CORRECTION TO THE PAPER

ON THE ZEROS OF POLYNOMIALS
OVER DIVISION RINGS

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On p. 226 (116 (1965), 218–226) it is stated that the curve $\xi_2^3 = (\xi_1 + 1)^3 - \xi_1^3$ has exactly nine points in $Q(\sqrt{3})$. However, one of these is at infinity; hence $N(f) = 16$ rather than 18 for this example. An example of a quadratic polynomial over $D$ with $16 < N(f) < \infty$ is

$$f(x) = (\xi_3 - \xi_2^2)e_1 + (\xi_1^2 - \xi_2\xi_3 - 1)e_2 + (\xi_2^2 - 1)e_3.$$  

To find the roots of $f(x)$ we consider the system

$$\begin{align*}
\xi_3 &= \xi_2^2, \\
\xi_1^2 &= \xi_2\xi_3 + 1, \\
\xi_4 &= 1.
\end{align*}$$  

Eliminating $\xi_3$ we obtain

$$(1) \quad \xi_1^2 = \xi_2^3 + 1.$$  

Euler [1] proved in 1738 that the only solutions of (1) in $Q$ are

$$(\xi_1, \xi_2) = (0, -1), (\pm 1, 0), \text{ and } (\pm 3, 2).$$  

Hence by a theorem of Billing [2], there are only finitely many solutions of (1) in $Q(\sqrt{3})$. These include the eleven pairs $(\xi_1, \xi_2) = (0, -\omega), (\pm 1, 0), \text{ and } (\pm 3, 2\omega)$, where $\omega$ is any cube root of unity. Corresponding to each $(\xi_1, \xi_2)$ there is a unique value of $\xi_3$, and two values for $\xi_4$. Thus $22 \leq N(f) < \infty$.

REFERENCES


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