ANALYTIC EXTENSIONS AND SELECTIONS

BY

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Abstract. Let $G$ be a closed subset of the closed unit disc in $C$, let $F$ be a closed subset of the unit circle of measure 0 and let $\Phi$ map $G$ into the class of all open subsets of a complex Banach space $X$. Under suitable additional assumptions on $\Phi$ we prove that given any continuous function $f: F \to X$ satisfying $f(z) \in \text{closure}(\Phi(z))$ ($z \in F \cap G$) there exists a continuous function $f$ from the closed unit disc into $X$, analytic in the open unit disc, which extends $f$ and satisfies $f(z) \in \Phi(z)$ ($z \in G - F$). This enables us to generalize and sharpen known dominated extension theorems for the disc algebra.

Let $p$ be a real valued positive continuous function on the unit circle $T$ in $C$ and let $F \subset T$ be a closed set of Lebesgue measure zero. Given any continuous function $f: F \to C$ satisfying $|f(s)| < p(s)$ ($s \in F$) there exists a function $\tilde{f}$ in the disc algebra which extends $f$ and satisfies $|\tilde{f}(t)| < p(t)$ ($t \in T$). This simple dominated extension theorem is a special case of a more general theorem proved by E. Bishop [1]. See [1]-[3], [6], [7], [10] for such theorems in general spaces of continuous functions and see [7] for the most general dominated extension theorem in the disc algebra.

Writing $\Phi(t) = \{z \in C: |z| < p(t)\}$ ($t \in T$) the above theorem becomes a selection theorem: Given any continuous function $f: F \to C$ satisfying $f(s) \in \Phi(s)$ ($s \in F$) there exists a function $\tilde{f}$ in the disc algebra which extends $f$ and satisfies $\tilde{f}(t) \in \Phi(t)$ ($t \in T$).

In the present paper we use some ideas of [5] to prove a selection theorem for the disc algebra which generalizes and sharpens known results on dominated extensions.

Throughout, we denote by $\Delta$, $\overline{\Delta}$ and $\partial\Delta$ the open unit disc in $C$, its closure and its boundary, respectively. If $X$ is a complex Banach space and $r > 0$ we write $\mathcal{B}_r(X) = \{x \in X: \|x\| < r\}$. Let $x \in X$ and $S, T \subset X$. We write $x + S = \{x + u: u \in S\}$ and $S + T = \{u + v: u \in S, v \in T\}$ and denote by $\overline{S}$ the closure of $S$. By $A(\Delta, X)$ we denote the Banach space of all continuous functions from $\Delta$ to $X$ which are analytic on $\Delta$ and by $A$ we denote the disc algebra $A(\Delta, C)$. We write $I = \{t: 0 < t < 1\}$ and denote the set of all positive integers by $N$.

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Suppose that \( \{ p_\alpha; \alpha \in \mathcal{A} \} \) is a family of nonempty open subsets of \( X \). For each \( \alpha \in \mathcal{A} \) let \( x_\alpha \in \overline{p_\alpha} \). We say that the sets \( p_\alpha \) (\( \alpha \in \mathcal{A} \)) are equi-locally connected at the points \( x_\alpha \) if given any \( \epsilon > 0 \) there is some \( \delta > 0 \) such that for every \( \alpha \in \mathcal{A} \), the set \( (x_\alpha + B_\delta(X)) \cap p_\alpha \) is contained in a connected component of \( (x_\alpha + B_\epsilon(X)) \cap p_\alpha \) [9]. We call any such \( \delta(\cdot) \) a modulus of equi-local connectedness of the sets \( p_\alpha \) (\( \alpha \in \mathcal{A} \)) at the points \( x_\alpha \).

Let \( S \subset \overline{\Delta} \) be a closed set and let \( X \) be a Banach space. We call the graph of a map \( \Phi: S \to 2^X \) the set of all pairs \( (z, x) \in S \times X \) such that \( x \in \Phi(z) \). We say that \( \Phi \) is open if its graph is open in \( S \times X \); equivalently, \( \Phi \) is open if given any \( z \in S \) and any \( x \in \Phi(z) \) there is some \( \epsilon > 0 \) such that \( x + B_\epsilon(X) \subset \Phi(u) \) (\( u \in S; |u - z| < \epsilon \)). In particular, if \( \Phi \) is open then \( \Phi(z) \) is open for every \( z \in S \).

Our main result is the following:

**Theorem.** Let \( X \) be a complex Banach space and let \( G \subset \overline{\Delta} \) be a closed set. Assume that \( \Phi: G \to 2^X \) is an open map such that \( g(z) \in \Phi(z) \) (\( z \in G \)) for some \( g \in A(\Delta, X) \). Let \( F \subset \partial \Delta \) be a closed set of measure 0 and let \( f: F \to X \) be a continuous function such that \( f(s) \in \Phi(s) \) (\( s \in G \cap F \)). Assume that \( \Phi(s) \) is connected for each \( s \in G \cap F \) and that the sets \( \Phi(s) \) (\( s \in G \cap F \)) are equi-locally connected at the points \( f(s) \). Then there exists an extension \( \tilde{f} \in A(\Delta, X) \) of \( f \) which satisfies \( \tilde{f}(z) \in \Phi(z) \) (\( z \in G - F \)).

**Lemma.** Under the assumptions of the theorem with \( G = \overline{\Delta} \), let \( U \subset \overline{\Delta} \) be a neighbourhood of \( F \) and let \( \epsilon > 0 \). Suppose that \( F = \bigcup_{i=1}^{m} F_i \) where the \( F_i \) (\( 1 \leq i \leq m \)) are pairwise disjoint nonempty closed sets. Assume that \( u, v: F \to X \) are two functions such that \( v(s) \in \Phi(s) \) (\( s \in F \)) and such that \( u|F_i \) and \( v|F_i \) are constant for each \( i \) (\( 1 < i < m \)). Suppose that there is some \( \tilde{u} \in A(\Delta, X) \) which extends \( u \) and satisfies \( \tilde{u}(z) \in \Phi(z) \) (\( z \in \overline{\Delta} \)). Then there is an extension \( \tilde{v} \in A(\Delta, X) \) of \( v \) which satisfies \( \tilde{v}(z) \in \Phi(z) \) (\( z \in \overline{\Delta} \)) and \( \| \tilde{u}(z) - \tilde{v}(z) \| < \epsilon \) (\( z \in \overline{\Delta} - U \)). If, in addition, \( R > 0 \) and

\[
\| u(s) - f(s) \| < \delta(R), \quad \| v(s) - f(s) \| < \delta(R) \quad (s \in F)
\]

where \( \delta(\cdot) \) is a modulus of equi-local connectedness of the sets \( \Phi(s) \) (\( s \in F \)) at the points \( f(s) \) then one may choose \( \tilde{v} \) so that \( \| \tilde{u} - \tilde{v} \| < 4R \).

**Proof.** We prove both assertions; the proof can be easily adapted to prove only the first assertion.

Observe first that by compactness of \( \overline{\Delta} \), by the continuity of \( \tilde{u} \) and by the fact that \( \Phi \) is open there is some \( \eta > 0 \) such that \( \tilde{u}(z) + B_\eta(X) \subset \Phi(z) \) (\( z \in \overline{\Delta} \)). For the moment, fix \( s \in F \). The assumptions imply that there is a path \( p: I \to \Phi(s) \) such that \( p(0) = u(s), p(1) = v(s) \) and \( \| p(t) - f(s) \| < R \) (\( t \in I \)). Since \( \Phi \) is open and since \( p(I) \) is compact there is some \( r' > 0 \) such that \( p(I) + B_r(X) \subset \Phi(z) \) (\( z \in \Delta, |z - s| < r' \)).
Since \( u \) and \( v \) are constant on \( F_i \) for \( 1 < i < m \) there are paths \( p_i: I \to X \) \((1 < i < m)\) and an \( r > 0 \) such that
\[
p_i(0) = u(s), \quad p_i(1) = v(s) \quad (s \in F_i, \ 1 < i < m),
\]
\[
p_i(I) + B_r(X) \subset \Phi(z) \quad (z \in \bar{A}, \ \text{dist}(z, F_i) < r, \ 1 < i < m),
\]
\[
\text{diam} \ p_i(I) < 2R \quad (1 < i < m),
\]
and \( r < \min\{\epsilon, R, \eta\} \). By the continuity of \( \tilde{u} \) we may choose pairwise disjoint neighbourhoods \( U_i \subset U \) of \( F_i \), respectively, such that
\[
p_i(I) + B_r(U_i) \subset \Phi(z), \quad ||\tilde{u}(z) - p_i(0)|| < r \quad (z \in U_i, \ 1 < i < m).
\]
By [4, Lemma 4] there exists for each \( i \) \((1 < i < m)\) an \( h_i \in A(\Delta, X) \) such that
\[
h_i|F_i = (v - u)|F_i \quad (1 < i < m),
\]
\[
h_i|F_j = 0 \quad (1 < j < m, j \neq i),
\]
\[
h_i(\bar{A}) \subset -p_i(0) + p_i(I) + B_r(U_i) \quad (1 < i < m)
\]
\[
||h_i(z)|| < r/m \quad (z \in \bar{A} - U_i, \ 1 < i < m).
\]
Put \( \tilde{\delta} = \tilde{u} + \sum_{i=1}^n h_i \). As in [5, p. 375] it is easy to see that \( \tilde{\delta} \) has all the required properties. Q.E.D.

PROOF OF THE THEOREM. It suffices to prove the Theorem in the case when \( G = A \) (otherwise define \( \Psi(z) = \Phi(z) \) \((z \in G)\), \( \Psi(z) = X \) \((z \in \bar{A} - G)\), observe that \( \Psi \) is an open map and apply the theorem to \( \Psi \)). Further, it suffices to prove the Theorem in the special case when \( g = 0 \) (otherwise define \( \Psi(z) = -g(z) + \Phi(z) \) \((z \in G)\), observe that, by the continuity of \( g \), \( \Psi \) is open, apply the theorem to \( \Psi \) and to the function \( s \to h(s) = -g(s) + f(s) \) and finally put \( \tilde{f} = g + \tilde{h} \)). So assume that \( G = \bar{A} \) and \( g = 0 \).

Let \( \delta(\cdot) \) be a modulus of equi-local connectedness of \( \Phi(s) \) \((s \in F)\) at the points \( f(s) \) and let \( \delta_n \) be a decreasing sequence of positive numbers converging to 0 and satisfying \( \delta_n < \delta\left(\frac{1}{4}, 2^{-n}\right) \) \((n \in N)\). As in the proof of Lemma 5, [5, pp. 371–372] it follows from our assumptions that for each \( n \) there is a decomposition \( F = \bigcup_{i=1}^n F_i \) where the \( F_i \) are pairwise disjoint nonempty compact sets, and a function \( f_n: F \to X \) such that \( f_n|F_i \) is constant for each \( i \) \((1 < i < m)\) and such that
\[
(a) \ f_n(s) \in \Phi(s) \quad (s \in F, \ n \in N),
\]
\[
(b) \ ||f_n(s) - f(s)|| < \delta_n \quad (s \in F, \ n \in N).
\]
We may assume that for all \( n \), each element of \((n + 1)\)st decomposition is contained in an element of \( n \)th decomposition. Since \( \Phi \) is open, \( 0 \in \Phi(z) \) for all \( z \in \bar{A} \), and since \( \bar{A} \) is compact there is some \( \epsilon_0 > 0 \) such that \( B_{\epsilon_0}(X) \subset \Phi(z) \) \((z \in \bar{A})\). Let \( U_n \subset \bar{A} \) be a decreasing sequence of neighbourhoods of \( F \)
such that \( F = \cap_{n=1}^{\infty} U_n \). Assume that there exist a decreasing sequence \( \varepsilon_n \) of positive numbers satisfying \( \varepsilon_n < \varepsilon_0 \) (\( n \in N \)) and a sequence \( g_n \in A(\Delta, X) \), \( g_0 = 0 \), with the following properties:

1. \( g_n|F = f_n|F \) (\( n \in N \)),
2. \( \| g_n - g_{n-1} \| < 1/2^{n-1} \) (\( n \in N, n > 2 \)),
3. \( \| g_n(z) - g_{n-1}(z) \| < \varepsilon_n/2^m (z \in \Delta - U_n, n \in N) \),
4. \( g_n(z) + B_1(X) \subset \Phi(z) \) (\( z \in \Delta, n \in N \)).

By (ii), \( g_n \) converge uniformly on \( \Delta \) so putting \( \tilde{f}(z) = \lim_{n \to \infty} g_n(z) \) (\( z \in \Delta \)) we have \( \tilde{f} \in A(\Delta, X) \). By (i) and (b) \( \tilde{f}|F = f \). Further, let \( z \in \Delta - U_1 \). Writing \( \tilde{f}(z) = \sum_{n=0}^{\infty} (g_{n+1}(z) - g_n(z)) \) it follows by (iii) that \( \tilde{f}(z) \in B_1(X) \subset \Phi(z) \).

Finally, if \( z \in U_1 - F \) then for some \( n \in N \) we have \( z \notin U_j \) (\( j > n \)) so by (iii) and (iv) it follows that

\[
\tilde{f}(z) = g_n(z) + \sum_{j=n}^{\infty} [g_{j+1}(z) - g_j(z)] \in g_n(z) + B_1(X) \subset \Phi(z)
\]

and consequently \( \tilde{f} \) has all the required properties.

It remains to prove the existence of \( \varepsilon_n \) and \( g_n \) with the above properties. Put \( g_0 = 0 \). By the first part of the Lemma there is some \( g_1 \in A(\Delta, X) \) satisfying \( g_1(z) \in \Phi(z) \) (\( z \in \Delta \)) and such that (i) and (iii) hold for \( n = 1 \). Since \( \Phi \) is open, \( g_1(z) \in \Phi(z) \) for all \( z \in \Delta \) and since \( \Delta \) is compact there is some \( \varepsilon_1 \) with \( 0 < \varepsilon_1 < \varepsilon_0 \) such that \( g_1(z) + B_{\varepsilon_1}(X) \subset \Phi(z) \) (\( z \in \Delta \)). Let \( m \in N \) and assume that \( g_m \in A(\Delta, X) \) satisfies (i) and (iv) for some \( \varepsilon_m > 0 \). By the Lemma (a) and (b) imply that there is some \( g_{m+1} \in A(\Delta, X) \) satisfying (i)–(iii) for \( n = m + 1 \) and such that \( g_{m+1}(z) \in \Phi(z) \) (\( z \in \Delta \)). Again, since \( \Phi \) is open, \( g_{m+1}(z) \in \Phi(z) \) (\( z \in \Delta \)) and since \( \Delta \) is compact there is some \( \varepsilon_{m+1} \) with \( 0 < \varepsilon_{m+1} < \varepsilon_m \) such that \( g_{m+1}(z) + B_{\varepsilon_{m+1}}(X) \subset \Phi(z) \) (\( z \in \Delta \)). Q.E.D.

Next we present some simple applications. In Corollaries 1–6 below \( G \) can be either \( \Delta \) or \( \partial \Delta \).

**Corollary 1.** Let \( X \) be a complex Banach space and let \( p: G \to (0, \infty) \) be a lower semicontinuous function. Given any closed set \( F \subset \partial \Delta \) of measure 0 and any continuous function \( f: F \to X \) satisfying \( \| f(s) \| < p(s) \) (\( s \in F \)) there exists \( \tilde{f} \in A(\Delta, X) \) which extends \( f \) and satisfies \( \| \tilde{f}(z) \| < p(z) \) (\( z \in G - F \)). Moreover, if \( z_j \in \Delta \) (\( 1 < j < k \)) and if \( n_j \) (\( 1 < j < k \)) are positive integers, \( \tilde{f} \) can be chosen to have a zero at \( z_j \) of order at least \( n_j \).

**Proof.** Since \( p \) is lower semicontinuous the map \( z \to \Phi(z) = \{ x \in X: \| x \| < p(z) \} \) is open on \( G \). Further, since \( p \) is lower semicontinuous there is some \( \delta > 0 \) such that \( p(z) > \delta \) (\( z \in G \)). Consequently \( g \in A(\Delta, X) \) defined by \( g(z) = 0 \) (\( z \in \Delta \)) satisfies \( g(z) \in \Phi(z) \) (\( z \in G \)). Further, it is easy to see that any family \( \{ P_a; a \in \mathcal{A} \} \) of nonempty open convex subsets of \( X \) is equi-locally connected at the points \( x_a \) for any \( x_a \in P_a \) (\( a \in \mathcal{A} \)) so the sets
The functions \( \Phi(s) \) for \( s \in F \) are equi-locally connected at \( f(s) \). Now the first assertion follows by the Theorem. To prove the second assertion, multiply \( \tilde{f} \) by \( \varphi \in A \) which satisfies \( \varphi|F = 1, |\varphi(z)| < 1 \) \( (z \in \Delta) \) and has a zero at \( z_j \) of order at least \( n_j \) \([2, \text{Theorem, pp. 284–285}] \). Q.E.D.

Corollary 1 sharpens and generalizes \([2, \text{Theorem, pp. 284–285}] \). Note that in the case when \( X = C \) Corollary 1 is an easy consequence of \([6, \text{Theorem 3}] \).

**Corollary 2.** Let \( X \) be a complex Banach space and let \( P \subset X \) be a nonempty open connected set which is locally connected at every point of \( \overline{P} \). Let \( \varphi: G \to C \) be a continuous function such that \( \varphi(z)g(z) \in P \) \( (z \in G) \) for some \( g \in A(\Delta, X) \). Given any closed set \( F \subset \partial \Delta \) of measure 0 and any continuous function \( f: F \to X \) satisfying \( \varphi(s)f(s) \in \overline{P} \) \( (s \in F) \) there exists \( \tilde{f} \in A(\Delta, X) \) which extends \( f \) and satisfies \( \varphi(z)\tilde{f}(z) \in P \) \( (z \in G - F) \).

**Proof.** Define \( \Phi(z) = \{x \in X: \varphi(z)x \in P\} \) \( (z \in G) \). Since \( P \) is open and since \( \varphi \) is continuous \( \Phi \) is an open map; since \( P \) is connected \( \Phi(z) \) is connected for each \( z \in G \). By the continuity of \( \varphi \) and \( f \) the set \( S = \{\varphi(s)f(s), s \in F\} \subset \overline{P} \) is compact and consequently \( P \) is uniformly locally connected on \( S \) \([5]\). Since \( \varphi \) is bounded on \( F \) it follows easily that the sets \( \Phi(s) \) \( (s \in F) \) are equi-locally connected at the points \( f(s) \). Now the assertion follows by the Theorem. Q.E.D.

Similarly we prove

**Corollary 3.** Let \( X \) be a complex Banach space and let \( P \subset X \) be a nonempty open connected set which is locally connected at every point of \( \overline{P} \). Let \( h: G \to X \) be a continuous function such that \( h(z) + g(z) \in P \) \( (z \in G) \) for some \( g \in A(\Delta, X) \). Given any closed set \( F \subset \partial \Delta \) of measure 0 and any continuous function \( f: F \to X \) satisfying \( h(s) + f(s) \in \overline{P} \) \( (s \in F) \) there exists \( \tilde{f} \in A(\Delta, X) \) which extends \( f \) and satisfies \( h(z) + \tilde{f}(z) \in P \) \( (z \in G - F) \).

Next we present some dominated extension theorems for the disc algebra.

**Corollary 4.** Let \( p: G \to [0, \infty) \) be an upper semicontinuous (USC) function and let \( q: G \to (0, \infty) \) be a lower semicontinuous (LSC) function such that \( p(z) < |g(z)| < q(z) \) \( (z \in G) \) for some \( g \in A \). Given any closed set \( F \subset \partial \Delta \) and any continuous function \( f: F \to C \) satisfying \( p(s) < |f(s)| < q(s) \) \( (s \in F) \) there is an \( \tilde{f} \in A \) which extends \( f \) and satisfies \( p(z) < |\tilde{f}(z)| < q(z) \) \( (z \in G - F) \).

**Proof.** Define \( \Phi(z) = \{\xi \in C: p(z) < |\xi| < q(z)\} \) \( (z \in G) \). Let \( z_0 \in G \) and let \( \xi_0 \in \Phi(z_0) \). For some \( \epsilon > 0 \) we have \( p(z_0) + \epsilon < |\eta| < q(z_0) - \epsilon \) for all \( \eta \in \xi_0 + B_\epsilon(C) \). Since \( p \) is USC and since \( q \) is LSC there is a
neighbourhood $U \subset G$ of $z_0$ such that $p(z) < p(z_0) + \varepsilon$, $q(z) > q(z_0) - \varepsilon$ ($z \in U$) and consequently $\Phi$ is open on $G$. Clearly $\Phi(z)$ is connected for every $z \in G$. Let $0 < r < R$ and let $S = \{z \in C: r < |z| < R\}$. It is easy to see that $(z + B_r(C)) \cap S$ is connected for every $z \in S$ and for every $\varepsilon > 0$. Consequently the sets $\Phi(s)$ ($s \in F$) are equi-locally connected at the points $f(s)$. Now the assertion follows by the theorem. Q.E.D.

**Remark.** To prove Corollary 4 in the case when $G = \partial \Delta$ one needs to assume only that $p(z) < q(z)$ ($z \in G$) and one does not need the existence of $g \in A$. Namely, by [10, Theorem 5.3, p. 15] there exists a continuous function $\varphi: G \to R$ satisfying $p(z) < \varphi(z) < q(z)$ ($z \in G$) and since $G$ is compact there is some $\varepsilon > 0$ such that $p(z) + \varepsilon < \varphi(z) < q(z) - \varepsilon$ ($z \in G$). By [11, p. 216] $A$ approximates in modulus on $\partial \Delta$ so there is some $g \in A$ such that $\varphi(z) - \varepsilon < |g(z)| < \varphi(z) + \varepsilon$ ($z \in G$) and consequently $p(z) < |g(z)| < q(z)$ ($z \in G$).

**Corollary 5.** Let $p_1$, $q_1: G \to R$ be two upper semicontinuous functions and let $p_2$, $q_2: G \to R$ be two lower semicontinuous functions such that $p_1(z) < \text{Re} g(z) < p_2(z)$, $q_1(z) < \text{Im} g(z) < q_2(z)$ ($z \in G$) for some $g \in A$. Given any closed set $F \subset \partial \Delta$ of measure 0 and any continuous function $f: F \to C$ satisfying $p_1(s) < \text{Re} f(s) < p_2(s)$, $q_1(s) < \text{Im} f(s) < q_2(s)$ ($s \in F$) there exists an $\hat{f} \in A$ which extends $f$ and satisfies $p_1(z) < \text{Re} \hat{f}(z) < p_2(z)$, $q_1(z) < \text{Im} \hat{f}(z) < q_2(z)$ ($z \in G - F$).

**Proof.** Define $\Phi(z) = \{z \in C: p_1(z) < \text{Re} \xi < p_2(z)$, $q_1(z) < \text{Im} \xi < q_2(z)$ ($z \in G$) and observe that by the semicontinuity of $p_1$, $p_2$, $q_1$, $q_2$, $\Phi$ is open on $G$. Further, since $\Phi(z)$ is convex for every $z \in G$ it follows that the sets $\Phi(s)$ ($s \in F$) are equi-locally connected at the points $f(s)$. Now the assertion follows by the theorem. Q.E.D.

**Corollary 6.** Let $p: G \to (0, \infty)$ be a lower semicontinuous function. Given any closed set $F \subset \partial \Delta$ of measure 0 and any continuous function $f: F \to C$ satisfying $|f(s)| < p(s)$, $\text{Re} f(s) > 0$ ($s \in F$) there exists an $\hat{f} \in A$ which extends $f$ and satisfies $|f(z)| < p(z)$, $\text{Re} f(z) > 0$ ($z \in G - F$).

**Proof.** Define $\Phi(z) = \{z \in C: |\xi| < p(z)$, $\text{Re} \xi > 0\}$ ($z \in G$). Since $p$ is LSC, $\Phi$ is open on $G$. Since $p$ is LSC and positive and since $G$ is compact there is some $\delta > 0$ such that $p(z) > \delta$ ($z \in G$). Define $g \in A$ by $g(z) = \delta/2$ ($z \in \Delta$). Clearly $g(z) \in \Phi(z)$ ($z \in G$). Since $\Phi(z)$ is convex for every $z \in G$ it follows that the sets $\Phi(s)$ ($s \in F$) are connected and equi-locally connected at the points $f(s)$. Now the assertion follows by the theorem. Q.E.D.

Corollary 6 with $G = \partial \Delta$ sharpens [7, Corollary 4.5]. Corollary 6 with $G = \Delta$ answers a question in [7, p. 294].
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