CORRECTION TO "ON THE FREE BOUNDARY OF A QUASIVARIATIONAL INEQUALITY ARISING IN A PROBLEM OF QUALITY CONTROL"

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In the proof of Theorem 4.1 of [1] we stated that $\frac{\partial z}{\partial y} > 0$ on $\partial C_\delta$; this is true if $\partial C_\delta$ lies in $\Omega_\delta$, but may possibly be false at points of $\partial C_\delta \cap \partial \Omega_\delta$. Thus the proof of Theorem 4.1 yields only the weaker result:

**Theorem 4.1'.** If (4.1), (4.2) hold then $\bar{C}$ cannot lie in the interior of $R_{n-1}^+$, that is, $\bar{C}$ must intersect $\partial R_{n-1}^+$.

Stronger assertions about the continuation set $C$ can be proved under additional restrictions on the $q_{ij}$. Recall that we are dealing with the variational inequality for $w = u - \psi$:

\[
-\mathcal{L}w < \frac{1}{Y} \sum_{i=1}^{n} B_i y_i \left( Y = \sum_{i=1}^{n} y_i, y_1 \equiv 1 \right),
\]

\[
 w \left( -\mathcal{L}w - \frac{1}{Y} \sum_{i=1}^{n} B_i y_i \right) = 0 \quad (1)
\]

in $R_{n-1}^+$ ($y = (y_2, \ldots, y_n)$ belongs to $R_{n-1}^+$ if and only if $y_i > 0$ for $2 < i < n$) under the assumptions:

\[
B_i > 0 \quad \text{if} \ 2 < i < n, \quad B_1 < 0 \quad (2)
\]

(if $B_1 > 0$ then $C = \emptyset$), where

\[
\mathcal{L}w = \frac{1}{2} \sum_{i,j=2}^{n} \mu_{ij} y_i y_j \frac{\partial^2 w}{\partial y_i \partial y_j} + \sum_{j=2}^{n} b_j \frac{\partial w}{\partial y_j} - \alpha w,
\]

\[
b_j = \frac{1}{Y} \sum_{i=2}^{n} \mu_{ij} y_i y_j + \sum_{i=1}^{n} (q_{i,j} - q_i) y_i,
\]

\[
\mu_j = (\lambda_i - \lambda_1) \cdot (\lambda_j - \lambda_1).
\]

We shall now assume that

\[
\mu_{ij} = \mu_i \delta_{ij}, \quad \mu_i > 0, \quad (3)
\]

\[
q_{i,j} = 0 \quad \text{if} \ i > j. \quad (4)
\]

Then (see (4.5) of [1]), each $w_1 = \frac{\partial z}{\partial y_i}$ ($z = Yw$) satisfies in $C_\delta$ an inhomogeneous elliptic equation to which the minimum principle applies, provided we already know that $w_{i+1} > 0$, $\ldots$, $w_n > 0$. Thus, we can establish by induction that $w_n > 0$.

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\[ w_{n-1} > 0, \ldots, w_k > 0 \] provided we can verify these inequalities on the boundary of \( \Omega_\delta \).

To do this we specialize to
\[ \Omega_\delta = \left\{ y; y_i > \delta \text{ for } 2 \leq i \leq n, \sum_{i=2}^n y_i = \frac{1}{\delta} \right\}. \]

Denote by \( \partial_0 \Omega_\delta \) the part of \( \partial \Omega_\delta \) where \( \Sigma y_i = \delta \), and by \( \partial_1 \Omega_\delta \) the remaining part of \( \partial \Omega_\delta \). We chose the boundary values of \( z \) as follows:
\[ z = 0 \text{ on } \partial_0 \Omega_\delta, \]
\[ z = \gamma \left( \sum_{j=2}^n y_j - \frac{1}{\delta} \right) \text{ on } \partial_1 \Omega_\delta, \quad \gamma > 0. \] (5)

Then
\[ z < 0, \quad \frac{\partial z}{\partial y_i} > 0, \quad \frac{\partial^2 z}{\partial y_i^2} = 0 \text{ on } \partial_1 \Omega_\delta \cap \{ y_j = \delta \}, \quad j \neq i. \] (6)

We choose \( \gamma \) sufficiently small so that \( \gamma |q_{ij}| < \frac{1}{2} B_j (2 < j < n) \). Then
\[ \sum_{i=2}^n \sum_{j=1, j \neq i}^n q_{i,j} \frac{\partial z}{\partial y_j} + \frac{1}{2} \sum_{i=2}^n B_j y_i^2 > 0 \text{ on } y_i = \delta. \] (7)

Further, if \( \delta \) is sufficiently small,
\[ -\alpha z + \frac{1}{2} \sum_{i=2}^n B_i y_i > -B_1 \text{ on } \partial_1 \Omega_\delta. \] (8)

Since
\[ \sum_{j=2}^n q_{i,j} = \sum_{j=1}^n q_{i,j} = 0 \text{ if } i > 1, \quad \sum_{j=2}^n q_{1,j} > 0, \]
we have
\[ -\frac{z}{Y} \sum_{i=1}^n \sum_{j=2}^n q_{i,j} y_i > 0. \] (9)

We finally recall [1] that \( z \) satisfies
\[ \frac{1}{2} \sum_{i=2}^n \mu_i \frac{\partial^2 z}{\partial y_i^2} + \sum_{i=1}^n \sum_{j=2}^n q_{i,j} y_i \frac{\partial z}{\partial y_j} - \frac{z}{Y} \sum_{i=1}^n \sum_{j=2}^n q_{i,j} y_i = \alpha z - \sum_{i=1}^n B_i y_i. \] (10)

Assume now that
\[ q_{i,n} = 0 \quad (1 < i < n). \] (11)

Since \( z = 0 \text{ on } \partial_0 \Omega_\delta, z < 0 \text{ in } \Omega_\delta, \) we have \( w_n = \frac{\partial z}{\partial y_n} > 0 \text{ on } \partial_0 \Omega_\delta \). On \( y_i = \delta, \) \( i < n, \) \( w_n > 0 \) by (6). Since \( w_n \) cannot take a negative minimum in \( C_\delta \), it follows that \( w_n > 0 \) in \( C_\delta \) if we can show that \( w_n \) cannot take a negative minimum at any point \( y^0 \in \bar{C}_\delta \) which lies in \( y_n = \delta \). Suppose it does; then \( w_n(y^0) < 0 \) and
\[ \frac{\partial^2 z}{\partial y_n^2} > 0 \text{ at } y^0. \] (12)

In view of (11) the coefficient of \( \frac{\partial z}{\partial y_n} \) in (10) vanishes. Using also (12), (7) with
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\( l = n, (6), (8), (9), \) we conclude that the left-hand side of (10) is larger than the right-hand side; a contradiction.

We have thus proved that \( w_n > 0 \) in \( C_8 \). Similarly we can prove that if \( q_{i,n-1} = 0 \) \( (1 < i < n) \) then \( w_{n-1} > 0 \), etc. We sum up:

**Theorem 1.** Assume that (3), (4) hold and that \( q_{i,m} = 0 \) if \( 1 < i < n, k < m < n \). Then for any \( j, k < j < n \), there exists a function \( \Psi_j(y_2, \ldots, y_{j-1}, y_{j+1}, \ldots, y_n) \) such that \( y = (y_2, \ldots, y_n) \) belongs to \( C \) if and only if

\[
y_j < \Psi_j(y_2, \ldots, y_{j-1}, y_{j+1}, \ldots, y_n).
\]

**Corollary 2.** If \( q_{i,j} = 0 \) \( (1 < i, j < n) \) then the assertions of Corollary 4.2 and Theorem 5.1 of [1] are valid.

REFERENCES


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