ABSTRACT. Let \( E \) be a locally convex space and let \( T \) be a semigroup of semicharacters on an idempotent semigroup. It is shown that there exists an isomorphism between the space of \( E \)-valued functions on \( T \) and the space of all \( E \)-valued finitely additive measures on a certain algebra of sets. The space of all \( E \)-valued functions on \( T \) which are absolutely continuous with respect to a positive definite function \( F \) is identified with the space of all \( E \)-valued measures which are absolutely continuous with respect to the measure \( m_F \) corresponding to \( F \). Finally a representation is given for the operators on the set of all \( E \)-valued finitely additive measures on an algebra of sets which are absolutely continuous with respect to a positive measure.

Introduction. Functions of bounded variation and absolutely continuous functions have been studied by several authors including [1], [2], [5], [6], [7], [8] and [9]. In [6] a representation is obtained of the linear functionals on \( AC(I) \) which are continuous in the bounded-variation norm in terms of the \( v \)-integral. In [8] and [9] the concepts of bounded variation and absolutely continuous are developed on idempotent semigroups. In [1] and [2] the results of [8] and [9] are used to extend the \( v \)-integral characterization of functionals in [6] to a \( v \)-integral characterization of normed vector space-valued operators on normed vector space-valued absolutely continuous functions on an idempotent semigroup.

In this paper we study the space of functions from a semigroup \( T \) of semicharacters on an idempotent semigroup \( S \) into a locally convex space \( E \). We identify \( E \)-valued functions on \( T \) with \( E \)-valued finitely additive measures on a certain algebra of subsets of \( S \), and then represent operators on this set of finitely additive measures. To this end we adopt the notation and development in [8] and [9].
1. Definitions. Let $S$ be an abelian idempotent semigroup and let $T$ be a semigroup of semicharacters on $S$ containing the identity semicharacter. Let $A$ denote the algebra of subsets of $S$ generated by the sets $J_f$, $f \in T$ (see [8]). Assume that $E$ is a real locally convex Hausdorff space and let $\{p: p \in I\}$ be a generating family of continuous seminorms on $E$ which is directed, i.e., given $p_1$, $p_2$ in $I$ there exists $p \in I$ such that $p \geq p_1$, $p_2$ (pointwise). For each $p \in I$, let $BV(T, E)_p$ denote the space of all functions $G: T \to E$ for which

$$\|G\|_{BV,p} = \sup \sum \tau p(L(Z, \tau)) < \infty$$

where the supremum is taken over the collection of all finite subsets $Z$ of $T$. Set

$$BV(T, E) = \bigcap_{p \in I} BV(T, E)_p.$$ 

Clearly $BV(T, E)$ is a real vector space. Let $\theta$ denote the locally convex topology on $BV(T, E)$ generated by the family of seminorms $\{\|\cdot\|_{BV,p}: p \in I\}$. If $G$ is a real-valued function on $T$ and $x \in E$, then $Gx$ is defined on $T$ by $(Gx)(f) = G(f)x$. Clearly

$$\{Gx: G \in BV(T), x \in E\} \subset BV(T, E).$$

Let now $F$ be positive definite. We denote by $AC(T, E, F)$ the $0$-closure of the space spanned by $Gx$, $x \in E$ and $G$ in the space $AC(T, E)$ of all real functions on $T$ which are absolutely continuous with respect to $F$. If $G \in AC(T, E, F)$, we say that $G$ is absolutely continuous with respect to $F$ and write $G \ll F$.

2. Finitely additive $E$-valued measures on $A$. Let $p \in I$. We denote by $M_p(A, E)$ the collection of all finitely additive $E$-valued measures $m$ on $A$ for which $\sup \Sigma_{j=1}^\infty p(m(B_j)) = \|m\|_p < \infty$ where the supremum is taken over all finite partitions $\{B_j\}$ of $S$ into sets in $A$. Since every element of $A$ is a disjoint union of sets of $B$-type, it suffices to take the supremum over partitions of $S$ into sets of $B$-type. Let

$$M(A, E) = \bigcap_{p \in I} M_p(A, E).$$

We denote by $\omega$ the locally convex topology on $M(A, E)$ generated by the seminorms $\{\|\cdot\|_p: p \in I\}$.

Let now $\mu$ be a real-valued finitely additive set function on $A$ and let $m$ be an $E$-valued finitely additive measure on $A$. Let $\Sigma$ denote the algebra of subsets of $S \times S$ generated by the sets $B_1 \times B_2$, $B_1$ and $B_2$ in $A$. Every element of $\Sigma$ can be written as a finite disjoint union of sets of the form $B_1 \times B_2$ with $B_1, B_2$ in $A$. It is not hard to show that there exists a unique $E$-valued finitely additive
measure $\mu \times m$ on $\Sigma$ such that $\mu \times m(B_1 \times B_2) = \mu(B_1)m(B_2)$ for all $B_1, B_2$ in $A$. Let $\pi: S \times S \to S$ be defined by $\pi(s, t) = st$. If $B \in A$, then $\pi^{-1}(B) \in \Sigma$.

We define the convolution $\mu \ast m$ on $A$ by $\mu \ast m(B) = \mu \times m(\pi^{-1}(B))$. It follows easily that $\mu \ast m$ is finitely additive. Moreover, we have the following:

**Lemma 1.** If $\mu$ is bounded and $m \in M_p(A, E)$, then $\mu \ast m \in M_p(A, E)$.

**Proof.** Let $Z = \{f_1, \ldots, f_n\} \subset T$ and $B_\tau = B(Z, \tau)$. By [8] we have

$$\pi^{-1}(B_\tau) = \bigcup \{B_{\tau_1} \times B_{\tau_2} : \tau_1, \tau_2 \text{ in } T, \tau_1 \wedge \tau_2 = \tau \}.$$

Hence

$$p(\mu \ast m(B_\tau)) \leq \sum_{\tau_1 \wedge \tau_2 = \tau} |\mu(B_{\tau_1})| p(m(B_{\tau_2})).$$

Thus

$$\sum_{\tau \in T_n} p(\mu \ast m(B_\tau)) \leq \sum_{\tau_1} |\mu(B_{\tau_1})| \sum_{\tau_2} p(m(B_{\tau_2}))$$

$$\leq \|m\|_p \sum_{\tau_1} |\mu(B_{\tau_1})| \leq \|m\|_p \|\mu\|.$$

This implies that $\|\mu \ast m\|_p \leq \|\mu\| \|m\|_p$.

For $\mu$ a real-valued finitely additive measure on $A$ and for $x$ in $E$ we define $\mu x$ on $A$ by $(\mu x)(B) = \mu(B)x$. If $\mu$ is bounded, then $\|\mu x\|_p = \|\mu\| p(x)$ for each $p$ in $I$. Let now $m$ be a nonnegative finitely additive measure on $A$. A real-valued bounded finitely additive measure $\mu$ on $A$ is said to be absolutely continuous with respect to $m$, and write $\mu \ll m$, if for every $e > 0$ there exists $\delta > 0$ such that $m(B) < \delta$ implies $|\mu|(B) < e$. An $E$-valued measure $\lambda$ on $A$ is said to be absolutely continuous with respect to $m$, and we write $\lambda \ll m$, if $\lambda$ is in the $\omega$-closure of the subspace of $M(A, E)$ spanned by $\mu x$, where $x \in E$ and $\mu$ a bounded real-valued measure on $A$ with $\mu \ll m$.

3. Relationship between $E$-valued functions on $T$ and finitely additive measures on $A$. For each finitely additive $E$-valued measure $m$ on $A$, we define $\hat{m}: T \to E$, $\hat{m}(f) = m(A_f) = \int f dm$. This gives us a map from the set of all finitely additive measures on $A$ into the set of all functions from $T$ into $E$. The following theorem gives the properties of this map.

**Theorem 1.** The map $m \to \hat{m}$, from the space of all finitely additive $E$-valued measures on $A$ into the space of all $E$-valued functions on $T$, is linear one-to-one and onto. Moreover the following hold:

(a) $m \in M_p(B, E)$ iff $\hat{m} \in BV(T, E)_p$.
(b) \( \|m\|_p = \|\hat{m}\|_{BV,p} \).
(c) \( m \ll \alpha \) iff \( \hat{m} \ll \hat{\alpha} \).
(d) \( \hat{\mu} \chi = \hat{\mu} x \) for every real measure \( \mu \) on \( A \) and every \( x \in E \).
(e) \( \mu \ast m = \hat{\mu} \hat{m} \).
(f) If \( h: (M(A, E), \omega) \rightarrow (BV(T, E), \theta) \), \( h(m) = \hat{m} \), then \( h \) is a topological isomorphism.

**Proof.** The proof that the map \( m \rightarrow \hat{m} \) is linear, one-to-one and onto is similar to the one in the scalar case (see [8]). Since \( m(B(Z, \tau)) = L(Z, \tau)\hat{m} \) for each set \( B(Z, \tau) \) of \( B \)-type, it follows easily that \( \|m\|_p = \|\hat{m}\|_{BV,p} \) and hence \( m \in M_p(B, E) \) iff \( \hat{m} \in BV(T, E)_p \). Also, from the \( \hat{\mu} x(f) = (\mu x)(A) = \mu (A) x = \hat{\mu} x \theta \), it follows that \( \hat{\mu} x = \hat{\mu} x \) for each real-valued measure \( \mu \) on \( A \) and each \( x \in E \).

Assume next that \( \alpha \) is a positive measure on \( A \) and \( m \) an \( E \)-valued measure on \( A \). Suppose that \( m \ll \alpha \) and let \( \rho \in I \) and \( \varepsilon > 0 \). There are bounded real-valued measures \( \mu_1, \ldots, \mu_n \) on \( A \), with \( \mu_i \ll \alpha \), and \( x_1, \ldots, x_n \in E \) such that \( \|\Sigma \mu_i x_i - m\|_p < \varepsilon \). Thus \( \|\Sigma \mu_i x_i - \hat{m}\|_{BV,p} = \|\Sigma \mu_i x_i - m\|_p < \varepsilon \). Since \( \hat{\mu_i} \ll \hat{\alpha} \), this shows that \( \hat{m} \ll \hat{\alpha} \). The proof of the converse is similar. Finally, from

\[
\hat{\mu} \ast \hat{m}(f) = \mu \ast m(A) = \mu \times m(\pi^{-1}(A)) = \mu \times m(A_f \times A_f)
\]

it follows that \( \hat{\mu} \ast \hat{m} = \hat{\mu} \hat{m} \).

For \( G \) a real-valued or \( E \)-valued function on \( T \) we denote by \( m_G \) the measure on \( A \) such that \( \hat{m}_G = G \).

Let now \( F \) be a positive definite function on \( T \). For each \( E \)-valued simple function \( s \) we define \( v_s: A \rightarrow E \) by

\[
v_s(B) = \int_B s dm_F.
\]

Clearly \( v_s \ll m_F \). By Theorem 1 we have \( \delta_s \ll F \). The function \( \delta_s = p_s \) is called a polygonal function.

**Theorem 2.** If \( AC(T, F, F) \) is equipped with the \( \theta \)-relative topology, then the collection of all polygonal functions is dense.

**Proof.** Let \( G \in AC(T, E, F) \), \( \rho \in I \) and \( \varepsilon > 0 \). There exist \( G_1, \ldots, G_n \in AC(T, F) \) and \( x_i \in E \) such that \( \|\Sigma G_i x_i - G\|_{BV,p} < \varepsilon/2 \). Each \( G_i \) is of the form \( G_i = \hat{\mu}_i \), \( \mu_i \ll m_F \). By Darst [3], there are simple functions \( s_i \) such that

\[
\sum \|\mu_i - \lambda_i\|_p(x_i) < \varepsilon/2
\]
where \( \lambda_i \) is defined on \( A \) by \( \lambda_i(B) = \int_B s_i \, dm_F \). Let \( s = \Sigma s_i x_i \). Then for the polygonal function \( p_s \) we have

\[
\left\| \sum G_i x_i - p_s \right\|_{BV,p} = \left\| \sum \mu_i x_i - v_s \right\| \leq \sum \left\| \mu_i x_i - v_{s_i} x_i \right\|_p \\
\leq \sum \left\| \mu_i - \lambda_i \right\| \left\| p(x_i) - \lambda_i \right\| < \epsilon/2.
\]

By the triangle inequality we have \( \left\| G - p_s \right\|_{BV,p} < \epsilon \) which completes the proof.

We omit the proof of the following easily established lemma.

**Lemma 2.** If \( E \) is complete, then \( (M(A, E), \omega) \) is complete.

**Theorem 3.** Let \( \hat{E} \) be the completion of \( E \). Then the completion of the space \( AC(T, E, F) \) under the relative \( \theta \)-topology is the space \( AC(T, \hat{E}, F) \).

**Proof.** First of all we observe that \( AC(T, \hat{E}, F) \) is complete as a closed subspace of the complete space \( (BV(T, \hat{E}), \theta) \). For \( p \in I \) let \( \hat{p} \) denote the unique continuous extension of \( p \) to \( \hat{E} \). Let \( G = \Sigma G_i x_i \), with \( x_i \in \hat{E} \) and \( G_i \ll F \), and let \( \epsilon > 0 \) and \( p \in I \). Since \( E \) is dense in \( \hat{E} \) there exist \( y_1, \ldots, y_n \) in \( E \) such that

\[
\sum \left\| G_i \right\|_{BV} \hat{p}(x_i - y_i) < \epsilon.
\]

If \( H = \Sigma G_i y_i \), then \( \left\| G - H \right\|_{BV,\hat{p}} \leq \Sigma \left\| G_i \right\|_{BV} \hat{p}(x_i - y_i) < \epsilon \). This shows that \( G \) is in the closure of \( AC(T, E, F) \) in \( AC(T, \hat{E}, F) \). Since the set \( \{ \Sigma G_i s_i; G_i \in AC(T, F), s_i \in \hat{E} \} \) is dense in \( AC(T, \hat{E}, F) \), the result follows.

4. Representation of continuous operators on spaces of vector measures.

By Theorem 1, the space \( AC(T, E, F) \) can be identified with the space of all \( E \)-valued measures on \( A \) which are absolutely continuous with respect to the positive measure \( m_F \). Therefore, to describe continuous operators on \( AC(T, E, F) \) it suffices to describe continuous operators on the space of measures which are absolutely continuous with respect to \( m_F \). In this section we will study the problem more generally.\(^{(1)}\) Let \( \Sigma \) be an algebra of subsets of a set \( X \). Denote by \( M(\Sigma, E) \) the space of all \( E \)-valued measures \( m \) on \( \Sigma \) such that for each \( p \in I \) we have \( \left\| m \right\|_p = \sup \Sigma p(m(F_i)) < \infty \) where the supremum is taken over the class of all finite partitions \( \{F_i\} \) of \( X \) into sets in \( \Sigma \). Let \( \omega \) denote the locally convex topology on \( M(\Sigma, E) \) generated by the family of seminorms \( \left\{ \left\| \cdot \right\|_p; p \in I \right\} \). Let \( \mu \neq 0 \) a fixed nonnegative finitely additive measure on \( \Sigma \) and let \( AC(\Sigma, E, \mu) \) denote the \( \omega \)-closure in \( M(\Sigma, E) \) of the space spanned by the class of all measures of the form \( \lambda x \) where \( x \in E \) and \( \lambda \) runs through the family of all bounded real-

\(^{(1)}\) The author wishes to thank the referee for suggesting that he look into the problem in this general form.
valued measures on $\Sigma$ which are absolutely continuous with respect to $\mu$. Let $\tau$ be the relative $\omega$-topology on $AC(\Sigma, E, \mu)$. We will represent the continuous linear operators from $(AC(\Sigma, E, \mu), \tau)$ into a locally convex space $H$.

For $s$ an $E$-valued simple function, we define $m_s: \Sigma \to E$ by $m_s(F) = \int_X s \, d\mu$. It is clear that $m_s \in AC(\Sigma, E, \mu)$.

**Lemma 3.** The class of all measures of the form $m_s$ is $\tau$-dense in $AC(\Sigma, E, \mu)$.

**Proof.** Let $m \in AC(\Sigma, E, \mu)$, $p \in I$ and $\epsilon > 0$. By definition there are real-valued measures $\mu_1, \ldots, \mu_n$, which are absolutely continuous with respect to $\mu$, and $x_i \in E$ such that $\|\sum \mu_i x_i - m\|_p < \epsilon/2$. By Darst [3] there are real-valued simple functions $s_i$ such that $\sum \mu_i s_i - X_i \|p(x_i) < \epsilon/2$ where $X_i$ is defined on $\Sigma$ by $X_i(B) = \int_B s_i \, d\mu$. Let $s = \sum s_i x_i$. Then

$$\|m_s - m\|_p \leq \|m - \sum \mu_i x_i\|_p + \sum \|\mu_i - \mu_i\|p(x_i) < \epsilon.$$ 

The lemma is proved.

Let now $H$ be another real locally convex space. We will represent the continuous linear operators from $(AC(\Sigma, E, \mu), \tau)$ into $H$.

**Definition.** Let $K$ be a function on $\Sigma$ with values in the space $L(E, H)$ of all linear operators from $E$ into $H$. Then $K$ is called convex relative to $\mu$ if, whenever $\{B_j\}$ is a finite partition of $\Sigma$ into sets in $\Sigma$, then $K(B) = \sum \alpha_j K(B_j)$ where $\alpha_j = \mu(B_j)/\mu(B)$. We take $0/0 = 0$. According to this convention it follows that $K(B) = 0$ whenever $\mu(B) = 0$. The function $K$ is called bounded if, for each continuous seminorm $q$ on $H$, there exists $p \in I$ such that

$$\sup q(K(B)x) = \|K\|_{p,q} \leq \infty$$

where the supremum is taken over the family of all sets $B$ in $\Sigma$ and all $x \in E$ with $p(x) \leq 1$.

**Definition.** Let $K$ be a convex (relative to $\mu$) bounded $L(E, H)$-valued function on $\Sigma$. The $v$-integral with respect to $K$ of an $E$-valued measure $m$ on $\Sigma$ is defined to be the limit $\Sigma K(B_j)m(B_j)$, when it exists, where the limit is taken over the collection of all finite $\Sigma$-partitions (i.e., partitions into members of $\Sigma$) $\{B_j\}$ of $X$. In this case we say that $m$ is $v$-integrable with respect to $K$ and we denote the integral by $\int m \, dK$.

We omit the proof of the following easily established lemma.

**Lemma 4.** Let $K$ be convex and bounded and let $s$ be an $E$-valued simple function. Then $m_s$ is $v$-integrable. Moreover there exists a finite $\Sigma$-partition $\{B_j\}$ of $X$ such that
\[
v \int m \, dK = \sum K(F_j) m_s(F_j)
\]
for every finite \(\Sigma\)-partition \(\{F_j\}\) of \(X\) which is a refinement of \(\{B_i\}\).

**Lemma 5.** Let \(K\) be convex and bounded and assume that \(H\) is complete. Then:

1. Every \(m\) in \(AC(\Sigma, E, \mu)\) is \(v\)-integrable with respect to \(K\).
2. The map \(m \rightarrow \phi_K(m) = v \mid m \, dK\), from \((AC(\Sigma, E, \mu), \tau)\) into \(H\), is a continuous linear operator. Furthermore, if \(\|K\|_{p,q} < \infty\), then

\[
\|K\|_{p,q} = \sup \{q(\phi_K(m)) : m \in AC(\Sigma, E, \mu), \|m\|_p \leq 1\}.
\]

**Proof.** First of all we observe that, if \(m\) is \(v\)-integrable and if \(\|K\|_{p,q} < \infty\), then

\[
q(\phi_K(m)) \leq \|K\|_{p,q} \|m\|_p.
\]

Let now \(m \in AC(\Sigma, E, \mu), q\) a continuous seminorm on \(H\) and \(\epsilon > 0\). Let \(p \in I\) be such that \(\|K\|_{p,q} = d < \infty\). Let \(s\) be an \(E\)-valued simple function such that \(\|m - s\|_p < \epsilon/2d\).

By Lemma 4, there exists a finite \(\Sigma\)-partition \(\{B_i\}\) of \(X\) such that \(\phi_K(m_s) = \Sigma K(F_j) m_s(F_j)\) for each finite \(\Sigma\)-partition \(\{F_j\}\) of \(X\) which is a refinement of \(\{B_i\}\). Now if \(\{F_j\}\) and \(\{G_i\}\) are both \(\Sigma\)-partitions of \(X\) which are refinements of \(\{B_i\}\), then

\[
q\left(\sum K(F_j) m(F_j) - \sum K(G_i) m(G_i)\right)
\leq q\left(\sum K(F_j) m(F_j) - \sum K(F_j) m_s(F_j)\right) + q\left(\sum K(G_i) m_s(G_i) - \sum K(G_i) m(G_i)\right)
\leq d \|m - m_s\|_p + d \|m - m_s\|_p < \epsilon.
\]

This shows that the net \(\{\Sigma K(F_j) m(F_j)\}\) is a Cauchy net in \(H\) and hence convergent. This proves (1).

The inequality (*) implies that \(\phi_K\) is a continuous linear map and that

\[
\|K\|_{p,q} \geq \|\phi_K\|_{p,q} = \sup \{q(\phi_K(m)) : m \leq \mu, \|m\|_p \leq 1\}.
\]

Let now \(\epsilon > 0\) be given. There exists \(F \in \Sigma\) and \(x \in E\), with \(p(x) \leq 1\), such that

\[
q(K(F)x) > \|K\|_{p,q} - \epsilon = a.
\]

Let \(s = \chi_F x\). Then \(\|m_s\|_p = p(x) \mu(F) \leq \mu(F)\). Also \(K(F) m_s(F) = \mu(F) K(F)x\).

Thus

\[
q(\phi_K(m_s)) = \mu(F) q(K(F)x) \geq a \mu(F).
\]
Hence \( \|K\|_{p,q} \geq a = \|K\|_{p,q} - \epsilon \). Since \( \epsilon > 0 \) was arbitrary, the result follows.

**Lemma 6.** If \( \phi \) is a continuous linear operator from \( AC(\Sigma, E, \mu) \) into \( H \), then there exists a unique convex bounded function \( K \) such that \( \phi = \phi_K \).

**Proof.** For each \( F \) in \( \Sigma \), let \( \lambda_F \) be defined on \( \Sigma \) by
\[
\lambda_F(B) = \mu(B \cap F)/\mu(F).
\]
Clearly \( \lambda_F \) is a bounded real-valued measure on \( \Sigma \) and \( \lambda_F \ll \mu \). We define \( K \) on \( \Sigma \) by
\[
K(F)x = \phi(\lambda_F x), \quad x \in E.
\]
Then \( K \) is an \( L(E, H) \)-valued function. It is easy to see that \( K \) is convex relative to \( \mu \). Also \( K \) is bounded. In fact, let \( q \) be a continuous seminorm on \( H \). Since \( \phi \) is continuous there exist \( p \in I \) and \( M > 0 \) such that \( q(\phi(m)) \leq M \) whenever \( \|m\|_p \leq 1 \). Thus \( q(K(B)x) \leq M \) whenever \( p(x) \leq 1 \) since \( \|\lambda_B x\|_p \leq 1 \) whenever \( p(x) \leq 1 \). This proves that \( K \) is bounded. Next we show that \( \phi = \phi_K \). To this end, it suffices, by Lemma 3, to show that \( \phi(m_s) = \phi_K(m_s) \) for each \( E \)-valued simple function \( s \). Since \( m_{s_1 + s_2} = m_{s_1} + m_{s_2} \), it suffices to prove the claim for any \( s \) of the form \( s = x\chi_F, F \in \Sigma, x \in E \). But, for \( s = x\chi_F \), we have \( m_{s} = \mu(F)\lambda_F x \). Thus
\[
\phi(m_s) = \mu(F)\phi(\lambda_F x) = \mu(F)K(F)x = K(F)m_s(F) = \phi_K(m_s).
\]
It follows that \( \phi = \phi_K \). Finally, assume that \( K_1 \) is another convex bounded function such that \( \phi = \phi_{K_1} \). If \( K_2 = K - K_1 \), then \( \phi_{K_2} = 0 \). We claim that \( K_2 = 0 \). Assume the contrary and let \( F \in \Sigma \) be such that \( K_2(F) \neq 0 \). Choose \( x \in E \), with \( K_2(F)x \neq 0 \), and set \( s = \chi_F x \). Then \( 0 = \phi_{K_2}(m_s) = \mu(F)K_2(F)x \neq 0 \) since \( \mu(F) \neq 0 \). This contradiction completes the proof.

Combining the preceding lemmas we get the following representation theorem.

**Theorem 4.** If \( H \) is complete, then the map \( K \mapsto \Phi_K \) is a one-to-one linear map from the space of all convex (relative to \( \mu \)) bounded \( L(E, H) \)-valued functions onto the space of all continuous linear operators from \( AC(\Sigma, E, \mu) \) into \( H \).

**Bibliography**


DEPARTMENT OF MATHEMATICS, SOUTHERN ILLINOIS UNIVERSITY, CARBONDALE, ILLINOIS 62901

Current address: Instituto de Matemática, Universidade Estadual de Campinas, Caixa Postal 1170, 13100 Campinas, São Paulo, Brazil