Lemma 4.2]. □

The above Lemmas 3.1–3.3 imply Theorem 3.3, the main result of this section. The proof of Theorem 3.3 is the same as that of [7, Theorem 4.1].

Theorem 3.3. *Let g be a positive payoff function that satisfies Assumption 3.1. Assume that there exists x_1 such that $A_q g$ is a non-zero positive measure in the interval $(0, x_1)$ and a non-positive measure in (x_1, ∞). Then $G = [x^*, \infty)$ with $x^* \geqslant x_1$. Moreover, the moment when the process X crosses the level x^* is the optimal stopping time.*

4. Concluding remarks

The optimal exercise problem is considered in this paper for an American contingent claim in the Lévy model of a financial market. Sufficient conditions are found for the stopping domain to be non-empty and to have the threshold structure.

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