ADDENDUM TO
"BEHNKE-STEIN THEOREM FOR ANALYTIC SPACES"

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ABSTRACT. A very simple argument shows that Theorem 3.1 in my paper
Behnke-Stein theorem for analytic spaces, (these Transactions, 199 (1974), pp. 317
326) is enough, via a Narasimhan result, to obtain information about the torsion of
the homology groups of a Runge pair of Stein spaces.

Let \((X, Y)\), with \(Y \subset X\), \(Y\) open, be a pair of reduced complex analytic
spaces of (complex) dimension \(n\). Andreotti and Narasimhan proved in [1], among
many others, the following result:

(1.1) If \((X, Y)\) is a Runge pair of Stein spaces (a 1-Runge pair in the term-

ology of [3]) and every singularity of \(X\) outside \(Y\) is isolated, then

\[ H_r(X \mod Y, \mathbb{Z}) = 0 \]

for \(r \geq n + 1\).

We wish to show the following statements (1.2) and (1.3),

(1.2) (Narasimhan [2, Theorem 3]): if \(X\) is a Stein space, then:

\[ H_r(X, \mathbb{Z}) = 0 \quad \text{for} \quad r \geq n + 1 \]

and

\[ H_n(X, \mathbb{Z}) \text{ is without torsion;} \]

(1.3) (Silva [3, Theorem 3.1]): if \((X, Y)\) is a Runge pair of Stein spaces
(or, equivalently, in the terminology of [3], a 1-Runge pair of cohomologically
1-complete spaces) then

\[ H_{n+1}(X \mod Y, \mathbb{C}) = 0; \]

make us able to remove from (1.1) the assumption that the singularities of \(X\) out-
side \(Y\) are isolated.

Indeed, if we write the exact homology sequence for the pair \((X, Y)\):

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\[ \cdots \longrightarrow H_r(X, Z) \longrightarrow H_r(X \mod Y, Z) \xrightarrow{\partial_r} H_{r-1}(Y, Z) \longrightarrow \cdots \]

from (1.2) we obtain that \( H_r(X \mod Y, Z) = 0 \) for \( r > n + 1 \). Suppose now \( r = n + 1 \). (1.3) implies that \( H_{n+1}^+(X \mod Y, Z) \) is a torsion group. If we look again at the exact homology sequence for \( (X, Y) \) we see that the morphism:

\[ \varphi_{n+1} : H_{n+1}^+(X \mod Y, Z) \longrightarrow H_n(Y, Z) \]

is injective, so that, \( H_n(Y, Z) \) being without torsion, we must have \( H_{n+1}^+(X \mod Y, Z) = 0 \).

In conclusion we have obtained the following

**Theorem.** If \((X, Y)\) is a Runge pair of Stein spaces, then

\[ H_r(X \mod Y, Z) = 0, \]

for \( r \geq n + 1 \).

**BIBLIOGRAPHY**