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Z_p ACTIONS ON SYMPLECTIC MANIFOLDS

BY

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ABSTRACT. A bordism classification is studied for periodic maps of prime period p preserving a symplectic structure on a smooth manifold. In sharp contrast to the corresponding oriented bordism, this theory contains nontrivial p-torsion even when p is odd. Calculation gives an upper limit on the size of this p-torsion.

1. **Introduction.** Let p be a prime. This note considers the bordism classification of smooth Z_p actions preserving a symplectic structure. Since the coefficient ring, the symplectic bordism ring Sp_*, is not completely known, we cannot expect a complete classification. However, we will discover that symplectic equivariant bordism differs in significant ways from oriented equivariant bordism. Thus the subject is probably worth further study.

This paper began in a conversation with R. E. Stong, who observed that Proposition 3 is the correct description of the fixed point classification for symplectic Z_p actions. I am indebted to Professor Stong for his patience in discovering several errors in preliminary versions of the paper.

2. **Symplectic group actions.** Conner and Floyd defined the notion of a unitary group action in [3, p. 576]. We can easily extend their ideas to define a symplectic group action.

Specifically, let $G \times M \longrightarrow M$ be a smooth action of the finite group G on an n-manifold M. Let τ be the tangent bundle of M, and for $k > n/4$ let $\tau(k)$ be the Whitney sum of τ and a trivial $(4k - n)$-plane bundle. The manifold M is then symplectic if and only if the classifying map $M \longrightarrow BO(4k)$ for $\tau(k)$ lifts to $BSp(k)$ for all sufficiently large k.

Given such a lifting f, there exist bundle automorphisms I and J on $\tau(k)$, covering the identity map of M, such that $I^2 = J^2 = -1$ and $IJ = -JI$. The homotopy class of f determines the homotopy classes of I and J. Conversely, the existence of I and J implies that $\tau(k)$ is quaternionic and hence that some lifting f exists.

Every element $g \in G$ acts on $\tau(k)$ via dg on τ and the identity map on the trivial bundle. Suppose that for suitable f, and for every $g \in G$, this mapping

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$dg \times 1$ commutes with I and J. We then say that the action of G preserves the symplectic structure of M given by f.

Let $F' \subset F$ be families of subgroups of G, as defined by [6, p. 3], with F' possibly empty. Then $Sp_*(G, F, F')$ is the bordism module (over Sp_*) of structure-preserving G actions on symplectic manifolds M, such that the isotropy subgroup G_m is in F for all $m \in M$, and in F' for all $m \in \partial M$. For a full definition see [6, §2]. We write *free* for the family $\{\{1\}\}$ and *all* for the family of all subgroups of G. We write

$$\sigma\colon Sp_*(G, F, F') \longrightarrow SO_*(G, F, F')$$

for the homomorphism that forgets that a G action preserves symplectic structure, but remembers that it is orientation preserving.

PROPOSITION 1. *For any finite group G, there is an isomorphism $Sp_*(G, \mathit{free}) \cong Sp_*(BG)$, which assigns to a free G action on M the map $M/G \longrightarrow BG$ classifying the quotient map $M \longrightarrow M/G$.*

Since M/G is clearly a symplectic manifold, the proof is exactly like that of [2, (19.1)].

3. **Maps of prime period.** We specialize to the case $G = Z_p$, where p is a prime.

PROPOSITION 2. *Let $\partial[M, \phi] = [\partial M, \phi | G \times \partial M]$ for any action $\phi\colon G \times M \longrightarrow M$. Then there is a long exact sequence*

$$\begin{aligned} \ldots \longrightarrow Sp_*(Z_p, \mathit{free}) &\xrightarrow{r} Sp_*(Z_p, \mathit{all}) \\ &\xrightarrow{s} Sp_*(Z_p, \mathit{all}, \mathit{free}) \xrightarrow{\partial} Sp_*(Z_p, \mathit{free}) \longrightarrow \ldots \end{aligned}$$

in which r and s are the forgetful homomorphisms.

The proof is standard; see [6, Proposition 2.2].

Given a sequence $(n) = (n_1, n_2, \ldots, n_{(p-1)/2})$ of nonnegative integers, write $N = \Sigma_k n_k$ and $BU((n)) = \Pi_k(BU(n_k))$.

PROPOSITION 3. *If $p = 2$, then there is an isomorphism*

$$Sp_m(Z_2, \mathit{all}, \mathit{free}) \cong \sum_{4k \leq m} Sp_{m-4k}(BSp(k)). \tag{1}$$

If p is odd, then there is an isomorphism

$$Sp_m(Z_p, \mathit{all}, \mathit{free}) \cong \sum_{4N \leq m} Sp_{m-4N}(BU((n))), \tag{2}$$

where the sum is over all sequences (n) having $4N \leq m$.

PROOF. Let p be any prime, and consider a Z_p action on M, preserving a symplectic structure described by bundle maps I and J. Then I also describes an underlying weakly complex structure on M.

Let F be a component of the fixed set of Z_p. Then F is a submanifold [2, §22] and the embedding of a tubular neighborhood converts its normal bundle ν into a bundle with Z_p action. We have $\tau(k)|F = \tau_F \oplus \nu \oplus (4k - n)$. Since I and J are equivariant, ν is invariant under I and J. Thus ν is quaternionic and F is symplectic.

For $p = 2$ this is all we need to know. Classifying the bundles ν gives a homomorphism from the left side of (1) to the right side. It is an isomorphism, since M is equivariantly bordant to the disjoint union of the tubular neighborhoods $D\nu$, where the latter have antipodal Z_2 action.

For p odd, ν splits as a sum of complex bundles $\nu_1 \oplus \nu_2 \oplus \cdots \oplus \nu_{p-1}$, where the action of a generator T of Z_p on ν_k is multiplication by $b^k = \exp(2\pi ik/p)$. Each ν_k is invariant under I, of course. However, J is an isomorphism from ν_k to ν_{p-k} for each k, for if $T(v) = \exp(2\pi ik/p)v$ then

$$T(Jv) = \exp(2\pi ik/p)Jv = J(\exp(2\pi i(p-k)/p)v).$$

Thus a homomorphism from the left side of (2) to the right side is given by classifying the ν_k, $1 \leqslant k < p/2$. In the other direction, given complex bundles ν_k, let ν be the direct sum of the $\nu_k \oplus \bar{\nu}_k$, with T acting on ν_k as multiplication by b^k and on $\bar{\nu}_k$ as multiplication by b^{-k}, and with $J =$ conjugation. Then $D\nu$ is a manifold with symplectic Z_p action.

Remarks. As a result of (1), $Sp_*(Z_2, \mathit{all}, \mathit{free})$ is known from work of P. S. Landweber [4, Theorem 4.1]. The right side of (2) is more mysterious. For odd p, notice the effect of the homomorphism

$$\sigma\colon Sp_*(Z_p, \mathit{all}, \mathit{free}) \longrightarrow SO_*(Z_p, \mathit{all}, \mathit{free}).$$

If we combine (2) with §38 of [2], we see that the class of $\nu \oplus \bar{\nu}$ in $Sp_*(BU((n)))$ is sent to the class of $\nu \oplus \nu$ in $SO_*(BU((n)))$.

4. Maps of odd prime period. For the rest of the paper, p will be an odd prime.

Proposition 4. *The homomorphism*

$$Sp_*(BZ_p, *) \longrightarrow SO_*(BZ_p, *),$$

on the reduced bordism groups of BZ_p, *is an isomorphism.*

Proof. We know Sp_* and SO_* are isomorphic modulo 2-torsion [5]. Hence the same is true of $Sp_*(X, A)$ and $SO_*(X, A)$, for any CW-pair (X, A). On the other hand, $SO_*(BZ_p, *)$ contains only p-torsion [2, p. 90], and by similar considerations this is also true of $Sp_*(BZ_p, *)$.

The SO_*-module structure of $SO_*(BZ_p, *)$ is described completely by [2, Theorem (36.5)]. Let $\mu\colon Sp_*(BZ_p, *) \longrightarrow H_*(BZ_p, *)$ be the homomorphism

$\mu[M, f] = f_*[M]$, where $[M] \in H_*(M)$ is the orientation class. Then $Sp_*(BZ_p, *)$ has one Sp_*-module generator in each odd dimension, and a generator $x_j \in Sp_{2j-1}(BZ_p, *)$ is characterized by $\mu(x_j) \neq 0$.

Write $\eta = \exp(2\pi i/p)$ and let a generator $t \in Z_p$ act on a unit vector in $\mathbf{C}^j$ by

$$t(z_1, \ldots, z_j) = (\eta z_1, \ldots, \eta z_j). \tag{3}$$

This free Z_p action on S^{2j-1} yields a bordism element in BZ_p with $\mu \neq 0$, but it is only symplectic if j is even.

In dimensions $4m + 1$ we need new generators. If n is odd, $\mathbf{C}P(n)$ is a symplectic manifold. Let $\xi \longrightarrow \mathbf{C}P(n)$ be the canonical complex line bundle and write $\nu = \xi + k\mathbf{C}$. Then $\nu + \bar{\nu} \longrightarrow \mathbf{C}P(n)$ is a symplectic bundle, isomorphic as an oriented bundle to $2\nu \longrightarrow \mathbf{C}P(n)$. Thus

$$\partial\sigma[\nu + \bar{\nu}] = [S\lambda_k, \theta] \in SO_{2(n+k)+1}(Z_p, \mathit{free}),$$

where $\lambda_k \longrightarrow \mathbf{C}P(2\nu)$ is the canonical complex line bundle and $\theta(t, -)$ is multiplication by η in the fibers of $S\lambda_k$.

Proposition 5. *If* $n = 1$ *and* $k = m - 1$, *then*

$$\mu[S\lambda_k, \theta] \neq 0 \in H_{4m+1}(BZ_p, *).$$

Proof. There is the following commutative diagram:

$$\begin{tikzcd}
 & S\lambda_k \arrow[r] \arrow[d, "\pi_1"'] & S^\infty \arrow[d] \\
 & S_k/Z_p \arrow[r, "f_k"] \arrow[d, "\pi_2"'] & BZ_p \arrow[d, "\pi'"] \\
\mathbf{C}P(1) & \mathbf{C}P(2\nu) \arrow[l, "q"'] \arrow[r, "g"] & BU(1)
\end{tikzcd}$$

Here f_k and g classify the bundles π_1 and $\pi_2\pi_1$, respectively, and q is the obvious projection. Let $\alpha_1 \in H^2(BU(1))$ be the universal Chern class. In the cohomology of $\mathbf{C}P(2\nu)$ there is the relation

$$g^*(\alpha_1)^{2m} = q^*c_1(2\nu)g^*(\alpha_1)^{2m-1},$$

whence

$$g^*(\alpha_1)^{2m}[\mathbf{C}P(2\nu)] = \pm c_1(2\nu)[\mathbf{C}P(1)] = \pm 2c_1(\nu)[\mathbf{C}P(1)] \neq 0.$$

Thus $g_*[\mathbf{C}P(2\nu)] \neq 0 \in H_{4m}(BU(1))$. It follows, for example by considering the spectral sequences of π_2 and π', that $(f_k)_*[S\lambda_k/Z_p] \neq 0$, as required. This completes the proof.

Summarizing, we make the following choice of Sp_*-module generators for $Sp_*(BZ_p, *)$. In dimension $4m - 1$ there is the usual inclusion $S^{4m-1}/Z_p \subset BZ_p$. If $m \geqslant 1$ there is in dimension $4m + 1$ the example f_k: $S\lambda_k/Z_p \longrightarrow BZ_p$ just constructed. In dimension 1 we may take $[S^1, i]$, where i: $S^1 \longrightarrow BZ_p$ is inclusion.

5. **Which free actions bound?** In this section, we will determine what we can of the homomorphism

$$Sp_* \oplus Sp_*(BZ_p, *) \cong Sp_*(Z_p, free) \xrightarrow{r} Sp_*(Z_p, all).$$

First, the restriction of r to the summand Sp_* sends $[M]$ to the class of $Z_p \times M$, where Z_p acts by multiplication on itself, and acts trivially on M. This must be a monomorphism, since Sp_* has no elements of order p.

Second, let θ: $Z_p \times S^1 \longrightarrow S^1$ by $\theta(t, z) = \eta z$. If $(S^1, \theta) = \partial(M, \phi)$ then the fixed set of Z_p in M would have to have codimension at least 4. Thus r: $Sp_1(BZ_p, *) \longrightarrow Sp_1(Z_p, all)$ is a monomorphism. In particular, $Sp_*(Z_p, all)$ has nontrivial p-torsion.

PROPOSITION 6. *The odd torsion in $Sp_*(Z_p, all)$ is the Z_p-vector space consisting of multiples $[M][S^1, \phi]$ for $[M] \in Sp_*$.*

PROOF. As in the proof of Proposition 4,

$$Sp_*(Z_p, all, free) \cong \sum SO_*(BU((n))) \quad \text{modulo 2-torsion.}$$

But the latter is free of odd torsion by [2, Theorem (18.1)]. Thus all odd torsion in $Sp_*(Z_p, all)$ lies in the image of r, by Proposition 2. The actions on S^{4m-1} are certainly sent to zero by r, and the examples $[S\lambda_k, \theta]$ were constructed in the image of ∂, so we see that of the p-torsion classes only multiples of $[S^1, \theta]$ can survive under r.

Thus we should consider the homomorphism $Sp_*/pSp_* \longrightarrow Sp_*(Z_p, all)$, which sends $[M]$ to $[M][S^1, \theta]$. Now $Sp_*/pSp_* \cong SO_*/pSO_*$ is a Z_p-polynomial algebra with one generator in each dimension divisible by four.

PROPOSITION 7. *For each $j \geqslant 4$ there exists a symplectic manifold M^{4j} so that $[M^{4j}]$ is indecomposable in SO_{4j}/pSO_{4j}, and $[M^{4j}][S^1, \theta] = 0 \in Sp_{4j+1}(Z_p, all)$.*

PROOF. First we define characteristic numbers

$$h_\omega: Sp_{4j+1}(BZ_p, *) \longrightarrow Z_p.$$

Let $\alpha \in H^1(BZ_p; Z_p) = Z_p$ be nonzero. Given $[M, f] \in Sp_{4j+1}(BZ_p, *)$ and a partition ω of j, let $p_\omega \in H^{4j}(M; Z_p)$ be the mod p reduction of the Pontrjagin class corresponding to ω. Then

$$h_\omega[M, f] = \langle p_\omega \alpha, [M] \rangle \in Z_p.$$

If $\lambda \longrightarrow N$ is a complex line bundle over a $4j$-manifold, and if π: $S\lambda \longrightarrow N$ is the projection of its sphere bundle, then the tangent bundles $\tau(S\lambda)$ and $\tau(N)$ are related by the isomorphism $\tau(S\lambda) = \pi^*\tau(N) + \mathbf{R}$. If f: $S\lambda/Z_p \longrightarrow BZ_p$ classifies $S\lambda \longrightarrow S\lambda/Z_p$, we will then have an equality $h_\omega[S\lambda/Z_p, f] = p_\omega[N]$. In particular, for the generators we chose in dimensions $4m + 1$, $m \geqslant 1$,

$$h_\omega[S\lambda_k/Z_p, f_k] = p_\omega[\mathbf{C}P(2\nu \longrightarrow \mathbf{C}P(1))]. \tag{4}$$

LEMMA 1. *The characteristic numbers* (4) *vanish for all* ω.

We defer the proof of this lemma, and of two subsequent lemmas, temporarily.

Now any p-torsion class $[M, \phi] \in Sp_{4j+1}(Z_p, free)$ can be expanded, in our chosen basis, so that

$$[M, f] = [N][S^1, \theta] + \text{a linear combination of the } [S\lambda_k/Z_p, f_k].$$

Therefore $h_\omega[M, f] = p_\omega[N]$, by Lemma 1.

Next, let n and k be odd positive integers, and suppose $\nu = \xi + \tfrac{1}{2}(k-1)\mathbf{C} \longrightarrow \mathbf{C}P(n)$, where ξ, as before, is the canonical line bundle. Then $M(n, k) = \mathbf{C}P(\nu + \bar{\nu})$ is a symplectic $2(n + k)$-manifold. As an oriented manifold, $M(n, k)$ is diffeomorphic to $\mathbf{C}P(2\nu)$.

LEMMA 2. *Let p be an odd prime, and let $n + k \geqslant 8$, $n + k$ even. There exists an odd positive integer n so that $M(n, k)$ is indecomposable in $SO_{2(n+k)}/pSO_{2(n+k)}$.*

Assuming this lemma also, we choose such an $M(n, k)$ and let $\lambda \longrightarrow M(n, k)$ be the canonical line bundle over $\mathbf{C}P(2\nu)$. Then $h_\omega[S\lambda/Z_p, f] = p_\omega[M(n, k)]$. We need one last lemma.

LEMMA 3. *If $p_\omega[V] = 0$ for all ω, then* $[V][S^1, \theta] \in \operatorname{Im} \partial\sigma$.

If θ' is the usual action on $S\lambda$, it follows that

$$[S\lambda, \theta'] \equiv [M(n, k)][S^1, \theta] \quad \text{modulo Im } \partial\sigma,$$

for both sides have the same characteristic numbers $h_\omega = p_\omega[M(n, k)]$. Since $\sigma^{-1}[S\lambda, \theta'] \in \operatorname{Im} \partial$, this completes the proof of the proposition.

We shall now prove the lemmas.

PROOF OF LEMMA 1. Let $1 + a'$ and $1 - b$ be the Chern classes of the canonical line bundles over $CP(1)$ and $CP(2\nu)$, respectively. Let π: $CP(2\nu) \longrightarrow CP(1)$ be the projection, and let $a = \pi^* a'$. The Chern class of $CP(2\nu)$ is then

$$c = (1 + a)^2(1 + b + a)^2(1 + b)^{2m-1}.$$

Since $a^2 = 0$, the Pontrjagin class is

$$p = (1 + (b + a)^2)^2(1 + b^2)^{2m-1}.$$

If $C(-, -)$ denotes the binomial coefficient, the rth Pontrjagin class may be computed:

$$\begin{aligned} p_r &= C(2m, r)b^{2r} + 4C(2m - 1, r - 1)ab^{2r-1} \\ &= \frac{2}{r} C(2m - 1, r - 1)[mb^{2r} + 2rab^{2r-1}]. \end{aligned}$$

Now suppose $\omega = (r_1, r_2, \ldots, r_t)$, that is, $r_1 + r_2 + \cdots + r_t = m$. Then

$$p_\omega = p_{r_1} \cdots p_{r_t} = \frac{2^t}{r_1 r_2 \cdots r_t} \prod_j C(2m - 1, r_j - 1)[mb^{2r_j} + 2r_j ab^{2r_j - 1}]$$

$$= \frac{2^t}{r_1 r_2 \cdots r_t}\left[\prod_j C(2m - 1, r_j - 1)\right][m^t b^{2m} + 2m^t ab^{2m-1}].$$

However, $b^{2m} + 2ab^{2m-1} = 0$, so $p_\omega = 0$ for all ω.

PROOF OF LEMMA 2. This is a straightforward (if laborious) application of P. E. Conner's calculations [1]. In his notation, we have to choose an odd integer n so that

$$S_{n+k}[M(n, k)] \neq \begin{cases} 0 \bmod p, & \text{if } n + k \neq p^z - 1, \\ 0 \bmod p^2, & \text{if } n + k = p^z - 1. \end{cases}$$

Let $1 + c \in H^*(CP(n))$ be the Chern class of ξ. We will need the characteristic class $s_i(2\nu) = 2s_i(\nu) = 2c^i$, and the dual Chern class $\bar{\nu}_i$ of 2ν. Since 2ν has Chern class $1 + 2c + c^2$, $\bar{\nu}_i = (-1)^i(i + 1)c^i$.

Now we apply Theorem 4.1 of [1]:

$$S_{n+k}[M(n, k)] = \pm\left[-(k + 1)(n + 1) + 2\sum_{i=1}^{n} C(n + k, i)(-1)^{n-i}(n - i + 1)\right].$$

With n odd,

$$\begin{aligned} \sum_{i=1}^{n} C(n + k, i)(-1)^{n-i}(n - i + 1) &= n + 1 + \sum_{j=0}^{n} (-1)^j(j + 1)C(n + k, j + k) \\ &= C(n + k - 2, n) + n + 1, \end{aligned}$$

so

$$S_{n+k}[M(n,k)] = 2C(n+k-2,n) - (n+1)(k-1),$$

up to a sign, which we neglect hereafter.

Let $m = n + k$, and write $S(n,k) = S_{n+k}[M(n,k)]$. Since $S(3, m-3) = (1/3)(m+1)(m-4)(m-6)$, we may take $n = 3$ if $8 \leqslant m \leqslant p+1$, and also when $m \equiv 0 \bmod p$, provided $p > 3$.

If $m \geqslant p + 3$, we may write $m - 2 = a_r p^r + a_{r-1}p^{r-1} + \cdots + a_1 p + a_0$ with $r > 0$, $0 \leqslant a_i < p$ for all i, and $a_r > 0$. If $1 \leqslant t \leqslant r$,

$$S(p^t, m - p^t) \equiv 2a_t - (a_0 + 1) \mod p.$$

In most cases, we can then take $n = p^t$ for some t.

This procedure fails if all the $a_t = a$ and $2a \equiv a_0 + 1 \bmod p$. Since $a > 0$, we must have $m \not\equiv 1 \bmod p$. If $m \not\equiv 0, \pm 1 \bmod p, p > 3$, we take $n = p - 2$. Then

$$C(m-2, p-2) \equiv 0 \mod p, \quad \text{and}$$

$$S(p-2, m-p+2) \equiv -(p-1)(m+1) \not\equiv 0 \mod p.$$

If $p = 3$ and $m \equiv 0 \bmod 3$, take $n = 7$. Then $a_0 = 0, a = 2$, and $m \equiv 6 \bmod 9$, so $C(m-2, 7) \equiv 0 \bmod 3$, and $S(7, m-7) \equiv -8(m-8) \not\equiv 0 \bmod 3$.

Finally, suppose $m \equiv -1 \bmod p$. Then $a_0 = p - 3$ and $a = p - 1$, so $m = p^{r+1} - 1$. We take $n = p^r$. Since

$$C(p^{r+1} - 1, p^r) \equiv (p-1) \mod p^2,$$

and

$$(p^{r+1} - 1)(p^{r+1} - 2)C(p^{r+1} - 3, p^r)$$
$$= (p^{r+1} - p^r - 1)(p^{r+1} - p^r - 2)C(p^{r+1} - 1, p^r),$$

$$C(p^{r+1} - 3, p^r) \equiv (1/2)(p^r + 1)(p^r + 2)(p - 1) \mod p^2.$$

Thus,

$$S(p^r, p^{r+1} - p^r - 1)$$
$$\equiv (p^r + 1)(p^r + 2)(p-1) - (p^r + 1)(-p^r - 2) \mod p^2$$
$$\equiv p(p^r + 1)(p^r + 2) \equiv 2p \mod p^2.$$

This proves the lemma.

PROOF OF LEMMA 3. Recall the powerful information provided by [2, Theorem (46.3)]. Let I_* be the ideal of elements $x \in SO_*$ such that $p_\omega(x) = 0 \bmod p$ for all ω. Then I_* is generated by $p = [M^0] \in SO_0$ and by certain classes $[M^{4k}] \in SO_{4k}$ for each $k \geqslant 1$. Furthermore, if $\alpha_{2j-1} \in SO_{2j-1}(Z_p, \mathit{free})$ is represented by S^{2j-1} with the action (3), then

$$p\alpha_{2j-1} + [M^4]\,\alpha_{2j-5} + [M^8]\,\alpha_{2j-9} + \ldots = 0. \tag{5}$$

Of course, $p[S^1, \theta] = 0$. By Proposition 6, there exist elements $b_{4k} \in SO_{4k}$ such that

$$\alpha_{4k+1} \equiv b_{4k}[S^1, \theta] \quad \mathrm{mod\ Im}\ \partial\sigma.$$

Then (5) implies the relations

$$(pb_{4k} + b_{4k-4}[M^4] + \cdots + b_4[M^{4k-4}] + [M^{4k}])[S^1, \theta] \equiv 0,$$

modulo $\mathrm{Im}\ \partial\sigma$. By an obvious inductive argument, $[M^{4k}]\,[S^1, \theta] \in \mathrm{Im}\ \partial\sigma$ for all $k \geqslant 0$. This proves the lemma, and finishes the proof of Proposition 7.

We have shown that the p-torsion in $Sp_*(Z_p, \mathit{all})$ is (after a dimension shift) some quotient of a Z_p-polynomial algebra on four generators, corresponding to the cases $m = 0, 2, 4, 6$. It remains to be determined what other relations may lie in the kernel of $Sp_* \longrightarrow Sp_*(Z_p, \mathit{all})$.

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