A CORRECTION OF SOME THEOREMS
ON PARTITIONS

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Theorem 4 in [1] gives a convergent series representation for \( p_a(n) \), the number of partitions of a positive integer \( n \) into positive summands of the form \( mp \pm a_j \). Here \( p \) is an odd prime and \( a_j \) is an element of a set \( a \) consisting of \( r \) positive residues of \( p \) each of which is less than \( p/2 \). It is stated that the theorem holds for \( n \geq A/12p \), where \( A = rp^2 - 6 \sum_{j=1}^{r} a_j(p-a_j) \). In the proof of this theorem the estimate \( O(n^{1/3}k^{2/3} + \delta) \) is used for certain complicated exponential sums. The proof of this estimate given in Theorem 2 of [1] depends on the fact that \( (A - 12pn, k) = O(n) \). This is clearly false (in general) if \( A = 12pn \) since \( (0, k) = k \). Thus, Theorem 4 of [1] has been established only if \( n > A/12p \).

Similar remarks apply to Theorem 6 in [2] in which a convergent series is obtained for \( q_a(n) \), the number of partitions of \( n \) into distinct positive summands of the form \( mp \pm a_j \). Here it is asserted that the theorem holds for \( n \geq -A/12p \). However, the proof given is valid only if \( n > -A/12p \). For the argument used to establish the required estimate \( O(n^{1/3}k^{2/3} + \delta) \) for the exponential sums involved does not hold if \( A = -12np \). Thus, until (if ever) the necessary estimates contained in Theorems 2 and 3 of [1] and Theorems 2 through 5 of [2] can be justified for \( n = \pm A/12p \) we must exclude these values of \( n \) from consideration.

We conclude by giving a simple necessary condition for \( A = \pm 12pn \). From the definition of \( A \) given above and the fact that either \( a_j \) or \( p - a_j \) is even we see that if \( A = \pm 12pn \) then \( 12|r \).

REFERENCES


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Received by the editors September 21, 1964.

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