INERTIAL AUTOMORPHISMS OF A CLASS OF WILDLY RAMIFIED $v$-RINGS

BY
NICKOLAS HEEREMA(°)

I. Introduction. Let $R$ be a ramified $v$-ring with ramification $e$. That is, $R$ is a complete, discrete, rank one, valuation ring having characteristic zero with residue field $k$ of characteristic $p \neq 0, 2$ and $pR$ is the $e$th power of the maximal ideal $M$ of $R$. Let $\mathfrak{O}$ represent the group of automorphisms of $R$, $e$ being the identity map. Then, for $i > 0$, $\mathfrak{O}_i = \{a | a \in \mathfrak{O}; a = e, \text{mod } M^i\}$ and $\mathfrak{S}_i = \{a | a \in \mathfrak{O}_i, a(m) - m \in M^{i+1} \text{ for } m \in M\}$. The ramification groups $\mathfrak{O}_i$ and $\mathfrak{S}_i$ are invariant in $\mathfrak{O}$. The object of this paper is to evaluate the factor groups of the series (1) of ramification groups in that case in which $e = p$. A second objective is the determination of those automorphisms in $\mathfrak{O}_1$ which are derivation automorphisms (see below).

Neggers has shown [3, Theorem 6] that for any $e$ and $i \geq (e+p)/(p-1)$, $\mathfrak{O}_i/\mathfrak{O}_{i+1}$ is isomorphic to $D(R)/\pi D(R)$ where $D(R)$ is the additive group of derivations on the ring $R$ and $\pi D(R) = \{\pi d | d \in D(R)\}$ where $\pi$ is a prime element in $R$. In addition he proved that $\mathfrak{S}_i/\mathfrak{O}_{i+1}$ is isomorphic to the additive group of those derivations on $k$ which lift to $R$ where again $i \geq (e+p)/(p-1)$. The map used by Neggers to evaluate $\mathfrak{O}_i/\mathfrak{O}_{i+1}$ also shows that if $i \geq (e+p)/(p-1)$, then $\mathfrak{O}_i/\mathfrak{S}_i$ is isomorphic to $D(R)/D^*(R)$ where $D^*(R) = \{d | d \in D(R), d(\pi) \in \pi R\}$ [3, proof of Theorem 6]. The principal tool of this investigation is the convergent higher derivation [2]. Let $D = \{D_i\}_{i=1}^{\infty}$ be a higher derivation on $R(D(R) \subset R$ for $i > 0)$. $D$ is convergent if, for $a \in R$, $\sum D_i(a)$ is a convergent series in the $\pi$-adic topology. If $D$ converges the map $a_D: a \rightarrow \sum_{i=0}^{\infty} D_i(a)(D_0(a) = a)$ is an inertial automorphism (see Theorem B). The group $\mathfrak{O}_D$ of all derivation automorphism $a_D$ is an invariant subgroup of $\mathfrak{O}$.

Throughout this paper $R$ will denote a $v$-ring in $R$ such that $[R:R] = e$, and $R$ is unramified. Thus $R$ has the same residue field $k$ as $R$. For $a$ in $R$, $\overline{a}$ will denote the image of $a$ under the natural map of $R$ onto $k$, $\pi$ always represents a prime element in $R$. We have

$$\pi^e + pu = 0, \quad \overline{u} \neq 0. \tag{2}$$

If $e = p$ and $\overline{u} \in k^p$ then $\pi$ can be chosen so that

$$\pi^p + p(1 + \overline{u}v) = 0, \quad t > 0, \quad \overline{v} \neq 0, \tag{3}$$

or $\pi^p + p = 0$. We note that the conditions

$$\overline{u} \notin k^p; \quad t = p \quad \text{and} \quad \overline{v} \notin k^p; \quad \text{as well as} \quad \overline{v} \text{ if } 1 \leq t < p$$

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are all independent of the choice of \( \pi \), assuming (3) to be satisfied for all but the first listed. Throughout this paper the symbols \( u \) and \( v \) will represent the quantities given in (2) and (3). The considerations (4) determine, to a large extent, the structure of the groups (1) as is seen in the following two theorems which summarize the results of this study. \( \Theta(R_\pi; R) \) is the group of automorphism of \( R_\pi \) over \( R \).

**Theorem 1.** Every inertial automorphism on \( R_\pi \) is a derivation automorphism, i.e. \( \Theta_1 = \Theta_D \), unless \( \bar{u} \in k^p \) and \( t = p - 1 \), in which case the following are equivalent.

(a) \( v \) is a \((p-1)\)th root in \( k \).
(b) \( R_\pi \) is Galois over \( R \).
(c) \( \Theta_2 \neq \Theta_2 \).
(d) \( \Theta_2 | \Theta_2 \) is the group of order \( p \).

If \( R_\pi \) is Galois over \( R \) then \( \Theta(R_\pi; R) \subset \Theta_D \) if and only if \( \bar{v} \notin k^p \). In any case, \( \Theta_1 = \Theta_D \cdot \Theta(R_\pi; R) \).

**Theorem 2.** If \( \bar{u} \notin k^p \) then for \( i \geq 1 \), \( \Theta_i / \Theta_{i+1} \) is isomorphic to the subgroup \( \bar{D} \) of those \( \delta \in D(k) \) which lift to \( R_\pi \). Also, \( \bar{D} = \{ \delta \mid \delta \in D(k), \delta(\bar{u}) = 0 \} \). In this case \( \Theta_i / \Theta_{i+1} \) is isomorphic to \( k^+ \), the additive group of \( k \).

If \( \bar{u} \in k^p \), then for \( i \geq 1 \), \( \Theta_i / \Theta_{i+1} \) is isomorphic to \( D(k) \) unless \( t = p \) and \( i = 1 \). If \( t = p, \Theta_i / \Theta_{i+1} \) is isomorphic to the subgroup of those \( \delta \in D(k) \) such that \( \delta(\bar{v}) = 0 \). Also, \( \Theta_i = \Theta_i, i \geq 1 \), unless \( t = p - 1, i = 2 \) and one of the four equivalent conditions of Theorem 1 holds.

By Neggers’ results referred to above [3,’proof of Theorem 6] we have

**Corollary.** \( \mathcal{D}(R_\pi) / \mathcal{D}^*(R_\pi) \) is isomorphic to \( k^+ \) if \( \bar{u} \notin k^p \) and is trivial if \( \bar{u} \in k^p \).

**II. Proofs.** For \( S \) a subring of \( R_\pi \), the symbols \( \mathcal{H}(S, R_\pi), \mathcal{H}(S, R_\pi) \) and \( \mathcal{H}_D(S, R_\pi) \) will stand for the set of all higher derivations, all convergent higher derivations, and all uniformly convergent higher derivations with domain \( S \) and range \( R_\pi \). We quote the following two theorems which will be used repeatedly. Theorem A provides the necessary freedom in the construction of \( D \in \mathcal{H}_D(R_\pi, R_\pi) \). Theorem B implies that if \( D \in \mathcal{H}_D(R_\pi, R_\pi) \), then \( \alpha_D \) is indeed an inertial automorphism.

**Theorem A** [2, Theorem 4]. Let \( \mathcal{F} \) be a \( p \)-basis for \( k \) and let \( \mathcal{S} \subset R \) be a set of representatives of the elements of \( \mathcal{F} \). If \( I \) is the set of positive integers and \( f \) is a mapping from \( \mathcal{S} \times I \) into \( R_\pi \) then there is one and only one \( D \in \mathcal{H}(R, R_\pi) \) such that \( D(s) = f(s, i) \) for all \( s \in \mathcal{S} \) and \( i \in I \). Moreover, \( D \) converges (uniformly) if and only if \( D \) converges (uniformly) on \( \mathcal{S} \).

**Theorem B** [2, Lemmas 1 and 5]. If \( D \) is in \( \mathcal{H}_D(R_\pi, R_\pi) \) then \( D(s) = \pi R_\pi \) and \( D(\pi R_\pi) = \pi^2 R_\pi \) for \( i > 0 \).

Theorems 1 and 2 will be proved by means of a series of lemmas.
Lemma 1. If \( \mathcal{P} \) is a set of representatives in \( R \) of a \( p \)-basis \( \mathcal{F} \) for \( k \) and \( D \) in \( \mathcal{H}(R, R_e) \) is such that \( D_j(\mathcal{P}) \subset \pi^j R_e \subset R_e \), \( j \geq 1 \), then \( D(R) \subset \pi^j R_e \) where

\[
q_j = \min_{t_1 + \cdots + t_i = 0} (t_1 + \cdots + t_i), \quad i \geq 1.
\]

Proof. For a given \( i \) we choose \( n \) sufficiently large so that \( D_j(\pi^n) \subset \pi^q R_e \) for \( j = 1, \ldots, i \) [2, Lemma 6] where \( \pi^n \) is the subring of \( R \) generated by the \( p^n \)th powers of elements in \( R \). Since every element in \( k \) has a representative in \( R^n[\mathcal{F}] \), it follows that \( R = R^n[\mathcal{F}] + \pi^n R \). If \( b = a s_1, \ldots, s_i \) where \( a \in R^n \) and \( s_1, \ldots, s_i \in \mathcal{F} \) then

\[
D_i(b) = \sum_{t_0 + \cdots + t_i = 1} D_{t_0}(a) D_i(s_1), \ldots, D_i(s_i)
\]

is seen to be in \( \pi^q R \) and hence, since \( D_j(\pi^n) = 0 \) for all \( j \), \( D(R) \subset \pi^q R_e \).

Let \( a \), an automorphism of \( R_e \), be in \( \mathcal{D}_i \), \( i \geq 1 \). Then \( a(a) = a + \pi^q a^*(a) \) and the mapping \( a^* \) induces a derivation \( \delta_a \) on \( k \). The mapping

\[
\phi_i: \alpha \rightarrow \delta_a
\]

is a homomorphism of \( \mathcal{D}_i \) into \( \mathcal{D}(k) \) with kernel \( \mathcal{D}_i + 1 \).

Lemma 2. If an automorphism \( \alpha \) of \( R_p \) is in \( \mathcal{D}_i \), then \( \delta_a(u) = 0 \). If \( \pi^p + p(1 + \pi^p v) = 0 \), then \( \delta_a(v) = 0 \) for \( \alpha \) in \( \mathcal{D}_i \).

Proof. Since \( \alpha(\pi) = \pi \in \pi^{i+1} R_p \) it follows that \( \alpha(\pi^p) - \pi^p \in \pi^{i+1} R_p \). Thus by (2) \( \alpha(u) = u \) is in \( \pi^{i+1} R_p \) or \( \alpha(u) = \pi R_p \) which implies \( \delta_a(u) = 0 \). In the remaining case let \( \alpha(\pi) = \pi + \pi^2 b \). Then \( \alpha(\pi^p) - \pi^p \equiv \pi^2 b^p \), mod \( \pi^{2p+1} R_p \), and \( \alpha(p(1 + \pi^p v)) - p(1 + \pi^p v) \equiv p\pi^{p+1} a^*(v) \), mod \( \pi^{2p+2} R_p \). Hence \( b \in \pi R_p \) and it follows that \( \alpha^*(v) \in \pi R_p \) or \( \delta_a(v) = 0 \).

Given \( D \) and \( H \) in \( \mathcal{H}(R_e, R_e) \), \( D \circ H \) in \( \mathcal{H}(R_e, R_e) \) is given by

\[
(D \circ H)_i = \sum_{j=0}^{i} D_i H_{i-j}.
\]

\( \mathcal{H}(R_e, R_e) \) is a group with respect to this composition and \( \mathcal{H}(R_e, R_e), \mathcal{H}_{\pi}(R_e, R_e) \) are subgroups [2, Theorems 1, 2]. Moreover, one can verify directly that, for \( D \) and \( H \) in \( \mathcal{H}(R_e, R_e) \), \( \alpha \circ D = \alpha \circ H \).

Lemma 3. Let \( (D^{(a)})_i \) be such that \( D^{(a)}(R_e) \subset \pi^i R_e \), \( i \geq 1 \), \( n \geq 1 \), and \( \lim_n s_n = \infty \). Let \( \alpha_a = \alpha a^{(1)}, \ldots, \alpha a^{(n)} \) and \( D^{(a)} = D^{(1)} \cdots D^{(n)} \). Then \( \lim_n \alpha_a(a) = \lim_n \alpha a^{(i)}(a) \) exist for all \( i > 0 \) and \( a \in R_e \). Moreover, \( \alpha : a \rightarrow \lim_n \alpha a(a) \) is an automorphism, \( D = \{D_i\} \) is in \( \mathcal{H}(R_e, R_e) \) where \( D_i(a) = \lim_n \alpha a^{(i)}(a) \) and \( \alpha = \alpha a \). If \( D^{(a)} \in \mathcal{H}_{\pi}(R_e, R_e) \), then \( D \in \mathcal{H}_{\pi}(R_e, R_e) \).

Proof. By definition of product in \( \mathcal{H}(R_e, R_e) \) we have

\[
D^{(m+1)}(a) - D^{(m)}(a) = \sum_{t_1 + \cdots + t_{m+1} = n; t_{m+1} \neq 0} D^{(1)} \cdots D^{(m+1)}(a),
\]
the right side of which is in \( \pi^{n+1}R_e \) since for \( D \in \mathcal{H}_e(R_e, R_e) \), \( D(\pi^nR_e) \subset \pi^nR_e \) for all \( i \) and \( n \) by Theorem B. Thus, \( \alpha_{n+1}(a) - \alpha_n(a) = \sum_{i=0}^{\infty} \mathcal{D}^{i+1}(a) - \sum_{i=0}^{\infty} \mathcal{D}^i(a) \in \pi^{n+1}R_e \). The rest of the lemma follows directly.

Since \( R_e \) is totally ramified over \( R \) and \( [R_p : R] = p \) then \( R_p = R[\pi] \) and the minimal polynomial \( f(x) \) of \( \pi \) over \( R \) is an Eisenstein polynomial, that is,

\[
f(x) = x^p + p \alpha_{p-1} x^{p-1} + \cdots + p \alpha_1 x + p \alpha_0
\]

and \( \alpha_0 \) is a unit. Clearly, \( \alpha_0 = \bar{u} \) (see (2)). Also, if \( \bar{u} \in k^p \) then \( \alpha_0 = b^p + pc \) where \( b \) and \( c \) are in \( R \). By replacing \( \pi \) with \( b^{-1} \pi \) we can assume that

\[
a_0 = 1 + pb_0.
\]

We note next that every \( D \in \mathcal{H}(R, R_e) \) extends uniquely to a higher derivation \( D \) of the quotient field of \( R_e \). Also, \( D(R_e) \subset R_e \) if and only if \( D(\pi) \in R_e \). If \( D \) converges on \( R \), \( D \) will converge on \( R_e \) if and only if \( D \) converges at \( \pi \) [2, Lemma 3].

Let \((r, s)\) denote an ordered set of \( r \) nonnegative integers whose sum is \( s \) and let \(|(r, s)|\) represent the largest integer in \((r, s)\). We let \( \sum_{(a, \bar{a})} D(a_1, \ldots, a_n) \) denote the sum of all products \( D_1(a_1)D_2(a_2) \cdots D_n(a_n) \) such that \( i_1 + \cdots + i_n = s \) and \( i_j \geq 0 \). Also, \( f'(x) \) and \( f^{(p)}(x) \) represent respectively the ordinary derivative of \( f \) and the polynomial obtained by replacing each coefficient in \( f \) with its image under \( D_i \). With these conventions it is useful to write the expression for \( D_i(\pi) \) derived from \( D_i(f(\pi)) = 0 \) as follows:

\[
f'(\pi)D_i(\pi) = f^{(p)}(\pi) + \sum_{(p, \bar{p}); |(p, \bar{p})| < 1} D(\pi, \ldots, \pi) + \sum_{(j, \bar{p}); |(j, \bar{p})| < 1} D(a_j, \pi, \ldots, \pi).
\]

Let \( v \) represent the exponential valuation on \( R_p \). Note that \( p \leq v(f'(\pi)) \leq 2p - 1 \).

**Lemma 4.** A given \( \delta \in \mathcal{D}(k) \) lifts to \( d \in \mathcal{D}(R_p) \) if and only if \( \delta(\bar{u}) = 0 \).

**Proof.** A derivation \( d \in \mathcal{D}(R_p) \) induces a derivation on \( k \) under the natural map of \( R_p \) onto \( k \) only if \( d(\pi) \subset \pi R_p \). But \( d(f(\pi)) = 0 \) means \( d(\pi) = -f'(\pi)f'(\pi) \). Thus \( f'(\pi) \in \pi^{p+1}R_p \) which means \( d(\alpha_0) \in \pi R_p \) and, hence, if \( d \) induces \( \delta \) on \( k \), \( \delta(\alpha_0) = \delta(\bar{u}) = 0 \). Conversely, every \( \delta \in \mathcal{D}(k) \) lifts to \( d' \) on \( R \) [1, Theorem 1]. If \( \delta(\bar{u}) = 0 \), \( d'(a_0) \in p R \) which means that \( f'(\pi)f'(\pi) \in \pi R_p \). Thus the extension \( d \), of \( d' \), to \( R_p \) is in \( \mathcal{D}(R_p) \) and induces \( \delta \) since \( d' \) does.

**Lemma 5.** Let \( D \in \mathcal{H}_c(R, R_e) \) where \( R_p = R[\pi] \) and \( f(x) = x^p + \sum_{\bar{a} \neq 0} p a_\bar{a} x^\bar{a} \) is the minimum function of \( \pi \) over \( R \). Let \( q > 2, n \geq 1 \), and \( m > p(n-1) \) be integers such that, using the same symbol for the extension of \( D \) to \( R_p \),

\[(9, 1) \quad D_j(\pi) \in \pi^2 R_p \quad \text{if} \quad j < n,
\]

\[(9, 2) \quad D_j(\pi) \in \pi^q R_p \quad \text{if} \quad n \leq j < m,
\]

and, if \( j \geq n \)

\[(9, 3) \quad D_j(a_n) \in f'(\pi)\pi^{q-p}.\]

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If \( q = 3 \) then (9, 3) is assumed to hold only for \( j \geq m \). Under these assumptions \( \sum_{j=n} D_j(\pi) \) converges and
\[
\sum_{j=n} D_j(\pi) \in \pi^q R_p.
\]

The proof of this lemma consists of checking the valuation \( v \) of the terms on the right side of (8). We show first that if \( j \geq m \),
\[
D_j(\pi) \in \pi^q R_p.
\]
Thus assuming (10) true for \( j < r \) where \( r \geq m \) we consider (8, r). By (9, 3) \( f^0(\pi) \) is in \( f'(\pi)\pi^q R_p \). The term \( D_n(\pi) \cdots D_p(\pi) \) of \( A_r = \sum_{i\neq j, j \neq r} D_i, \ldots, \pi \) is in \( \pi^q R_p \) in view of the fact that at least one \( i \) and another is different from zero.

The above term appears in \( A_r \) a multiple of \( p \) times unless \( i_1 = i_2 = \cdots = i_p = r/p \) in which case it is in \( \pi^q R_p \). Since \( v(f'(\pi)) \leq 2p-1 \) and \( p \neq 2 \). Thus \( A_r \in f'(\pi)\pi^q R_p \). A similar argument shows \( B_r \), the remaining term on the right side of (8, r), to be in \( f'(\pi)\pi^q R_p \). Thus by (8, r) \( D_i(\pi) \in \pi^q R_p \).

Given \( i \geq 0 \), we assume for some integer \( s \geq m \) that if \( j > s \), then \( D_j(\pi) \in \pi^{i+1} R_p \) and, for \( h = 0, \ldots, p-1 \)
\[
D_j(a_h) \in f'(\pi)\pi^{i+1} R_p.
\]
Let \( s' = ps \) and let \( j > s' \). Then \( f^0(\pi) \in f'(\pi)\pi^{i+1} R_p \) and, by an analysis like that above, \( A_j \) and \( B_j \) are seen to be in \( f'(\pi)\pi^{i+1} R_p \). Thus \( D_j(\pi) \in \pi^{i+1} R_p \). Since \( D \) converges on \( R \), given \( i \geq 0 \), there is an \( s \) such that (11) holds for \( j > s \). It follows that \( \sum_{j=n} D_j(\pi) \) converges, and in view of (10) \( \sum_{j=n} D_j(\pi) \in \pi^q R_p \).

**Lemma 6.** If \( \alpha \in S_t \), \( i \geq 1 \), then there is a \( D \in H(R_p, R_p) \) such that \( \alpha^{-1} a_0 \in G_{i+1} \). Moreover, \( S_t/\theta_{t+1} \) is isomorphic to the subgroup of those \( \delta \) in \( \mathcal{D}(k) \) for which \( \delta(\bar{u}) = 0 \) with the following exception. If \( \bar{u} \in k^p \) and, for suitable choice of \( \pi \) we have \( \pi^p = p(1 + \pi^u) \) then \( S_t/\theta_{t+1} \) is isomorphic to the subgroup of those \( \delta \) in \( \mathcal{D}(k) \) for which \( \delta(\bar{v}) = 0 \).

**Proof.** By Lemma 2, it will be sufficient to find \( D \in H(R_p, R_p) \) such that \( \phi(\sigma_D) \) (see (5)) is a given \( \delta \) for which \( \delta(\bar{u}) = 0 \), or, in the exceptional case, \( \delta(\bar{v}) = 0 \). Let (6) be the minimum function of \( \pi \) over \( R \).

1. Let \( \delta \) be any derivation on \( k \) for which \( \delta(\bar{u}) = \delta(\bar{a}_0) = 0 \) and let \( H = \{H_j\} \) be any higher derivation in \( H(R, R) \) satisfying the two conditions (a) \( H_1 \) induces \( \delta \), (b) \( H_j(a_0) \in p R_j, j = 1, \ldots, p-1 \). Specifically, every derivation on \( k \) lifts to \( R \) [1, Theorem 1] which fact makes \( H_1 \) available. Let \( H_j = H_j(j) \) for \( j = 2, \ldots, p-1 \). By Theorem A, maps \( H_j, j \geq p \), can be defined so that \( H = \{H_j\} \in H(R, R) \). Let \( D = \{D_j\} \) where
\[
D_j = \pi^j H_j.
\]

Clearly, \( D \in H(R, R) \). We now show that
\[
D_j(\pi) \in \pi^{j+1} R_p, \quad j \geq 1,
\]
and
\[
\sum D_j(\pi) \text{ converges,}
\]
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from which, with (12), it will follow that, using the same symbol for the extended higher derivation, \( D \in \mathcal{H}(R_p, R) \), \( a_0 \in \mathfrak{S}_1 \) and \( \Phi_1(a_0) = \delta \).

Let \( v(f'(\pi)) = p + r - 1 \). Thus \( r \) is the least positive integer such that \( a_i \) is a unit.

Looking to the conditions of Lemma 5 we note that \( f'(\pi)^{n+1-p-s}R_p = \pi^{n+1-r-s}R_p \). If \( s \geq r \), \( D_j(a_j) = \pi^{n+1-r-s}R_p \) for \( j \geq 1 \). If \( r > s > 0 \), \( a_i \in pR \); hence, \( H_i(a_i) \in pR \) and thus \( D_j(a_j) \in \pi^{n+1-r-s}R_p \) for \( j \geq 1 \). Finally, \( D_j(a_0) = \pi^{n+1}H_i(a_0) \). If \( j < p \), \( H_i(a_0) \in pR \) and \( \pi^{n+1}H_i(a_0) \in \pi^{n+1}R_p \). If \( j \geq p, \) \( i \geq l + r \). Hence, \( D_j(a_0) \in \pi^{n+1}R_p \) for \( j \geq 1 \). Thus conditions (9, 1) to (9, 3) are satisfied with \( q = i + 1 \) and \( n = m = 1 \). Hence (13) and (14) hold.

Case 2. \( v(f'(\pi)) < 2p - 2, i = 1 \). We define \( D \) as in Case 1 and note by inspection of (8, j) for \( j = 1, \ldots, p \) that \( D_j(\pi) = \pi^{n+1}R_p \), \( D_j(\pi) \in \pi^{n+1}R_p \) for \( j = 2, \ldots, p \). Also, in this case, \( f'(\pi)^{n+1-p-s}R_p = \pi^{n+1}R_p \). By (12) \( D_j(a_i) \in \pi^{n+1}R_p \) for \( j \geq p \) and \( s = 0, \ldots, p - 1 \). Thus conditions (9, 1) to (9, 3) are satisfied for \( n = 2, m = p + 1 \) and \( q = 3 \). Hence \( \sum D_j(\pi) \) converges and is in \( \pi^2R_p \). Thus, \( a_0 \in \mathfrak{S}_1 \) and \( \Phi_1(a_0) = \delta \).

Case 3. \( v(f'(\pi)) = 2p - 2, i = 1 \). We consider a number of subcases. In each case \( D \) is constructed by the method of Theorem A.

(3, i). \( \tilde{a}_{p-1} \notin k^p, \tilde{a}_0 \notin k^p, \tilde{a}_{p-1} \) and \( \tilde{a}_0 \) \( p \)-independent. As before, we initiate the construction of \( D \in \mathcal{H}_1(R, R_p) \) by letting \( D_j = \pi^jH_i, j = 1, \ldots, p - 1 \), where \( \{H_i\}_{t=1}^{t-1} \) are chosen such that \( H_i(\tilde{a}) = 0 \) and \( H_i(a_0) \in pR \). Let \( \mathfrak{S} \) be a set of representatives in \( R \) of a \( p \)-basis \( \mathcal{F} \) of \( k \). We may assume both \( a_0 \) and \( a_{p-1} \) in \( \mathfrak{S} \). By inspection of (8, 1) to (8, \( p - 1 \)), we have \( D_1(\pi) \in \pi^2R_p \) and \( D_j(\pi) \in \pi^3R_p, j = 2, \ldots, p - 1 \).

Considering (8, \( p \)), each summand of \( A_p \) and \( B_p \) is in \( p^2R_p \). Thus, \( [D_1(\pi)]^p \) which is in \( p^2R_p \) but not in \( p^2+1R_p \). Thus, we define \( D_p \) by \( D_p(\pi) = 0 \) and \( D_p(a_0) \) is so chosen that \( f'(\pi)^{n+1-p-s}R_p = \pi^{n+1}R_p \). Thus, \( D_p(\pi) \in \pi^2R_p \). Also, \( H_i(a_0) \in \pi^2R_p \) for \( j > p \) we let \( D_j(\pi) = 0 \) for \( s \leq p \). By Lemma 1 \( D_j(\pi) \in \pi^p+1R_p \) for \( j > p \). It follows that conditions (9) of Lemma 5 are fulfilled for \( n = 2, m = p + 1 \) and \( q = 3 \). Thus by Theorem A the extension of \( D \) to \( R_p \) converges uniformly \( a_0 \in \mathfrak{S}_1 \) and \( \Phi_1(a_0) = \delta \).

(3, iii) \( \tilde{a}_{p-1} \notin k^p, \tilde{a}_0 \notin k^p, \tilde{a}_{p-1} \) and \( \tilde{a}_0 \) \( p \)-dependent. Let \( H_1 \in \mathcal{D}(R) \) induce \( \delta \in \mathcal{D}_k \) such that \( \delta(\tilde{a}_0) = 0 \) (Lemma 4) and let \( \mathfrak{S} \) be a set of representatives of a \( p \)-basis for \( k \). Let \( H_j = \pi^jH_i, j = 1, \ldots, p - 1 \), where \( \{H_j\}_{j=1}^{p-1} \) are chosen such that \( H_j(\tilde{a}_0) = 0 \) and \( H_j(a_0) \in pR \). Let \( \mathfrak{S} \) be a set of representatives of a \( p \)-basis for \( k \) which contains \( a_0 \).

Thus, \( D_j(\tilde{a}_0) \) and \( D_j(\tilde{a}_{p-1}) \) are \( p \)-dependent. Thus by (8, 1) \( D_j(\pi) \in \pi^2R_p \). Also, \( \pi^2R_p \) for \( j \geq 1, s = 0, \ldots, p - 1 \). Thus, conditions (9) of Lemma 5 are satisfied for \( n = 2, m = 2, q = 3 \) and again \( D \in \mathcal{H}(R_p, R), a_0 \in \mathfrak{S}_1 \) and \( \Phi_1(a_0) = \delta \).

(3, iv) \( \tilde{a}_{p-1} \notin k^p, \tilde{a}_0 \notin k^p \). A higher derivation \( D \) in \( \mathcal{H}_1(R, R_p) \) is chosen as in (3, ii). Since \( \tilde{a}_{p-1} = b_{p-1} + pc, H_1(\tilde{a}_{p-1}) \in pR_p \). Thus \( D_j(\pi) \in \pi^2R_p \) and for the rest the argument of (3, ii) applies.
representatives of a $p$-basis for $k$. We can assume $a_{p-1}$ in $\mathcal{S}$. Let $H_1$ in $\mathcal{D}(R)$ induce $\delta$ in $\mathcal{D}(k)$. For $j=2, \ldots, p-1$ and $s \in \mathcal{S}$ we let $H_j(s)=0$. For $j=1, \ldots, p-1$ let $D_j=\pi^jH_j$. By (8, 1), $D_j(\pi) \in \pi^3R_p$ ($D_j(\pi) \in \pi^3R_p$ unless $D_1(a_{p-1}) \notin \pi^3R_p$). Also, $D_j(\pi) \in \pi^3R_p$ for $j=2, \ldots, p-1$. The terms $A_p+B_p$ of (8, p) have $[D_1(\pi)]^p$ as the unique summand of minimum valuation, if $D_1(\pi) \notin \pi^3R_p$. In any case, $D_1$ is defined by $D_1(s)=0$ for $s \in \mathcal{S}$, $s \neq a_{p-1}$ and $D_1(a_{p-1}) \in \pi^3R_p$ is chosen so that $D_1(\pi)$ is in $\pi^3R_p$. Finally $D_j(s)=0$ for $s \in \mathcal{S}$ and $j>p$. Again by Theorem A these conditions determine $D$ in $\mathcal{H}_1(R_p, R_p)$. By Lemma 1 $D_j(R) \subseteq \pi^3R_p$ for $j>p$. Again we invoke Lemma 5 with $n=2$, $m=p+1$ and $q=3$ to show that $D \in \mathcal{H}_1(R_p, R_p)$, $a_0 \in \mathfrak{S}_1$ and $\Phi_1(\alpha_0)=\delta$.

Case 4. $v(f'(\alpha))=2p-1$, $\alpha_0 \notin k^p$, $i>1$. Let $H \in \mathcal{H}(R, R)$ be chosen so that $H_j(\alpha_0) \in pR_p$ for $j=1, \ldots, p-1$ and $H_1$ induces a given $\delta \in \mathcal{D}(k)$ for which $\delta(\alpha_0)=0$. Let $D=\{\pi^jH_j\}$. Since by (8, 1) $D_j(\pi) \in \pi^3R_p$ and, by inspection, $D_j(\alpha_0) \in \pi^3R_p$, $s=0, \ldots, p-1$, $j \geq 1$, we see by Lemma 5 that $\sum D_j(\pi) \in \pi^3R_p$. Thus $a_0 \in \mathfrak{S}_1$ and $\Phi_1(\alpha_0)=\delta$. Clearly, $D \in \mathcal{H}_1(R_p, R_p)$.

Case 5. $v(f'(\pi))=2p-1$, $\alpha_0 \notin k^p$, $i>1$. We can assume that $a_0=1+p\beta_0$. Let $H \in \mathcal{H}(R, R)$ be such that $H_1$ induces a given $\delta \in \mathcal{D}(k)$. Let $D=\{\pi^jH_j\}$ and argue as above.

Case 6. $v(f'(\pi))=2p-1$, $\alpha_0 \notin k^p$, $i=1$. Let $\delta \in \mathcal{D}(k)$, $\delta(\alpha_0)=0$, and let $H_1$ in $\mathcal{D}(R)$ induce $\delta$. Let $\mathcal{S}$ be a set of representatives in $R$ of a $p$-basis for $k$ with $a_0$ in $\mathcal{S}$. We define $K_1 \in \mathcal{D}(R)$ as follows: $K_1(a_0)=\pi^p(\alpha_0)$ and $K_1(s)=0$ for $s \in \mathcal{S}$, $s \neq a_0$. By Theorem A, these conditions determine a derivation on $R$. The derivation $D_1=\pi H_1-\pi^p K_1$ has the property $D_1(a_0)=0$ and is the first map of $D \in \mathcal{H}_1(R_p, R_p)$. For the rest, we define $D_j(s)=0$ for $s \in \mathcal{S}$ and $j>1$. By Theorem A, $D \in \mathcal{H}_1(R_p, R_p)$. By Lemma 1 $D_1(R) \subseteq \pi R_p$ and $D_j(R) \subseteq \pi^3R_p$ for $j \geq 1$. The conditions of Lemma 5 are fulfilled for $n=1$, $m=1$ and $q=3$. Moreover, $\Phi_1(\alpha_0)=\delta$.

Case 7. $v(f'(\pi))=2p-1$, $\alpha_0 \notin k^p$, $i=1$. Again, $\pi$ is chosen so that $a_0=1+p\beta_0$. We have the situation (3) with $t=p$ and $b=\beta_0$. Thus, in deference to Lemma 2, we choose $\delta \in \mathcal{D}(k)$ so that $\delta(\beta_0)=0$ and let $H_1 \in \mathcal{D}(R)$ induce $\delta$. Let $H \in \mathcal{H}(R, R)$ be any higher derivation on $R$ with the given $H_1$ as the first map. Let $D=\{D_j\}$ where $D_j=\pi H_j$, $j \geq 1$. Let $n=m=1, q=3$ in Lemma 5 and we conclude that $\sum D_j(\pi) \in \pi^3R_p$. Again we have the desired conclusion and Lemma 6 is proved.

The next series of lemmas are concerned with automorphisms in the “gap” between $\mathfrak{S}_i$ and $\mathfrak{S}_i$.

**Lemma 7.** If $\pi$ is a prime element of $R_p$ and $\pi^p=-\alpha \in k^p$, then, given $i \geq 2$, there is a $D \in \mathcal{H}_i(R_p, R_p)$ such that $a_0 \in \mathfrak{S}_i$ and $a_0(\pi)=\pi^i + \alpha$ where $\alpha$ is any given element of $k$. Hence $\mathfrak{S}_i/\mathfrak{S}_{i-1}$ is isomorphic to $k^*$. 

**Proof.** We assume (6) to be the minimum function of $\pi$ over $R$ and thus $\alpha \notin k^p$. 

Let $\mathcal{S}$ be a set of representatives in $R$ of a $p$-basis for $k$ with $a_0 \in \mathcal{S}$. With $a$ chosen arbitrarily in $R_p$ we define a derivation $D_1$ mapping $R$ into $R_p$ by $D_1(a_0) = -p^{-1}f'(\pi)a$ and $D_1(s) = 0$ for $s \in \mathcal{S}, s \neq a_0$. Then $D_1(R) \subset f'(\pi)\pi^{t-p}R_p$ by Lemma 1 and by (8.1), $D_1(\pi) \equiv \pi a$, mod $\pi^{t+1}R_p$. Let $D_j(s) = 0, s \in \mathcal{S}, j = 2, \ldots, p-1$. If $i=2$, the term $[D_i(\pi)]^p$ in $A_p$ of (8, p) makes it necessary to consider cases.

Case 1. $i > 2$ or $v(f'(\pi)) < 2p - 2$. In this case we let $D_j(s) = 0, s \in \mathcal{S}, j > p-1$. Thus, by Lemma 1, $D_j(R) \subset f'(\pi)\pi^{t-p}R_p, j \geq 1$, and if $j > 1$, $D_j(R) \subset f'(\pi)\pi^{t-p}$ since $f'(\pi) \in \pi^pR_p$. The conditions of Lemma 5 are fulfilled for $n=2, m=p+1$ and $q=1+i+1$. Thus $D$ extends to $R_p$, is uniformly convergent on $R_p$ and $\sum_{j=1}^p D_j(\pi) \in \pi^{t+1}R_p$. In particular then, $\sum_{j=1}^p D_j(\pi) \equiv \pi a$, mod $\pi^{t+1}R_p$.

Case 2. $i = 2, v(f'(\pi)) = 2p - 2$. In this case we choose $D_p(s) = 0, s \in \mathcal{S}, s \neq a_0$ and $D_p(a_0) \in \pi^pR_p$ so that $D_p(\pi)$ will be in $\pi^3R_p$. Again, we let $D_j(s) = 0$ for $j > p$, $s \in \mathcal{S}$ and apply Lemma 5 with $n=2, m=p+1$ and $q=3$, obtaining the same conclusion as in Case 1.

The map $\tau_i: \mathcal{S}_i \to k^+$ given by $\tau_i(\alpha) = \bar{\alpha}$ where $\alpha(\pi) = \pi + \pi a$, is a homomorphism with kernel $\mathcal{K}_i$ and evidently maps onto $k^+$ if $i \geq 2$.

**Lemma 8.** If $\pi$ is a prime element of $R_p$, $\pi^p = -pu$, and $\bar{u} \in k^p$ then $\mathcal{S}_i = \mathcal{S}_i$ for $i > 1$ unless $i = 2$ and of (3) is $p-1$. If $t = p-1$ the following are equivalent.

(a) $\bar{u}$ has a $(p-1)$th root in $k$.

(b) $R_p$ is Galois over $R$.

(c) $\mathcal{S}_2 \neq \mathcal{S}_2$.

(d) $\mathcal{S}_2/\mathcal{S}_2$ is the group of order $p$.

**Proof.** Let $\alpha$ be in $\mathcal{S}_i$. Then $\alpha = e + \pi^i \alpha^*$. The relation

$$[\alpha(\pi)]^p - \pi^p = p[1 + [\alpha(\pi)]'a(v)] - p(1 + \pi^i v)$$

becomes

$$\pi^{t+p-1}\alpha^*(\pi) + \cdots + \pi^{ip}[\alpha^*(\pi)]^p$$

(15)

If $i > 2$ the unique term having minimal valuation on the left side of (15) is $\pi^{t+p-1}\alpha^*(\pi)$. If $p \parallel t$ the unique term of minimal valuation on the right is $pt\pi^{t-1+i}\alpha^*(\pi)$, unless $\alpha^*(\pi)$ is in $\pi R_p$. Thus, either $\alpha^*(\pi) = \pi R_p$ or $t+i-1 = p+i-1$, which cannot be. Thus, if $i > 2$ and $p \parallel t$, then $\alpha \in \mathcal{S}_i$ or $\mathcal{S}_i = \mathcal{S}_i$. If $p \parallel t$ and $i \geq 2$ the left side of (15) has valuation less than the right side unless $\alpha^*(\pi) \in \pi R_p$. Thus again $\mathcal{S}_i = \mathcal{S}_i$.

If $i=2$ and $p \parallel t$, the unique term of minimal valuation on the left side of (15) is $\pi^{2p}[\alpha^*(\pi)]^p$, assuming $\alpha^*(\pi)$ to be a unit. The corresponding term on the right is $pt\pi^{t-1+i}\alpha^*(\pi)v$. Thus, $2\pi = p+t+1$ or $t = p-1$. So, if $t \neq p-1$, then by (15), $\pi^{2p}[\alpha^*(\pi)]^p \equiv (p-1)\pi^p \alpha^*(\pi)v$, mod $\pi^{2p+1}R_p$, or, using (3), $[\alpha^*(\pi)]^{-1} \equiv (p-1)v$, mod $\pi R_p$. Thus, $(p-1)v$, or $\bar{v}$, is a $(p-1)$th root in $k$ and the residue
of \( \alpha^* (\pi) \) is a \((p-1)\)th root of \((p-1)\pi \). We have shown that \((c) \rightarrow (d) \rightarrow (a)\). A theorem of Wishart [4, Theorem 4.15] asserts that \((a) \rightarrow (b)\).

Suppose, finally, that \( \alpha \) in \( \mathfrak{G}_1 \) leaves \( R \) element-wise fixed. Then, if \( \alpha (\pi) = \pi + \pi^rb, \alpha \in \mathfrak{G}_1 \). Thus, if \( \alpha \neq e \), then \( \alpha \in \mathfrak{G}_1, \alpha \notin \mathfrak{S}_2 \) for some \( r > 1 \). Evidently, \( r = 2 \) and \((b) \rightarrow (c)\). This fact was also observed by Wishart [4, Corollary 4.16] who noted that if \( \bar{u} \in k^p \) then \( R_p \) is Galois over \( R \) if and only if \( \mathfrak{G}_2 \neq \mathfrak{S}_2 \). It follows from Lemma 7 that if \( \bar{u} \notin k^p \), then \( \mathfrak{G}_2 \) can be different from \( \mathfrak{S}_2 \) without \( R_p \) being Galois over \( R \).

**Lemma 9.** If \( \mathfrak{G}_2 \neq \mathfrak{S}_2 \), then, for each \( \alpha \in \mathfrak{G}_2 \), there is a \( D \) in \( \mathcal{H}_u (R_p, R_p) \) such that \( aD^{-1} \in \mathfrak{G}_2 \) if and only if, in \((3)\), \( \bar{v} \notin k^p \).

**Proof.** Assuming first that \( \bar{v} \notin k^p \) it follows from Lemma 8 that in \((3)\), \( t = p - 1 \) and \( \bar{v} \) is a \((p-1)\)th root in \( k \). Assuming \((6)\) to be the minimal polynomial of \( \pi \) over \( R \), relation \((3)\) with \( t = p - 1 \) implies that \( a_1, \ldots, a_{p-2} \) are in \( pR, \bar{a}_{p-1} (-\bar{v}) \) is a \((p-1)\)th root in \( k \), \( v(f'(\pi)) = 2p - 2 \) and \( a_0 = 1 + pb_0 \).

Let \( w \) be a unit in \( R_p \) such that \( \bar{w} \) is a \((p-1)\)th root of \( \bar{a}_{p-1} (-\bar{v}) \). We wish to construct \( D \in \mathcal{H}_u (R_p, R_p) \) such that \( aD \in \mathfrak{G}_2 \) and \( aD(\pi) = \pi^2w, \mod \pi^3R_p \).

Let \( \mathcal{S} \) be a set of representatives in \( R \) for a \( p \)-basis of \( k \) chosen to include \( a_{p-1} \). Then \( D_1 \) is defined by \( D_1(a_{p-1}) = -f'(\pi)\pi^2w/pn^{p-1}, D_1(s) = 0 \) for \( s \in \mathcal{S}, s \neq a_{p-1} \). By Lemma 1 \( D_1(R) \subseteq \pi^3R_p \) and by \((8.1)\) \( D_1(\pi) = \pi^2w, \mod \pi^3R_p \). For \( j = 2, \ldots, p - 1 \) and \( s \in \mathcal{S} \), \( D_1(s) = 0 \). By \((8, 2)\) to \((8, p - 1)\), \( D_1(\pi) = \pi^{p-1}R_p \). The term \( [D_1(\pi)]^p \) in \((8, p)\) leads us to define \( D_p \) by \( D_p(a_{p-1}) = -\pi^{p+1}w/pn^{p-1} \) and \( D_p(s) = 0, s \notin \mathcal{S}, s \neq a_{p-1} \). Since each term of \((8, p)\) in \( A_p + B_p \) is in \( \pi^{p+1}R_p \), save \([D_1(\pi)]^p \) and \([D_1(\pi)]^p \equiv \pi^2w, \mod \pi^{p+1}R_p \), we have \( D_p(\pi) = \pi^3R_p \). Finally, we let \( D_1(s) = 0 \) for \( s \in \mathcal{S} \) and \( j > p \). Then \( D_1(R) \subseteq \pi^3R_p \) for \( j > p \) and by Lemma 5 with \( n = 2, m = p + 1 \) and \( q = 3 \), we conclude that \( \sum D_1(\pi) \) converges and \( \sum_{j=2}^{p-3} D_1(\pi) = 0 \), \( D_j(\pi) = 0 \), \( j = 2, \ldots, p-1 \). By \((8, 2)\) \( D_1(\pi) = 0 \) for \( j > p \) and \( D_1(R) \subseteq \pi^3R_p \). Thus, \( \alpha_D \) is in \( \mathfrak{G}_2 \). We have shown that \( \alpha_D(\pi) = \pi^2R_p \) and it is shown below that

\[(16) \quad \alpha_D(s) - s = \pi^2R_p, \quad s \in \mathcal{S}.\]

If \( s \notin \mathcal{S}, s \neq a_{p-1} \) then \( \alpha_D(s) = s \) by definition of \( D \). Since \( D_1(a_{p-1}) = 0 \) for \( j > 1 \), \( p \) it is sufficient to show that \( D_1(a_{p-1}) + D_p(a_{p-1}) = \pi^3R_p \). Now, \( f'(\pi) = (p-1)a_{p-1} \pi^{p-2}, \mod \pi^{p-1}R_p \). Also \( \pi^{p-1} R_p = (p-1)a_{p-1}, \mod \pi^{p-1}R_p \). Using these facts as well as the congruence \( \pi^p \equiv -p, \mod \pi^{p+1}R_p \), leads to the conclusion \( D_1(a_{p-1}) + D_p(a_{p-1}) = -f'(\pi)\pi^2w/pn^{p-1} - \pi^2w/pn^{p-1} = \pi^3R_p \).

Since \( \alpha_D \) is inertial, \( \alpha_D(\pi^p) = \pi^pR \) and every unit in \( R \) is, mod \( pR \), a polynomial in elements of \( \mathcal{S} \) with coefficients in \( R_p \). It follows that \( \alpha_D(a) - a = \pi^3R_p \) for \( a \) in \( R \). Thus \( \alpha \) is in \( \mathfrak{G}_2 \).

It was shown in the proof of Lemma 8 that if \( \alpha \in \mathfrak{G}_2 \) then \( \alpha = \epsilon + \pi^2a^* \) and \( a \neq \mathfrak{S}_2 \) or the residue of \( \pi^* (\pi) \) is a \((p-1)\)th root of \((p-1)\pi = \bar{a}_{p-1} (-\bar{v}) \). Thus if we choose \( w \), in the construction of \( D \), to be \( \pi^* (\pi) \), then \( \alpha_D^{-1} \in \mathfrak{G}_2 \).

If \( \bar{u} \in k^p \) then \( \bar{v} = \bar{v}_0^p + \bar{v}_1 \). Thus \( \bar{a}_{p-1} = \bar{v}_0^p + \bar{b}_0 = \bar{v}_1 \). Since, again, \( a_0 = 1 + pb_0 \), we choose \( c_0 \) and \( c_1 \) in \( R \) so that \( a_{p-1} = c_0^p + pc_1 \). Let \( D \in \mathcal{H}_u (R_p, R_p) \) be such that...
There is, then, a first index \( j \) such that \( f^{D_j}(\pi) \notin f'((\pi)\pi^j R_p) \) and \( f^{D_j}(\pi) \notin f'(\pi)\pi^{j+1} R_p \). This requires that \( D_j(c_p + pc_1) \notin \pi \pi^{j+1} R_p \) and \( D_j(c_p + pc_1) \notin \pi^{j+1} R_p \). However, \( D_j(R) \subset \pi R_p \) and hence \( D_j(c_p + pc_1) \notin \pi^{j+1} R_p \). We have a contradiction. Thus \( \Theta_2 \cap \Theta_D \subset \Theta_2 \), and Lemma 9 is proved.

For \( i > 1 \) and \( \alpha \in \Theta_i \) there is a \( D \in \mathcal{H}_\alpha(R_p, R_p) \) such that \( D(R_p) \subset \pi^i R_p \) (see (12)) and \( \alpha \alpha_D \in \Theta_{i+1} \). Also if \( i > 2 \) and \( \alpha \in \Theta_i \) then \( \alpha \in \Theta_i \) or there is a \( D \in \mathcal{H}_\alpha(R_p, R_p) \) such that \( D(R_p) \subset \pi^i R_p \) and \( \alpha \alpha_D \in \Theta_i \). This follows from Lemma 7, Case 1 of the proof of Lemma 7 and Lemma 8. Thus, given \( \alpha \in \Theta_2 \) there is a sequence \( \{D^{(n)}\} \), \( D^{(n)} \in \mathcal{H}_\alpha(R_p, R_p) \) such that \( D^{(n)}(R_p) \subset \pi^n R_p \) where \( \lim_n s_n = \infty \), and \( \alpha = \alpha_{D_1} \alpha_{D_2} \cdots \alpha_{D_{2n}} \mod \Theta_{n+2} \). By Lemma 3, there is a \( D \in \mathcal{H}_\alpha(R_p, R_p) \) such that \( \alpha = \alpha_D \).

By Lemma 6 and Lemma 9, we conclude that \( \Theta_2 \) and \( \Theta(R_p, R) \) together generate \( \Theta \). If \( \beta \) is an automorphism on \( R \) and \( D \in \mathcal{H}_\beta(R, R) \) then \( H = \{H_i\} \) where \( H_i = \beta^{-1} D_i \beta \) is also in \( \mathcal{H}_\beta(R, R) \). If \( D \) converges uniformly so does \( H \). Thus \( \Theta_D \) is an invariant subgroup of \( G \) the automorphism group of \( R_p \). Hence \( \Theta_1 = \Theta_2 \cdot \Theta(R_p, R) \). These observations along with Lemmas 8 and 9 prove Theorem 1. Theorem 2 follows directly from Lemmas 2, 4, 6, 7 and 8.

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**Florida State University, Tallahassee, Florida**