ERRATUM TO A "CONSTRUCTIVE ERGODIC THEOREM"

BY

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Reinhard Lang has kindly pointed out to me that the inequality at the top of p. 129 of [1] is incorrect. The argument, beginning at the bottom of p. 128, should therefore be amended to read:

Let \( \varepsilon > 0 \). By the preceding, there is an increasing sequence of integers \( \{n(i) : i = 1, 2, 3, \ldots \} \) such that, for each positive integer \( i \), the set

\[
A_{n(i),m} = \{ x : \max [ |f^k(x) - f^j(x)| : n(i) < k, j < m ] > \varepsilon \cdot 2^{-i} \}
\]

has measure less than \( \varepsilon \cdot 2^{-i} \) for all integers \( m \geq n(i) \). As a consequence, the series \( \sum_{i=1}^{\infty} \mu(A_{n(i),n(i+1)}) \) converges so that the set

\[
A(\varepsilon) = \bigcup_{i=1}^{\infty} A_{n(i),n(i+1)}
\]

is integrable and has measure less than \( \varepsilon \).

Now if \( x \in -A(\varepsilon) \), then \( |f^k(x) - f^j(x)| \leq \varepsilon \cdot 2^{-i} \) for \( n(i) < j, k < n(i + 1) \) and for each positive integer \( i \). So far any positive integers \( k, j, p, q \) satisfying \( n(p+1) \geq k \geq n(p) \geq n(q+1) \geq j \geq n(q) \) and for each \( x \in -A(\varepsilon) \), we have the estimate

\[
|f^k(x) - f^j(x)| \leq |f^k(x) - f^p(x)|
\]

\[
+ \sum_{i=q+1}^{p-1} |f^{n(i+1)}(x) - f^n(x)| + |f^{n(q+1)}(x) f^j(x)|
\]

where the right-hand side is dominated by

\[
\sum_{i=q}^{p} \varepsilon \cdot 2^{-i} < \varepsilon \cdot 2^{-q+1}.
\]

This estimate implies that the sequence \( \{f^n\} \) is uniformly Cauchy on \( -A(\varepsilon) \). Since \( \varepsilon > 0 \) was arbitrary, we can conclude, in particular, that the sequence \( \{f^n\} \) is Cauchy almost everywhere.

REFERENCE