

SOME CANONICAL COHOMOLOGY CLASSES ON GROUPS OF VOLUME PRESERVING DIFFEOMORPHISMS

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ABSTRACT. We discuss some canonical cohomology classes on the space $\overline{B}\mathcal{D}iff_{\omega_0}^c M$, where $\mathcal{D}iff_{\omega_0}^c M$ is the identity component of the group of compactly supported diffeomorphisms of the manifold M which preserve the volume form ω . We first look at some classes $c_k(M)$, $1 \leq k \leq n = \dim M$, which are defined for all M , and show that the top class $c_n(M) \in H^n(\overline{B}\mathcal{D}iff_{\omega_0}^c M; \mathbf{R})$ is nonzero for $M = S^n$, n odd, and is zero for $M = S^n$, n even. When $H_c^i(M; \mathbf{R}) = 0$ for $0 \leq i < n$, the classes $c_k(M)$ all vanish and a secondary class $s(M) \in H^{n-1}(\overline{B}\mathcal{D}iff_{\omega_0}^c M; \mathbf{R})$ may be defined. This is trivially zero when n is odd, and is twice the Calabi invariant for symplectic manifolds when $n = 2$. We prove that $s(\mathbf{R}^n) \neq 0$ when n is even by showing that it is one of a set of nonzero classes which were defined by Hurder in [7].

1. Statement of main results. Let M be a connected oriented n -dimensional C^∞ -manifold without boundary and with smooth volume form ω . We write $\mathcal{G} = \mathcal{D}iff_{\omega_0}^c M$ for the identity component of the group of compactly supported C^∞ -diffeomorphisms of M which preserve ω , with the C^∞ -topology, and G for the group \mathcal{G} considered with the discrete topology. The homotopy fiber of the natural map $BG \rightarrow B\mathcal{G}$ is called $\overline{B}\mathcal{G}$.

We will think of $\overline{B}\mathcal{G}$ as the realization \mathcal{C} of the complex $\text{Sing } \mathcal{G}/G$. Here $\text{Sing } \mathcal{G}$ is the smooth singular complex of \mathcal{G} , and $\text{Sing } \mathcal{G}/G$ is its quotient by the action of G given by the multiplication on the right. Thus a p -simplex S in \mathcal{C} is given by a smooth (C^∞) map $t \mapsto h_t \in \mathcal{G}$, $t \in \Delta^p$, which is well defined up to composition on the right by an element $g \in G$. Such a simplex corresponds to a codimension n foliation $\mathcal{F}(S)$ on $\Delta^p \times M$ with typical leaf $\{(t, h_t(y)) : t \in \Delta^p\}$. The foliation $\mathcal{F}(S)$ is transverse to the fibers $t \times M$ of the projection $p: \Delta^p \times M \rightarrow \Delta^p$, and is trivial, with leaves $\Delta^p \times x$, for x outside of a compact subset of M . One can also describe $\mathcal{F}(S)$ as the pull-back of the point foliation of M by the map $f: (t, x) \mapsto h_t^{-1}(x)$. Hence $\Omega(S) = f^*(\omega)$ is a transverse volume form for $\mathcal{F}(S)$. In other words, $\Omega(S)$ is a closed n -form which defines $\mathcal{F}(S)$ in the sense that a tangent vector Y of $\Delta^p \times M$ is tangent to $\mathcal{F}(S)$ if and only if $i(Y)\Omega(S) = 0$. Clearly, $\Omega(S)$ is the only transverse volume form of $\mathcal{F}(S)$ which restricts to ω on each $t \times M$.

It is easy to check that the $\mathcal{F}(S)$ fit together to give a foliation \mathcal{F} of $\mathcal{C} \times M$. Similarly, the forms $\Omega(S)$ fit together to give a canonical closed n -form Ω on $\mathcal{C} \times M$.

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(By definition, a k -form Λ on $\mathcal{C} \times M$ is a collection of k -forms $\Lambda(S)$ on $\Delta^p \times M$, one for each p -simplex S in \mathcal{C} , such that $\Lambda(S)$ restricts to $\Lambda(S')$ on the face S' of S .)

Any form Λ on $\mathcal{C} \times M$ has a unique decomposition $\Lambda = \Lambda_0 + \Lambda_1 + \dots + \Lambda_n$, where the form Λ_i has degree i in the M -variables. In particular,

$$\Omega = \Omega_0 + \dots + \Omega_n$$

where Ω_n is the pull-back $\pi^*\omega$ of ω by the projection $\pi: \mathcal{C} \times M \rightarrow M$ and where each $\Omega_i, i < n$, is compactly supported. (This means that $\Omega_i(S)$ has compact support in $\Delta^p \times M$ for all S .) Therefore Ω gives rise to *canonical cohomology classes*

$$c_k(M) \in H^k(\overline{B}\mathcal{D}iff_{\omega_0}^c M; H_c^{n-k}(M; \mathbf{R}))$$

for $1 \leq i \leq n$, as follows. For any k -cycle K in $\mathcal{C} = \overline{B}\mathcal{D}iff_{\omega_0}^c M$, the value of $c_k(M)$ on K is the cohomology class of the closed, compactly-supported $(n - k)$ -form

$$x \mapsto \int_{K \times x} \Omega = \int_{K \times x} \Omega_{n-k}$$

on M which is obtained by integrating Ω over the fiber of the projection $\pi: K \times M \rightarrow M$. Explicit formulae for the $c_k(M)$ are given in Appendix 1.

Since $\pi_1\mathcal{C}$ is isomorphic to the universal cover $\widetilde{\mathcal{D}iff}_{\omega_0}^c M$ of $\mathcal{D}iff_{\omega_0}^c M$, the class c_1 corresponds to a homomorphism $\widetilde{\mathcal{D}iff}_{\omega_0}^c M \rightarrow H_c^{n-1}(M; \mathbf{R})$. Up to sign, it is just the flux homomorphism, which is defined in [1, II, §1] for example. See Appendix 1.

Notice that c_n vanishes when M is noncompact since $H_c^0(M; \mathbf{R}) = 0$ in this case. On the other hand $c_n(M)$ is nonzero when M is a compact Lie group. In fact, if H is a k -dimensional compact submanifold in $\mathcal{D}iff_{\omega_0}^c M$, then the ‘‘diagonal’’ foliation of $H \times M$ with leaves $\{(h, h(y)): h \in H\}$ gives rise to a k -cycle in \mathcal{C} . Moreover the transverse volume form on $H \times M$ is the pull-back of ω by the map $(h, y) \mapsto h^{-1}(y)$. Applying this to the action of the torus T^k on T^n one sees that $c_k(T^n) \neq 0$ for any k . Similarly taking $H \subseteq U(k)$ one has $c_n(S^n) \neq 0$ for n odd. Our first main result is

PROPOSITION 1. $c_n(S^n) = 0$ for n even.

This is proved in §3, together with some other calculations of $c_n(M)$.

REMARK. Fathi observed that there is a natural homomorphism

$$\psi^*: \tilde{H}^*(M; \mathbf{R}) \rightarrow \tilde{H}^*(\overline{B}\mathcal{D}iff_{\omega_0}^c M; \mathbf{R})$$

which corresponds to the $c_k(M)$ by means of the formula

$$\psi^*(a)(K) = (-1)^k \langle a, c_k(M)(K) \rangle.$$

Here K is a k -cycle in $\overline{B}\mathcal{D}iff_{\omega_0}^c M$, $a \in H^k(M; \mathbf{R})$ and $\langle a, b \rangle = \int_M \alpha \wedge \beta$, where α, β are forms representing the classes a and b . It is sometimes more natural to think in terms of the homomorphism ψ^* instead of the classes c_k . Note in particular that ψ^* may be defined on the cochain level. For it is easy to check that the formula

$$\psi(\alpha)(S) = \int_{\Delta^k \times M} \pi^*\alpha \wedge \tilde{\Omega}(S),$$

where α is a k -form on M and $\tilde{\Omega}$ is the compactly supported form $\Omega - \pi^*\omega$, defines a homomorphism of the de Rham complex Λ^*M to $C^*(\mathcal{C}; \mathbf{R})$. Evidently ψ induces ψ^* .

One important fact about Ω is that $\Omega^2 = 0$. Indeed, the square of any transverse volume form is zero because locally such a form is pulled back from \mathbf{R}^n .

PROPOSITION 2. *If M is a noncompact manifold such that $H_c^i(M; \mathbf{R}) = 0$ for all $i < n$, then the form Ω is exact. In fact, there is a form Φ on $\mathcal{C} \times M$ such that $d\Phi = \Omega$ and $\Phi\Omega$ has compact support.*

Observe that $d(\Phi\Omega) = \Omega^2 = 0$. Hence, by integrating $\Phi\Omega$ over the fibers $t \times M$, one gets a closed $(n - 1)$ -form on \mathcal{C} and thus an element $s(M)$ of $H^{n-1}(\overline{B}^{\mathcal{O}}\text{diff}_{\omega_0}^c M; \mathbf{R})$. We will see in §2 that this does not depend on the choice of Φ , provided that some natural restrictions are placed on the support of Φ . Hence $s(M)$ is canonically defined. When n is odd, $\Phi\Omega = \frac{1}{2}d(\Phi^2)$ and $s(M) = 0$.

PROPOSITION 3. *$s(\mathbf{R}^n) \neq 0$ when n is even.*

COROLLARY. *$s(M) \neq 0$ when n is even.*

We will see in §3 that Propositions 1 and 3 are related. Proposition 3 is proved by showing that $s(\mathbf{R}^n)$ coincides with a nonzero class in $H^{n-1}(\overline{B}^{\mathcal{O}}\text{diff}_{\omega_0}^c \mathbf{R}^n; \mathbf{R})$ which was discovered by Hurder [7]. His class may be described in the following way. Let $\overline{B}\Gamma_{sl}^n$ be the classifying space for codimension n foliations with transverse volume form and trivialised normal bundle. Then $\overline{B}\Gamma_{sl}^n$ is $(n - 1)$ -connected [4], and has $\pi_n \cong \mathbf{R}$. Let $u \in H^n(\overline{B}\Gamma_{sl}^n; \mathbf{R})$ be the “universal transverse volume form”. In other words, if \mathcal{F} is a foliation with transverse volume form α and trivial normal bundle which is classified by the map g , then $g^*(u) = [\alpha]$. It is shown in [10] that a map $g: S^n \rightarrow \overline{B}\Gamma_{sl}^n$ is null homotopic exactly when $g^*(u) = 0$. Therefore, if $e \in H^n(K(\mathbf{R}, n); \mathbf{R})$ is the fundamental class, there is a fibration

$$(*) \quad \overline{B}\Gamma_{sl}^n \xrightarrow{i} \overline{B}\Gamma_{sl}^n \xrightarrow{f} K(\mathbf{R}, n)$$

in which $\overline{B}\Gamma_{sl}^n$ is n -connected and $f^*(e) = u$. As mentioned above, $\alpha^2 = 0$. Hence $u^2 = 0$. However, $e^2 \neq 0$ when n is even. Therefore, for even n there must be at least one nonzero element of $H^{2n-1}(\overline{B}\Gamma_{sl}^n; \mathbf{R})$ which transgresses to e^2 in the spectral sequence of (*).

One such class may be described as follows. Let $g: M \rightarrow \overline{B}\Gamma_{sl}^n$ be a map of a manifold into $\overline{B}\Gamma_{sl}^n$. By [4], such a map determines a foliation \mathcal{F} with transverse volume form α on some bundle over M . Moreover α must be exact. Let $\alpha = d\beta$. Then the form $\beta\alpha$ is closed because α^2 is zero, and one can easily check that it represents a cohomology class $[\beta\alpha]$ in $H^{2n-1}(M; \mathbf{R})$ which depends only on the homotopy class of g . Evidently, there is a unique class $a \in H^{2n-1}(\overline{B}\Gamma_{sl}^n; \mathbf{R})$ such that $g^*(a) = [\beta\alpha]$ for all such maps g .

LEMMA 4. *The class a transgresses to $-e^2$ in the spectral sequence of (*).*

Now let

$$j: (\overline{B}^{\mathcal{O}}\text{diff}_{\omega_0}^c \mathbf{R}^n) \times \mathbf{R}^n \rightarrow \overline{B}\Gamma_{sl}^n$$

classify the canonical foliation on $(\overline{B}^{\mathcal{D}}\text{diff}_{\omega_0}^c \mathbf{R}^n) \times \mathbf{R}^n$. (We may assume that j maps into $\overline{B}\Gamma_{sl}^n$ since Ω is exact.) It is shown in [8] that the adjoint $\text{Ad } j$ of j ,

$$\text{Ad } j: \overline{B}^{\mathcal{D}}\text{diff}_{\omega_0}^c \mathbf{R}^n \rightarrow \mathcal{N}ap_{cpcr}(\mathbf{R}^n, \overline{B}\Gamma_{sl}^n) = \Omega^n \overline{B}\Gamma_{sl}^n,$$

induces an isomorphism on integer homology when \mathbf{R}^n has infinite ω -volume. (If \mathbf{R}^n has finite ω -volume one can either use [9] or, preferably argue as in §2 below.)

Let h be the composite

$$H^{2n-1}(\overline{B}\Gamma_{sl}^n) \rightarrow H^{n-1}(\Omega^n \overline{B}\Gamma_{sl}^n) \xrightarrow{(\text{Ad } j)^*} H^{n-1}(\overline{B}^{\mathcal{D}}\text{diff}_{\omega_0}^c \mathbf{R}^n).$$

Hurder pointed out that the class a is spherically supported. This implies that its image in $H^{n-1}(\Omega^n \overline{B}\Gamma_{sl}^n)$ is nonzero. Hence $h(a)$ is nonzero. We will see in §2 that $h(a) = s(\mathbf{R}^n)$. It follows that $s(\mathbf{R}^n)$ is also nonzero.

When $n = 2$ the class $s(\mathbf{R}^2)$ is a multiple of the Calabi invariant. This invariant is an element of $H^1(\overline{B}^{\mathcal{D}}\text{diff}_{\sigma_0}^c M; \mathbf{R})$, where $\mathcal{D}\text{diff}_{\sigma_0}^c M$ is the identity component of the group of compactly supported symplectic diffeomorphisms of M , and is defined for certain noncompact M . See [1, II, §4 and 11]. The symplectic case is considered further in Appendix 3. In particular we show that the restriction of $s(\mathbf{R}^{2m})$ to $\overline{B}^{\mathcal{D}}\text{diff}_{\sigma_0}^c \mathbf{R}^{2m}$ is zero when $m > 1$.

All the classes which are mentioned in this paper are smooth and so may be defined on the Lie algebra level. This is discussed in Appendix 2.

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2. The class s . Let A^* be the algebra of all smooth forms on $\mathcal{C} \times M$, and let A_c^* be the subalgebra of forms which are compactly supported with respect to M . We will write $A^{r,s}$, resp. $A_c^{r,s}$, for the subspace of $A^{r+s} = A^k$, resp. A_c^k , consisting of all k -forms with degree r in the T -variables. We will identify Δ^p with $\{(t_1, \dots, t_p) \in \mathbf{R}^p: 0 \leq t_1 \leq \dots \leq t_p \leq 1\}$. Then the restriction of $\Lambda \in A_c^{r,s}$ to the p -simplex S in \mathcal{C} is a form

$$(\#) \quad \Lambda(S) = \sum dt_{i_1} \wedge \dots \wedge dt_{i_r} \wedge \alpha_{i_1, \dots, i_r}(t)$$

on $\Delta^p \times M$, where each $\alpha_{i_1, \dots, i_r}(t)$, $t \in \Delta^p$, is a smooth family of compactly supported s -forms on M . Note also that the differential d on A^* is the sum $d = d_T + d_M$, where d_M and d_T differentiate with respect to the variables in M and Δ^p respectively.

LEMMA 5. *If $H_c^s(M; \mathbf{R}) = 0$, the sequence*

$$A_c^{r,s-1} \xrightarrow{d_M} A_c^{r,s} \xrightarrow{d_M} A_c^{r,s+1}$$

is exact.

PROOF. This is routine. Suppose that $\Lambda \in A_c^{r,s}$ is in $\ker d_M$, and suppose inductively that a solution of the equation $d_M \Psi = \Lambda$ has been constructed on $\mathcal{C}_{p-1} \times M$, where \mathcal{C}_{p-1} is the $(p-1)$ -skeleton of \mathcal{C} . Then we must define $\Psi(S)$ for each p -simplex S . The conditions on Λ and M imply that the forms $\alpha_{i_1, \dots, i_r}(t)$ in the

expression (#) for Λ are exact for each t . The problem now is to find smooth families $\beta_{i_1, \dots, i_p}(t)$ of compactly-supported $(s - 1)$ -forms on M such that $d_M \beta_{i_1, \dots, i_p}(t) = \alpha_{i_1, \dots, i_p}(t)$. Also, setting

$$\Psi(S) = \sum dt_{i_1} \wedge \dots \wedge dt_{i_r} \wedge \beta_{i_1, \dots, i_p}(t)$$

we need $\Psi(S)$ to agree with the previously defined Ψ on $\partial\Delta^p \times M$. However one can always find suitable families $\beta(t)$ by using de Rham's theorem with parameters. \square

PROPOSITION 6. *If $H_c^q(M; \mathbf{R}) = 0$ for $0 < q \leq k$, then any closed form $\Lambda \in A_c^p$ which has no component in $A_c^{p-s, s}$ for $s > k$ may be written*

$$\Lambda = d\Psi + \chi_0,$$

where $\Psi \in A_c^{p-1}$ and $\chi_0 \in A_c^{p,0}$.

PROOF. Since $H_c^n(M; \mathbf{R}) = \mathbf{R}$, we must have $k < n$. Then $\Lambda = \Lambda_0 + \dots + \Lambda_k$, where $\Lambda_i \in A_c^{p-i, i}$. Because Λ is closed we have

$$d_M \Lambda_k = 0, \quad d_T \Lambda_0 = 0 \quad \text{and} \quad d_M \Lambda_{q-1} + d_T \Lambda_q = 0, \quad 0 < q \leq k.$$

The argument is now completed by a standard diagram chase in the double complex:

$$\begin{array}{ccccccc} A_c^{0,2} & \xrightarrow{d_T} & A_c^{1,2} & \rightarrow & \dots & & \\ d_M \uparrow & & d_M \uparrow & & \uparrow & & \\ A_c^{0,1} & \xrightarrow{d_T} & A_c^{1,1} & \xrightarrow{d_T} & A_c^{2,1} & \rightarrow & \dots \\ d_M \uparrow & & d_M \uparrow & & d_M \uparrow & & \\ A_c^{0,0} & \xrightarrow{d_T} & A_c^{1,0} & \xrightarrow{d_T} & A_c^{2,0} & \rightarrow & \dots \quad \square \end{array}$$

COROLLARY 7. *If in addition M is noncompact, then $\chi_0 = 0$ and $\Lambda = d\Psi$.*

PROOF. Observe that χ_0 is closed. Hence,

$$\chi_0(S) = \sum \gamma_{i_1, \dots, i_p}(t, x) dt_{i_1} \wedge \dots \wedge dt_{i_p},$$

where the functions $\gamma_{i_1, \dots, i_p}(t, x)$ are constant with respect to $x \in M$. Since they are also compactly supported, they must vanish when M is noncompact. \square

PROOF OF PROPOSITION 2. We must show that the canonical form Ω is exact when $H_c^q(M; \mathbf{R}) = 0$ for $0 \leq q < n$. But $\Omega = \tilde{\Omega} + \Omega_n$ where $\Omega_n = \pi^*(\omega)$ is exact, and where $\tilde{\Omega} = \Omega_0 + \dots + \Omega_{n-1}$ is closed. Hence Ω is exact by Corollary 7. \square

Thus the equation $d\Phi = \Omega$ has a solution $\Phi = \tilde{\Phi} + \Phi_n$ where $\tilde{\Phi} \in A_c^{n-1}$ satisfies $d\tilde{\Phi} = \tilde{\Omega}$ and where $d\Phi_n = d_M \Phi_n = \Omega_n$. Clearly $\Phi\Omega \in A_c^{2n-1}$. Also $d(\Phi\Omega) = \Omega^2 = 0$. Therefore, we may define the cohomology class $s(M)$ as a cochain by setting

$$s(M)(S) = \int_{\Delta^{n-1} \times M} \Phi\Omega$$

for each $(n - 1)$ -simplex S . Since $\Phi\Omega = \frac{1}{2}d(\Phi^2)$ when n is odd, the class $s(M)$ is zero in this case. (Observe that Φ^2 is compactly supported.) When n is even, it will sometimes be useful to define s using the form $\tilde{\Phi}(\Omega + \Omega_n) = \Phi\Omega - d(\tilde{\Phi}\Phi_n)$. This

alternative definition makes it clear that the cohomology class s does not depend on the choice of $\tilde{\Phi} \in A_c^{n-1}$ or of Φ_n , provided that $d\tilde{\Phi} = \tilde{\Omega}$ and $d\Phi_n = \Omega_n$.

In the remainder of this section, we will fill in the details of the proof that $s(\mathbf{R}^n) \neq 0$ for n even which was sketched in §1. First we will prove Lemma 4 which says that the class $a \in H^{2n-1}(\overline{\overline{B}}\Gamma_{sl}^n)$ transgresses to $-e^2$ in the spectral sequence of the fibration

$$\overline{\overline{B}}\Gamma_{sl}^n \xrightarrow{i} \overline{B}\Gamma_{sl}^n \xrightarrow{f} K(\mathbf{R}, n).$$

PROOF OF LEMMA 4. Recall that the transgression τ is the composite $(\tilde{f}^*)^{-1} \circ \delta$ where

$$H^{2n-1}(\overline{\overline{B}}\Gamma_{sl}^n) \xrightarrow{\delta} H^{2n}(\overline{B}\Gamma_{sl}^n, \overline{\overline{B}}\Gamma_{sl}^n) \xleftarrow{\tilde{f}^*} H^{2n}(K(\mathbf{R}, n), *).$$

Therefore, because $e^2 \in \text{Im } \tau$ it will suffice to show that $\delta(a) = -\tilde{f}^*(e^2)$. This will follow if we show that $\delta(g^*a) = -g^*\tilde{f}^*(e^2)$ for any map g of a compact manifold with boundary $(M, \partial M)$ into the pair $(\overline{B}\Gamma_{sl}^n, \overline{\overline{B}}\Gamma_{sl}^n)$.

A map $g: M \rightarrow \overline{B}\Gamma_{sl}^n$ corresponds to a foliation \mathcal{F} of some bundle E over M with transverse volume form $\tilde{\alpha}$. We will identify M with the zero section of \underline{E} and will put $\alpha = \tilde{\alpha} | M$. Then the restriction of α to ∂M is exact because $g(\partial M) \subseteq \overline{\overline{B}}\Gamma_{sl}^n$. Let β be a form on M such that $d\beta = \alpha$ on ∂M . Then g^*a is by definition the class $[\beta\alpha]$ on ∂M . Since $\alpha = d\beta$ on ∂M and $\alpha^2 = 0$, we have

$$\delta(g^*a) = \delta[\beta\alpha] = \delta[2\beta\alpha - \beta d\beta] = [-(\alpha - d\beta)^2]$$

in $H^{2n}(M, \partial M)$. It follows that the class $[(\alpha - d\beta)^2]$ is independent of the choice of β . On the other hand, it is easy to check that as β varies the class $[\alpha - d\beta]$ runs over the set of elements of $H^n(M, \partial M)$ which map to $[\alpha] = g^*f^*(e)$ in $H^n(M)$. Therefore $g^*\tilde{f}^*(e) = [\alpha - d\beta]$ for some β , and $g^*\tilde{f}^*(e^2) = -\delta(g^*a)$ as required. \square

The next step is to show that a is spherically supported. In fact, let λ be a nonzero element of $\pi_n(\overline{B}\Gamma_{sl}^n)$ and let $\mu \in \pi_{2n-1}(\overline{B}\Gamma_{sl}^n)$ be taken by i_* to the Whitehead product $[\lambda, \lambda] \in \pi_{2n-1}(\overline{B}\Gamma_{sl}^n)$. Then $\langle u, \mathcal{H}(\lambda) \rangle \neq 0$, where \mathcal{H} is the Hurewicz homomorphism and $u \in H^n(\overline{B}\Gamma_{sl}^n; \mathbf{R})$ is the universal transverse volume form. It is now easy to check that $\langle a, \mathcal{H}(\mu) \rangle \neq 0$: for example, see [6].

Finally we must show that $h(a) = s(\mathbf{R}^n)$. Consider the commutative diagram

$$\begin{array}{ccc} H^{2n-1}(\overline{\overline{B}}\Gamma_{sl}^n) & \rightarrow & H^{n-1}(\Omega^n \overline{\overline{B}}\Gamma_{sl}^n) \\ \downarrow j^* & & \downarrow (Ad j)^* \\ H^{2n-1}(\mathcal{C} \times \mathbf{R}^n) & \xrightarrow{\mathcal{G}} & H^{n-1}(\mathcal{C}) \end{array}$$

where \mathcal{G} is integration over the fiber $t \times \mathbf{R}^n$ and where j is as in §1. It is clear from the definition of a that $j^*(a)$ is represented by the form $\Phi\Omega$ on $\mathcal{C} \times \mathbf{R}^n$. Hence $h(a) = s(\mathbf{R}^n)$ as claimed.

This completes the proof of Proposition 3. The results of [9] were used here in order to show that j_* is an isomorphism when \mathbf{R}^n has finite ω -volume. This may be avoided by arguing as follows. Let \mathcal{G}_λ be the subgroup of $\mathcal{D}iff_{\omega_0}^c \mathbf{R}^n$ consisting of

diffeomorphisms with support in the open ball of radius λ . If $\text{vol}_{\tilde{\omega}} \mathbf{R}^n \leq \text{vol}_{\omega} \mathbf{R}^n = \infty$, there is by [2] an embedding $i: \mathbf{R}^n \hookrightarrow \mathbf{R}^n$ such that $i^*\omega = \tilde{\omega}$. We may also assume that $\mathcal{G}_\lambda \subset i_*(\mathcal{D}iff_{\tilde{\omega}0}^c \mathbf{R}^n)$ for some $\lambda > 0$. Therefore, in order to show that $s(\mathbf{R}^n, \tilde{\omega}) \neq 0$ it suffices to show that the restriction s_λ of $s(\mathbf{R}^n, \omega)$ to $\overline{B}\mathcal{G}_\lambda$ is nonzero for all λ . Because $\mathcal{D}iff_{\omega 0}^c \mathbf{R}^n = \lim_{\rightarrow} \mathcal{G}_\lambda$, some s_λ is certainly nonzero. And it is easy to check that the isomorphism $r: \mathcal{G}_\mu \rightarrow \mathcal{G}_\lambda$ which is induced by multiplication by $\mu^{-1}\lambda$ takes s_μ to a nonzero multiple of s_λ . A similar argument proves the corollary to Proposition 3.

3. The class c_n . We first prove Proposition 1 which states that $c_n(S^n) = 0$ when n is even.

PROOF OF PROPOSITION 1. By Proposition 6 there are forms $\tilde{\Phi}$ and χ_0 on $\mathcal{C} \times S^n$ such that $\tilde{\Omega} = d\tilde{\Phi} + \chi_0$ and $\chi_0 \in A_c^{n,0}$. Note also that if K is any n -cycle in \mathcal{C} , the integral of Ω over $K \times x$ is independent of x . Hence

$$c_n(K) = \int_{K \times x} \Omega = \lambda \int_{K \times S^n} \Omega \Omega_n$$

where $\lambda^{-1} = \int_{S^n} \omega$. But

$$\Omega \Omega_n = (\tilde{\Omega} - \chi_0 + \Omega_n) \Omega_n + \chi_0 \Omega_n = d(\tilde{\Phi} \Omega_n) + \chi_0 \Omega_n.$$

Also, because $\tilde{\Omega}(\Omega + \Omega_n) = \Omega^2 - \Omega_n^2 = 0$ when n is even, we have

$$2\chi_0 \Omega_n = (\chi_0 - \tilde{\Omega})(\Omega + \Omega_n) - \chi_0 \tilde{\Omega} = -d(\tilde{\Phi}(\Omega + \Omega_n)) - \chi_0 \tilde{\Omega}.$$

Therefore

$$2c_n(K) = -\lambda \int_{K \times S^n} \chi_0 \tilde{\Omega} = 0,$$

since all the terms in $\chi_0 \tilde{\Omega}$ have degree $< n$ in the x -variables. \square

It would be interesting to understand which compact manifolds M have $c_n(M) = 0$. Clearly the above proof applies when $H^i(M; \mathbf{R}) = 0, 0 < i < n$, and n is even. The examples given in §1 of manifolds such that $c_n(M) \neq 0$ were constructed using the action of $\mathcal{D}iff_{\omega 0} M$ on M and were really examples with $i^*c_n(M) \neq 0$ where $i: \mathcal{D}iff_{\omega 0} M \rightarrow \overline{B}\mathcal{D}iff_{\omega 0} M$ is the natural map. In fact, a cycle $\gamma: K \rightarrow \mathcal{D}iff_{\omega 0} M$ in $\mathcal{D}iff_{\omega 0} M$ gives rise to a foliation on $K \times M$ with leaves $\{(k, \gamma(k)y): k \in K\}$. Its transverse volume form is the pull-back of ω by the map $(k, y) \mapsto \gamma(k)^{-1}y$. It follows that the value of the class $i^*c_n(M)$ on the cycle K is the constant function on M which equals $\langle e^*[\omega], K \rangle$, where $e: K \rightarrow M$ is the evaluation map $k \mapsto \gamma(k)^{-1}y$. Gottlieb proves in [3, Theorem A] that $\chi(M) \cdot e^*([\omega]) = 0$, where $\chi(M)$ is the Euler characteristic of M . Hence we have shown

LEMMA 8. *If M is a compact manifold with $\chi(M) \neq 0$, then $i^*c_n(M) = 0$.*

In particular, this implies that $i^*c_2(X) = 0$ when X is an oriented surface of genus > 1 .

LEMMA 9. *If X is an oriented surface of genus > 0 , then $c_2(X) \neq 0$.*

PROOF. Let $Y = X - \{x_0\}$. We will write \mathcal{C}_X for $\overline{B}\mathcal{D}iff_{\omega 0} X$ and \mathcal{C}_Y for $\overline{B}\mathcal{D}iff_{\tilde{\omega} 0} Y$ where $\tilde{\omega} = \omega|_Y$. Let $p: \mathcal{C}_X \rightarrow \overline{B}\Gamma_{s'}^2$ classify the germ at $\mathcal{C}_X \times x_0$ of the canonical

foliation of $\mathcal{C}_X \times X$. It is shown in [9] that the homotopy fiber of p has the same integral homology as \mathcal{C}_Y . Therefore there is a spectral sequence

$$H^r(\overline{B}\Gamma_{sl}^2; H^s(\mathcal{C}_Y)) \Rightarrow H^{r+s}(\mathcal{C}_X)$$

where coefficients in \mathbf{R} are understood. Since $\overline{B}\Gamma_{sl}^2$ is 1-connected, this gives rise to an exact sequence

$$(\#) \quad 0 \rightarrow H^1(\mathcal{C}_X) \xrightarrow{j^*} H^1(\mathcal{C}_Y) \xrightarrow{\tau} H^2(\overline{B}\Gamma_{sl}^2) \xrightarrow{p^*} H^2(\mathcal{C}_X)$$

where τ is the transgression and j^* is induced by the inclusion $Y \subseteq X$.

Banyaga shows in [1] that $H_1(\mathcal{C}_X; \mathbf{Z}) \cong H^1(X; \mathbf{R})$. Further, because the form $\langle \alpha, \beta \rangle = \int_Y \alpha \wedge \beta$ does not vanish identically for $\alpha, \beta \in H_c^1(Y; \mathbf{R})$, the results of Rousseau [11] imply that $H_1(\mathcal{C}_Y; \mathbf{Z}) \cong H_c^1(Y; \mathbf{R})$. Thus there is a commutative diagram

$$\begin{array}{ccc} H_1(\mathcal{C}_Y; \mathbf{Z}) & \rightarrow & H_c^1(Y; \mathbf{R}) \\ \downarrow & & \downarrow \\ H_1(\mathcal{C}_X; \mathbf{Z}) & \rightarrow & H^1(X; \mathbf{R}) \end{array}$$

where the horizontal maps are isomorphisms given by the flux and the vertical maps are induced by the inclusion of Y in X . It follows that the map j^* in the exact sequence (#) is an isomorphism and that p^* is injective. But if $u \in H^2(\overline{B}\Gamma_{sl}^2)$ is the “universal transverse volume form”, p^*u is represented by the restriction of Ω to $\mathcal{C}_X \times x_0$ and so equals $c_2(X)$. Hence $c_2(X) \neq 0$ as claimed. \square

In general, if $Y = X - x_0$ one has an exact sequence

$$H^{n-1}(\mathcal{C}_Y) \xrightarrow{\tau} H^n(\overline{B}\Gamma_{sl}^n) \xrightarrow{p^*} H^n(\mathcal{C}_X)$$

in which $p^*u = c_n(X)$. Therefore the vanishing of $c_n(X)$ implies that u is in the image of τ . In particular, consider the case $X = S^n$, for n even.

LEMMA 10. $\tau(s(\mathbf{R}^n))$ is a nonzero multiple of u .

PROOF. Recall that τ is the composite $(\bar{p}^*)^{-1} \circ \delta$ where

$$H^{n-1}(\mathcal{C}_Y) \xrightarrow{\delta} H^n(\mathcal{C}_X, \mathcal{C}_Y) \xleftarrow{\bar{p}^*} H^n(\overline{B}\Gamma_{sl}^n).$$

Therefore, it will suffice to show that $\delta(s(\mathbf{R}^n))$ is a nonzero multiple of \bar{p}^*u . Let $\tilde{\Phi}$ and χ_0 be forms on $\mathcal{C}_X \times X$ as in the proof of Proposition 1. Clearly we may assume that the restriction $\tilde{\Phi}_Y$ of $\tilde{\Phi}$ to $\mathcal{C}_Y \times Y$ is compactly supported and satisfies $d\tilde{\Phi}_Y = \tilde{\Omega}$. Thus $\tilde{\Phi}$ is zero on some neighbourhood of $\mathcal{C}_Y \times x_0$ in $\mathcal{C}_Y \times X$, and $\chi_0 = 0$ on $\mathcal{C}_Y \times X$. The class \bar{p}^*u is represented by the restriction of the form Ω to $(\mathcal{C}_X \times x_0, \mathcal{C}_Y \times x_0)$. Therefore, if L is an n -cycle in $(\mathcal{C}_X, \mathcal{C}_Y)$ we have

$$\bar{p}^*u(L) = \int_{L \times x_0} \Omega = \int_{L \times x_0} (d\tilde{\Phi} + \chi_0) + \Omega_n = \int_{\partial L \times x_0} \tilde{\Phi} + \int_{L \times x_0} (\chi_0 + \Omega_n).$$

But $\check{\Phi} = 0$ on $\partial L \times x_0$ since $\partial L \subset \mathcal{C}_Y$. Also $\int_{L \times x_0} \Omega_n = 0$. Therefore, because χ_0 is independent of x (see Corollary 7) we have

$$\tilde{p}^*u(L) = \int_{L \times x_0} \chi_0 = \lambda \int_{L \times X} \chi_0 \Omega_n \quad \text{where } \lambda^{-1} = \int_X \omega.$$

But

$$\begin{aligned} \delta(s(\mathbf{R}^n))(L) &= s(\mathbf{R}^n)(\partial L) = \int_{\partial L \times Y} \check{\Phi}(\Omega + \Omega_n) = \int_{\partial L \times X} \check{\Phi}(\Omega + \Omega_n) \\ &= \int_{L \times X} (\tilde{\Omega} - \chi_0)(\Omega + \Omega_n) = - \int_{L \times X} 2\chi_0 \Omega_n \end{aligned}$$

as in the calculation of $c_n(X)$ in Proposition 1. \square

This result suggests that one should be able to extend s to a natural nonzero class $s(Y) \in H^{n-1}(\overline{B^0} \text{Diff}_{\omega_0}^c Y; \mathbf{R})$ which is defined for all even dimensional manifolds of the form $Y = X - x_0$, where X is a compact manifold with $c_n(X) = 0$. (To say that s is natural means of course that $j^*s(Y') = s(Y)$ for all embeddings $j: (Y, \omega) \hookrightarrow (Y', \omega')$.) Similarly, one might expect that s would always be zero on odd dimensional manifolds and hence that $c_n(X) \neq 0$ for all odd dimensional X . Note that these suggestions are consistent with our results in dimension 2, for Rousseau's work says exactly that the only noncompact 2-dimensional manifold on which s can be defined is \mathbf{R}^2 .

Appendix 1. Explicit formulae for the c_k . Consider a p -simplex S in \mathcal{C} . Let $\tilde{X}_1(t), \dots, \tilde{X}_p(t)$ be the vector fields on $\Delta^p \times M$ which are tangent to the leaves of the foliation $\mathcal{F}(S)$ and which lie above the fields $\partial/\partial t_1, \dots, \partial/\partial t_p$ on $\Delta^p = \{(t_1, \dots, t_p): 0 \leq t_1 \leq \dots \leq t_p \leq 1\}$. Then for each $t \in \Delta^p$ we may write

$$\tilde{X}_i(t) = X_i(t) + \partial/\partial t_i,$$

where $X_i(t)$ is an ω -preserving vector field on M . (Compare [1, §I] where it is shown that $X_i(t, x) = \partial h_i / \partial t_i (h_i^{-1}(x))$.) The vectors $[\tilde{X}_i(t), \tilde{X}_j(t)]$ are tangent to the leaves of $\mathcal{F}(S)$ and project to $[\partial/\partial t_i, \partial/\partial t_j] = 0$ on Δ^p . Since the projection of $\Delta^p \times M$ on Δ^p maps each leaf of $\mathcal{F}(S)$ diffeomorphically onto Δ^p , it follows that $[\tilde{X}_i(t), \tilde{X}_j(t)] = 0$. This proves

LEMMA A1 [1, Proposition I.1.1]. For each $t \in \Delta^p$ we have

$$[X_i(t), X_j(t)]_M = \frac{\partial}{\partial t_j} (X_i(t)) - \frac{\partial}{\partial t_i} (X_j(t)).$$

For short we will write this equation as

$$[X_i, X_j] = \partial_j X_i - \partial_i X_j.$$

Next, suppose that Y_1, \dots, Y_k are ω -preserving vector fields on M and consider the $(n - k)$ -form

$$\omega(Y_1, \dots, Y_k; \dots) = i(Y_k) \cdots i(Y_1) \omega$$

on M . When $k = 1$ this form is closed. In general one has

LEMMA A2.

$$d_M \omega(Y_1, \dots, Y_k; \dots) = \sum_{i < j} (-1)^{i+j+k} \omega([Y_i, Y_j], Y_1, \dots, \hat{i}, \hat{j}, \dots, Y_k; \dots).$$

PROOF. This follows easily by induction on k , using the formulae

$$di(X) + i(X)d = \mathcal{L}_X,$$

and

$$\mathcal{L}_X(\omega(Y_1, \dots, Y_k; \dots)) = \omega([X, Y_1], Y_2, \dots, Y_k; \dots) + \dots + \omega(Y_1, \dots, [X, Y_k]; \dots). \quad \square$$

PROPOSITION A3. For each p -simplex S we have

$$\Omega_{n-k}(S) = (-1)^k \sum_{1 \leq i_1 < \dots < i_k \leq p} dt_{i_1} \wedge \dots \wedge dt_{i_k} \wedge \omega(X_{i_1}, \dots, X_{i_k}; \dots)$$

where the fields $X_i(t)$ are as defined above.

PROOF. Let $\Lambda(S) = \Lambda_0(S) + \dots + \Lambda_n(S)$ be the form defined by the right-hand side of the above equations. Note that $\Lambda_k(S) = 0$ if $k < n - p$. We must check that $\Lambda(S)$ is closed, that it defines $\mathcal{F}(S)$ and that it restricts to ω on each fiber $t \times M$. The last two statements are easy to verify, and will be left to the reader. Since $\Lambda_n(S) = \pi^*(\omega)(S)$ is closed, the first statement is equivalent to the equations

$$d_T \Lambda_k(S) + d_M \Lambda_{k-1}(S) = 0 \quad \text{for } 0 \leq k \leq n.$$

These follow easily from Lemmas A1 and A2. For example,

$$\begin{aligned} d_T \Lambda_{n-1}(S) &= d_T \left(- \sum_i dt_i \wedge \omega(X_i; \dots) \right) \\ &= - \sum_{i \neq j} dt_j \wedge dt_i \wedge \omega(\partial_j X_i; \dots) \\ &= \sum_{i < j} dt_i \wedge dt_j \wedge \omega([X_i, X_j]; \dots) = -d_M \Lambda_{n-2}(S). \quad \square \end{aligned}$$

COROLLARY A4. For each k -cycle K in \mathcal{C} we have

$$c_k(M)(K) = \int_K \Omega_{n-k} = (-1)^k \sum_{S \in K} \int_{\Delta^k} dt_1 \wedge \dots \wedge dt_k \wedge \omega(X_1, \dots, X_k; \dots).$$

It follows immediately that $-c_1(M)$ is the flux homomorphism.

Appendix 2. Lie algebra cohomology. In [5] Haefliger describes spaces such as $\overline{B} \mathcal{D}iff_{\omega_0}^c M$ in a slightly different context, emphasizing their connection with Lie algebras. In this appendix we will translate our results into his language. I am grateful to Haefliger for explaining how this may be done.

Let \mathfrak{g}_M be the Lie algebra of compactly supported vector fields on M which preserve ω . Consider the double complex $C^{r,s}(M) = C^r(\mathfrak{g}_M; \Lambda^s(M))$ of r -cochains on \mathfrak{g}_M with values in the s -forms on M where \mathfrak{g}_M acts trivially on $\Lambda^* M$, and let

$C^*(M)$ be the associated total complex. As in [5], one can construct a universal characteristic homomorphism

$$\chi: C^{r,s}(M) \rightarrow A_d^{r,s} \subseteq A^{r,s}$$

where $A^{r,s}$ is as in §2 and $A_d^{r,s}$ is its subcomplex of forms smooth under deformation. Define $\bar{\Omega}_{n-k} \in C^{k,n-k}$ by

$$\bar{\Omega}_{n-k}(X_1, \dots, X_k) = (-1)^k \omega(X_1, \dots, X_k; \dots)$$

and let $\bar{\Omega} = \sum \bar{\Omega}_{n-k} \in C^n(M)$. Then $\bar{\Omega}$ is closed. Moreover, because $\theta(\partial/\partial t_i) = X_i(t)$, the calculations of Appendix 1 show that $\chi(\bar{\Omega}) = \Omega$. Thus $\bar{\Omega}$ is the Lie algebra analogue of Ω .

Since $\bar{\Omega}$ is exact when $H_c^i(M; \mathbf{R}) = 0, i < n$, one can easily construct a class $\bar{s} \in H^{n-1}(\mathfrak{g}_M; \mathbf{R})$ which is taken by χ to s . There are also classes $\bar{c}_k \in H^k(\mathfrak{g}_M; H_c^{n-k}(M))$ which correspond to the c_k . As before, they come from a homomorphism

$$\bar{\psi}^*: H^*(M) \rightarrow H^*(\mathfrak{g}_M; \mathbf{R})$$

which is induced by the cochain map

$$\bar{\psi}: \Lambda^* M \rightarrow C^*(\mathfrak{g}_M; \mathbf{R})$$

given by

$$\bar{\psi}(\beta)(X_1, \dots, X_k) = \int_M \beta \wedge \omega(X_1, \dots, X_k; \dots), \text{ for } k \geq 1.$$

Appendix 3. The symplectic case. Let σ be a symplectic form on M and set $\omega = \sigma^m$ where $\dim M = 2m = n$. Then $\mathcal{C}_\sigma = \bar{B}^{\mathcal{D}}\text{diff}_{\sigma_0}^c M$ is a subcomplex of \mathcal{C} . The arguments of §1 show that $\mathcal{C}_\sigma \times M$ carries a canonical symplectic form Σ which is induced locally from \mathbf{R}^{2m} . Clearly $\Sigma^m = \Omega$ and $\Sigma^{m+1} = 0$. Moreover, one can show as in Appendix 1 that $\Sigma = \Sigma_0 + \Sigma_1 + \Sigma_2$ where $\Sigma_2 = \pi^*(\sigma)$, and

$$\Sigma_1(S) = - \sum_{1 \leq i \leq p} dt_i \wedge \sigma(X_i; \cdot),$$

$$\Sigma_0(S) = \sum_{1 \leq i < j \leq p} \sigma(X_i, X_j) dt_i \wedge dt_j$$

on any p -simplex S in \mathcal{C}_σ . The form Σ defines classes similar to the c_k .

If M is noncompact and if $H_c^1(M; \mathbf{R}) = 0$, then Corollary 7 implies that there is a form $\bar{\Psi}$ in $A_c^{1,0}$ such that $d\bar{\Psi} = \Sigma_0 + \Sigma_1$. It is easy to check that on any 1-simplex S we have $\bar{\Psi}(S) = \alpha(t, x) dt$, where, for each $t \in I$ the function $\alpha(t, x)$ is the unique compactly supported function on M such that $d_M \alpha(t, x) = \sigma(X(t); \cdot)$. The Calabi class $R \in H^1(\bar{B}^{\mathcal{D}}\text{diff}_{\sigma_0}^c M; \mathbf{R})$ is then given by

$$R(S) = \int_{I \times M} \alpha(t, x) dt \wedge \sigma^m = \frac{1}{2} \int_{I \times M} \bar{\Psi}(\Omega + \Omega_n)$$

for each 1-simplex S . See [1, II, §4]. This should be compared with the formula $s(M)(S) = \int_{\Delta^{n-1} \times M} \bar{\Psi}(\Omega + \Omega_n)$ for $s(M)$. Note that these formulae agree up to a constant when $m = 1$. Observe also that the form $\bar{\Psi}(\Omega + \Omega_n)$ is not closed. It defines

a cohomology class as above because the part of $d(\tilde{\Psi}(\Omega + \Omega_n))$ which lies in $A_c^{2,n}$ is d_M -exact and so has zero integral over M . To see this one uses the formula $2\Sigma_0\Sigma_2^m + m\Sigma_1^2\Sigma_2^{m-1} = 0$, which is the part of the equation $\Sigma^{m+1} = 0$ concerning forms in $A^{2,n}$, and observes that $\Sigma_1 = d_M\tilde{\Psi}$.

Let $\Lambda_j = \Sigma^j + \Sigma^{j-1}\Sigma_2 + \cdots + \Sigma_2^j$. Then $\tilde{\Psi}\Lambda_m$ is closed and its component in $A_c^{1,n}$ equals $(m+1)\tilde{\Psi}\Sigma_2^m$. Hence the class $(m+1)R$ may be defined by integrating $\tilde{\Psi}\Lambda_m$ over M . More generally one gets classes $R_j \in H^{2j+1}(\bar{B}^{\mathcal{O}}\mathcal{D}iff_{\sigma_0}^c M; \mathbf{R})$ by integrating $\tilde{\Psi}\Lambda_m\Sigma^j$ over M , $0 \leq j \leq m$. Notice also that $d(\tilde{\Psi}\Lambda_{m-1}) = \Sigma^m - \Sigma_2^m$. Hence $\Omega = \Sigma^m$ is exact on $\mathcal{C}_\sigma \times M$ and one can define a class s_σ in $H^{2m-1}(\bar{B}^{\mathcal{O}}\mathcal{D}iff_{\sigma_0}^c M; \mathbf{R})$ which corresponds to s . It is not hard to check that $s_\sigma = R_{m-1}$. Since the classes R_j , $j \geq 1$, vanish when Σ_2 and hence Σ are exact, this shows that the restriction of $s(\mathbf{R}^{2m})$ to $\bar{B}^{\mathcal{O}}\mathcal{D}iff_{\sigma_0}^c \mathbf{R}^{2m}$ is zero if $m > 1$.

Note finally that the R_j may be defined on $\bar{B}^{\mathcal{O}}\mathcal{D}iff_{\sigma_0}^{\Phi^c} M$ for any symplectic, noncompact M , where $\mathcal{D}iff_{\sigma_0}^{\Phi^c} M$ is the kernel of the flux homomorphism $\mathcal{D}iff_{\sigma_0}^c M \rightarrow H_c^1(M; \mathbf{R})/\Gamma$.

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