ERRATA, VOLUME 21

G. H. HARDY, On the representation of a number as the sum of any number of squares, and in particular of five.

Miss G. K. Stanley has applied the method of my memoir in vol. 21 of the Transactions to the seven-square problem (see Journal of the London Mathematical Society, vol 2 (1927), pp. 91–96), and has supplied me with the following list of errata. The most important is that on page 266, line 16, where a wrong convention is chosen for the meaning of $\sqrt{-k}$: the correction of this involves a considerable number of consequential corrections.

Page 263, (3. 134), read

$$\eta(T) = \eta \left(1 - \frac{1}{T}\right) \rightarrow \frac{\pi^3}{96}$$;

Page 265, line 21, insert (3. 2251);
Page 266, line 12, read $\sum' \sum \frac{(-1)^{\eta} T_{h,k}}{\sqrt{k} \{(h - kr)i\}^{3/2}}$;

Page 267, (3. 242), read

$$\chi(\frac{1}{\epsilon}) = \frac{\pi^2}{8} + \frac{\pi^2}{8} \left(\frac{i}{\tau}\right)^{-3/2} + \frac{1}{2} \sum' \frac{(-1)^{\eta} T_{H,K}}{\sqrt{K} \{(K - H/\tau)i\}^{3/2}},$$

where $\epsilon = 1$ unless $H$ and $K$ are both negative, when $\epsilon = -1$, and $\eta = 1$ if $H < 0$, $\eta = -1$ if $H > 0$, and . . . . .
(3. 243), read

\[
\left\{ \left( K - \frac{H}{\tau} \right) i \right\}^{5/2} = i \left\{ - \frac{1}{\tau} \right\}^{5/2} \left\{ (H - K\tau) i \right\}^{5/2},
\]

where \( i = 1 \) unless \( H \) and \( K \) are both positive, when \( i = -1 \), and where \ldots \);

line 16, read

\[ "- \frac{\pi}{2} < \gamma = \arg \{ (H - K\tau) i \} < - \frac{3\pi}{2}" ; \]

line 17, read \("\beta + \gamma \) lies between \(- \frac{3\pi}{2} \) and \( \frac{3\pi}{2} \)";

line 18, read \("\alpha = \beta + \gamma \)";

line 19, read

\[
\left\{ \left( K - \frac{H}{\tau} \right) i \right\}^{5/2} = \left( - \frac{1}{\tau} \right)^{5/2} \left\{ (H - K\tau) i \right\}^{5/2} ;
\]

line 20, omit;

(3. 244), read

\[
\chi \left( - \frac{1}{\tau} \right) = \frac{\pi^2}{8} + \frac{\pi^2}{8} \left( \frac{i}{\tau} \right)^{-5/2} + \frac{1}{2} \sqrt{\frac{i}{\tau}} \left( - \frac{1}{\tau} \right)^{-5/2} \sum' \sum' \frac{(-1)K}{\sqrt{K}} \frac{\lambda T_{H,K}}{\left\{ (H - K\tau) i \right\}^{5/2}},
\]

where \( \lambda = \eta \xi = 1 \) if \( K > 0 \), \( \lambda = -1 \) if \( K < 0 \), and where \ldots \);

Page 268, (3. 245), insert a factor \( \lambda \) in each sum;

(3. 251), read

\[
\chi \left( 1 - \frac{1}{T} \right) = \frac{\pi^2}{8} - \frac{1}{2} \sum' \sum' \frac{(-1)^H \eta}{\sqrt{H}} \frac{T_{H-K,H}}{\left\{ (H + K/T) i \right\}^{5/2}},
\]

where \( \eta = 1 \) if \( H > 0 \), \( \eta = -1 \) if \( H < 0 \), and \( H \ldots \);" line 12, read \("\arg \{ (H + K/T) i \} \)";

last formula of footnote, read

\[ \sqrt{i} (-1/\tau)^{-5/2} = - (i/\tau)^{-5/2} ; \]

Page 269, line 3, read \("h > 0, k < 0 \) ;

(3. 255), read

\[
\chi \left( 1 - \frac{1}{T} \right) = \frac{\pi^2}{8} - \frac{1}{2} \sqrt{-i} \sum' \sum' \frac{(-1)^H \eta}{\sqrt{H}} \frac{W_{H,K}}{\left\{ (H + K/T) i \right\}^{5/2}} ;
\]

line 6, omit;
line 8, replace \( e \) by \( \xi \), and continue: “where \( \xi = 1 \) unless \( H > 0 \), \( K < 0 \), when \( \xi = -1 \), and am \( T \ldots . . . \)”; 

line 11, read

\[
\chi \left( 1 - \frac{1}{T} \right) = -\frac{1}{2\sqrt{-iT^{3/2}}} \sum_{H} \sum_{K} \frac{(-1)^{\lambda}}{\sqrt{K}} \frac{W_{H,K}}{\{(H-KT)i\}^{3/2}},
\]

where \( \lambda = 1 \) when \( K > 0 \) and \( \lambda = -1 \) when \( K < 0 \), and the summation is . . . .”;

(3. 256), omit \( \pi^2/8 \);

line 16, read

\[
\sum_{K=1,3,5,\ldots} \frac{1}{\sqrt{K}} \sum_{K=1}^{H} \frac{(-1)^{H}e^{(i-1)^{H}i^{K}}}{\{(H-KT)i\}^{3/2}};
\]

line 20, read

\[
\sum_{H} \frac{(-1)^{H}e^{(i-1)^{H}i^{K}}}{\{(H-KT)i\}^{3/2}} = \sum_{H} \frac{(-1)^{H}e^{H\xi/i}}{\{(H-KT)i\}^{3/2}};
\]

Page 270, line 6, for “unity” read “\( \pi^2/8 \)”;

Page 282, line 14, read “if \( N \equiv 3 (\text{mod } 4) \) or if \( N \equiv 1 (\text{mod } 4) \) and \( \alpha \) is odd, while

\[
x_{1} = 1 - \frac{1}{4} - \frac{1}{4.8} - \ldots - \frac{1}{4.8^{\beta-1}} - \frac{1}{4.8^{\beta}} - ( -1)^{(1/4)(N-1)} \frac{1}{8^{\beta+1}}
\]

if \( N \equiv 1 (\text{mod } 4) \) and \( \alpha \) is even, \( \alpha \) being equal to \( 2 \beta+1 \) when odd and to \( 2 \beta \) when even”;

line 31, add “when \( N \equiv 3 \) or \( 7 \) (\text{mod } 8). If \( N \equiv 1 \) we must replace \( 4.8^{\beta-1} \) by \( 8^{\beta} \), and if \( N \equiv 5 \) by \( \xi \cdot 8^{\beta} \)”;

Page 283, line 1, read “If \( N \equiv 3 \) or \( 7 \), we have, by (7. 42)”;

line 4, omit the last equality;

line 7, read

\[
\psi \left( \frac{n}{4^{\beta}} \right) = 2\tilde{\tau}(n),
\]

and so

\[
\tilde{\tau}(n) = \frac{80}{\pi^2} n^{3/2} \sum \left( \frac{n}{m} \right) \frac{1}{m^2}.\]

When \( n = 1 \) or \( 5 \), it will be found that we obtain the same result.”