NAKAYAMA ALGEBRAS AND GRADED TREES

BY

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Abstract. Let $k$ be an algebraically closed field. We show that if $T$ is a finite tree, then there is a grading $g$ on $T$ such that $(T, g)$ is a representation finite graded tree, and such that the corresponding simply connected $k$-algebra is a Nakayama algebra (i.e. generalized uniserial algebra).

Introduction. Let $k$ be an algebraically closed field. A simply connected algebra $\Lambda$ over $k$ is an algebra which is representation-finite, connected, basic, finite-dimensional and has a simply connected Auslander-Reiten quiver $\Gamma_\Lambda$. In order to study the simply connected algebras, K. Bongartz and P. Gabriel introduced the notion of graded trees [2]. If $T$ is a finite tree, let $T_0$ denote the set of vertices of $T$. A grading of the tree $T$ is a function $g: T_0 \rightarrow \mathbb{N}$ ($\mathbb{N}$ is the nonnegative integers), satisfying the following conditions:

(a) $g(x) - g(y) \in 1 + 2\mathbb{Z}$, whenever $x$ and $y$ are neighbours in $T$ ($\mathbb{Z}$ the integers).

(b) $g^{-1}(0) \neq \emptyset$.

A graded tree is a pair $(T, g)$ formed by a tree $T$ and a grading $g$ of $T$.

K. Bongartz and P. Gabriel show that there is a bijection between the isomorphism classes of representation-finite graded trees and the isomorphism classes of simply connected algebras. For the benefit of the reader we give a summary of their results in §1. They also show in [2] that every tree $T$ admits only a finite number of representation-finite gradings. In this paper we show that for every tree $T$ it is possible to find a grading $g$ such that $(T, g)$ is representation-finite. This answers a question raised by P. Gabriel. In fact, what we show is that given a tree $T$ it is possible to find a grading $g$ such that the associated simply connected algebra is a Nakayama algebra. Conversely, given a noncyclic Kupisch series for a Nakayama $k$-algebra $\Lambda$, one may associate a graded tree $(T, g)$ such that the simply connected $k$-algebra obtained from $(T, g)$ is $\Lambda$.

1. Simply connected algebras and graded trees. Let $(T, g)$ be a graded tree. To this graded tree we associate a translation quiver $Q_T$ in the following way. The vertices of $Q_T$ are the points $(n, t) \in \mathbb{N} \times T_0$ such that $n - g(t) \in 2\mathbb{N}$, two such vertices $(m, s)$ and $(n, t)$ are joined by an arrow $(m, s) \rightarrow (n, t)$ if $s, t$ are neighbours in $T$ and $n = m + 1$. The projective vertices are the points $(g(t), t)$, the translate of a nonprojective vertex is defined by $\tau(n, t) = (n - 2, t)$. 

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For each graded tree $T = (T, g)$ there is a unique map $d: (Q_T)_0 \to N^{T_0}$ satisfying the following conditions:

(a) $d(g(t), t) = \delta_t + \sum_s d(g(t) - 1, s)$, where $s$ ranges over the neighbours $s$ of $t$ such that $g(s) < g(t)$ and $d(g(t) - 1, s) > 0$ (where a function is $> 0$ if all its values are $\geq 0$ and at least one of them if $> 0$), and the Kronecker function $\delta_t$ takes the value $1$ at $t$ and $0$ otherwise.

(b) $d(n, t) = \sum_s d(n - 1, s) - d(n - 2, t)$, whenever $(n, t)$ is a nonprojective vertex of $Q_T$ for which the functions $d(n - 2, t)$ and $\sum_s d(n - 1, s) - d(n - 2, t)$ are both $> 0$, when $s$ ranges over the neighbours of $t$ in $T$ such that $g(s) < n$.

(c) For any other vertex $(n, t)$ of $Q_T$ we have $d(n, t) = 0$.

Using these conditions, $d(n, t)$ can be computed by induction on $n$, starting with $n = g(t)$. $d$ is called the dimension map of $Q_T$. We denote by $R_T$ the full subtranslation-quiver of $Q_T$ formed by the vertices $(n, t)$ such that $d(n, t) > 0$. The grading $g$ is called admissible if $R_T$ is a connected subquiver of $Q_T$, and $T$ is then called an admissible graded tree. The grading is called representation-finite if it is admissible and $R_T$ is finite. $T$ is then called a representation-finite graded tree.

Remark. We are using a definition of $d$ different from the one given in [2, p. 356], since it was through our definition we saw the main result of this paper. Also with our definition the projective vertices in $R_T$ coincide with those in $Q_T$ regardless of the grading $g$. It is easy to see that the two definitions are the same when $R_T$ is connected.

Let $T$ be an admissible graded tree. Let $A^T$ be the finite-dimensional algebra $A^T = \prod_{p,q} k(R_T)(q, p)$, where $k(R_T)$ is the mesh category of $R_T$, and $p, q$ range over all projective vertices of $R_T$. Then each vertex $x$ of $R_T$ is associated with an $A^T$-module $M(x) = \prod_p k(R_T)(p, x)$, where $p$ ranges over all projective vertices of $R_T$, and it is shown in [2] that for every vertex $(n, t)$ of $R_T$, the $A^T$-module $M(n, t)$ is indecomposable and its dimension vector is $d(n, t)$, especially, $M(g(t), t)$ are the indecomposable projective modules, and if $M(n, t)$ is not projective, $DTr(M(n, t)) = M(n - 2, t) = M(t(n, t))$. In fact, if $(T, g)$ is representation-finite, then there is a translation-quiver isomorphism of the Auslander-Reiten quiver $\Gamma_A$ onto $R_T$.

If $\Gamma$ is a locally finite translation-quiver, and $x$ is a vertex of $\Gamma$, the set of all $n \in Z$ such that $\tau^n x$ is defined, is an interval $[\tau^n x, n \in \tau^n x]$ of $Z$. Then the set $x^\tau = \{\tau^n x, n \in \tau^n x\}$ is called the $\tau$-orbit of $x$. The vertex $x$ is stable if $[\tau^n x] = Z$, it is periodic if it is stable and has a finite $\tau$-orbit. The $\tau$-orbits of a connected component $E$ of the stable part $\Gamma_s$ of $\Gamma$ are either all finite or all infinite. In the first case we call $E$ a periodic component of $\Gamma$.

If $x \xrightarrow{\sigma} y$ is an arrow of $\Gamma$, where $y$ is not projective, there is a unique arrow $\tau y \rightarrow x$, which we denote $\sigma x$. The $\tau$-orbit of $\sigma$, denoted $\tau^\sigma$, is the set of all arrows of $\Gamma$ of the form $\sigma^\alpha x$.

The graph $G_{\Gamma}$ associated with $\Gamma$ has as vertices the nonperiodic $\tau$-orbits and the periodic components of $\Gamma$. To each periodic component, considered as a vertex of $G_{\Gamma}$, we associate a loop of $G_{\Gamma}$. Let $a^\sigma$ be a $\sigma$-orbit connecting $x^\tau$ and $y^\tau$. If both $x$ and $y$ are nonperiodic, we associate with $a^\sigma$ an edge connecting the vertices $x^\tau$ and $y^\tau$. If $y$ is not periodic and $x$ belongs to a periodic component $E$ we associate with $a^\sigma$ an edge of $G_{\Gamma}$ connecting $E$ and $y^\tau$. 

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Now, if $A$ is a simply connected algebra, and $\Gamma_A$ is the Auslander-Reiten quiver of $A$, then the graph $G_A$ associated with $\Gamma_A$ is a tree [2, Theorem 4.2]. Since $\Gamma_A$ is simply connected and finite, there is a unique quiver morphism $K_A: \Gamma_A \to \mathbb{Z}A_2$ such that $0 = \min K(x)$, the minimum taken over all vertices $x$ of $\Gamma_A$. Here $\mathbb{Z}A_2$ is the following translation quiver where $\rightarrow$ indicates the translation. Since $G_A$ is a tree, each $\tau$-orbit $t$ of $\Gamma_A$ contains exactly one projective vertex $p_t$. We set $g_A(t) = K_A(p_t) \in \mathbb{N}$. The function $g_A$ is then a grading of $G_A$, and $(G_A, g_A)$ is a graded tree. The maps $(T, g) \to A_T$ and $A \to (G_A, g_A)$ are inverse maps and therefore there is a bijection between the isomorphism classes of representation-finite graded trees and the isomorphism classes of simply connected algebras [2, 6.5].

![Diagram of trees and algebras]

2. The relation between Kupisch series and trees. In this section we examine the relation between the Nakayama algebras with noncyclic Kupisch series and trees. We show that given a tree $T$, it is possible to associate a Nakayama algebra $\Lambda$ to this tree such that the graph of $\Lambda$ is isomorphic to $T$. From this follows the main result of this paper: To every tree $T$ it is possible to find a grading $g$ such that $(T, g)$ is representation-finite. The Nakayama algebra $\Lambda$ is not uniquely given by the construction we use.

But first we show how to construct a tree $T_\Lambda$ from a Nakayama algebra $\Lambda$ with noncyclic Kupisch series. The construction determines $T_\Lambda$ uniquely up to isomorphism, and later we will see that $T_\Lambda$ is in fact the graph $G_A$ associated with $\Gamma_A$. Therefore this gives us an easy way to construct $G_A$ if $\Lambda$ is a Nakayama algebra.

We recall that the Kupisch series for an indecomposable Nakayama algebra $\Lambda$ is an ordered complete set of representatives $P_1, \ldots, P_n$ of the isomorphism classes of indecomposable projective $\Lambda$-modules, satisfying the following conditions:

(i) $P_i/rP_i \cong rP_{i+1}/r^2P_{i+1}$, or equivalently:

$$P_{i+1}/rP_{i+1} = TrD(P_i/rP_i).$$

(ii) $L(P_i) \geq 2$ for all $i$ such that $2 \leq i \leq n$.

(iii) $L(P_{i+1}) \leq L(P_i) + 1$ for $i = 1, \ldots, n$, and $L(P_1) \leq L(P_n) + 1$.

$L(M)$ = the length of the $\Lambda$-module $M$.

Any finite sequence of integers $c_1, \ldots, c_n$ satisfying (ii) and (iii) above when we put $c_i = L(P_i)$, is called an admissible sequence. Given an arbitrary admissible sequence, an algebra can be constructed such that its Kupisch series corresponds to this sequence. The Kupisch series is noncyclic if $L(P_1) = 1$. For details, see [4].

Let $\Lambda$ be an indecomposable Nakayama algebra with a noncyclic Kupisch series. Let $T_\Lambda$ be the following tree: The vertices of $T_\Lambda$ are the representatives of the isomorphism classes of indecomposable projective $\Lambda$-modules. For each $i$, let $t_i$ be the vertex corresponding to the projective $P_i$. If $i, j \in \{1, \ldots, n\}$, with $i \leq j$, there is an edge connecting $t_i$ and $t_j$ if $i$ is the greatest integer less than $j$ such that $L(P_i) = L(P_j) + 1$. $T_\Lambda$ is connected, since for every $j \in \{2, \ldots, n\}$ it follows from
(iii) above that there always exists such an i, and it is not difficult to see that $T_\Lambda$ really is a tree when constructed as above.

We define a walk in a tree $T$ to be a sequence of vertices $S_1 \cdots S_n$, connected by edges $\alpha_1 \cdots \alpha_{n-1}$ in such a way that for each $i \in \{1, \ldots, n - 1\}$, $S_i$ and $S_{i+1}$ are connected by the edge $\alpha_i$ of $T$.

If $u$ is a walk: $S_1 \overrightarrow{\alpha_1} S_2 \overrightarrow{\alpha_2} \cdots \overrightarrow{\alpha_{k-1}} S_k$ in a tree $T$, we define the length of $u$, $l(u) = k$. If $S_i$ and $S_j$ are two vertices of $T$, the shortest walk from $S_i$ to $S_j$ is the walk that does not pass through any vertex twice. It follows from the construction above that for any vertex $t_i$ in $T_\Lambda$, $L(P_i)$ is equal to the length of the shortest walk in $T_\Lambda$ from $t_i$ to $t_1$.

**Example.** Given the admissible sequence $\{1, 2, 3, 3, 4, 3\}$ the corresponding tree $T_\Lambda$ is:

![Diagram of tree]

Conversely, starting with a tree $T$, to this tree we can associate a noncyclic Kupisch-series for an indecomposable Nakayama algebra: Fix a point $t_1$ in the tree $T$ and a walk $V$ around the tree from $t_1$ to $t_1$ which passes through every edge in the tree exactly twice.

**Example.** If $T$ is the tree, then $V: t_1 \leftarrow B \rightarrow A \rightarrow B \rightarrow C \leftarrow D \rightarrow C \rightarrow E \rightarrow C \rightarrow B \rightarrow t_1 \leftarrow F \rightarrow t_1 \leftarrow G \rightarrow H \rightarrow G \rightarrow t_1 \leftarrow I \rightarrow t_1$ is such a walk.

![Diagram of walk]

The order in which $V$ passes through each vertex for the first time defines an ordering of the vertices of $T$, such that $t_i$ is the $i$th new vertex which occurs in $V$. In
the example above \( t_2 = B, t_3 = A, t_4 = C, t_5 = D, t_6 = E, t_7 = F, t_8 = G, t_9 = H, t_{10} = I \).

Suppose \( T_0 = \{t_1, \ldots, t_n\} \). Then for each \( i \in \{1, \ldots, n\} \), let \( C_i = l(U_i) \), where \( U_i \) is the shortest walk in \( T \) from \( t_i \) to \( t_i \). It is clear that \( C_1 = 1 \), and that \( C_i \geq 2 \) for \( i \geq 2 \). Further, if \( t_{i+1} \) is a neighbour of \( t_i \), then \( C_{i+1} = C_i + 1 \), because it is clear that there is only one neighbour \( t_k \) of \( t_i \) with \( l(U_k) < l(U_i) \), and it is the only neighbour with \( k < i \). If \( t_{i+1} \) is not a neighbour of \( t_i \), then \( t_{i+1} \) is a neighbour of a vertex \( t_j \) with \( l(U_j) < l(U_i) \). So in that case \( C_{i+1} < C_i + 1 \). Therefore we have that \( \{C_1, \ldots, C_n\} \) is an admissible sequence which corresponds to the noncyclic Kupisch series of an indecomposable Nakayama algebra.

We now claim that every indecomposable Nakayama algebra \( \Lambda \) with a noncyclic Kupisch series is simply connected. The ordinary quiver \( Q_\Lambda \) of an indecomposable Nakayama algebra \( \Lambda \) with a noncyclic Kupisch series is a tree of form \( \rightarrow \cdots \rightarrow \), therefore the fundamental group \( \pi(Q_\Lambda, x) = \{1\} \), and from [3,2.2] we know that there is a surjective group homomorphism \( \phi_\Lambda: \pi(Q_\Lambda, x) \rightarrow \pi(G_\Lambda, x) \). Therefore \( \pi(G_\Lambda, x) \) is trivial, and \( \Lambda \) is simply connected. See also [2,6.1].

Therefore, to every tree \( T \) one may associate a simply connected algebra \( \Lambda \), namely, the indecomposable Nakayama algebra constructed above. Remark that the Kupisch series of \( \Lambda \) depends on the choice of the basis point \( t_i \) and the walk \( V \), therefore given a tree \( T \), there is usually more than one choice of a corresponding Nakayama algebra \( \Lambda \). For our purposes, it is enough to look at one of these. Since \( \Lambda \) is simply connected, we know that the graph \( G_\Lambda \) is a tree [2, Theorem 4.2]. Because of the connection between simply connected algebras and graded trees, to show that the tree \( T \) has a representation-finite grading, it is enough to show that \( G_\Lambda \) is isomorphic to the tree \( T \). (Remark that we consider a tree to be completely determined by the vertices and the edges connecting them, such that for instance, are considered to be isomorphic.)

\[
T_1 = \quad T_2 = \quad
\]

The number of \( \tau \)-orbits is equal to the number of projective \( \Lambda \)-modules, so the number of vertices of \( G_\Lambda \) is equal to the number of vertices of \( T \). Now we define a map \( \theta: T_0 \rightarrow (G_\Lambda)_0 \) such that \( \theta(T_i) \) is the vertex representing the \( \tau \)-orbit of the projective \( \Lambda \)-module \( P_i \), with \( L(P_i) = C_i \), where \( C_i \) is as defined above. Then \( \theta \) is a bijection. Denote \( \theta(t_i) \) by \( S_i \). Since \( G_\Lambda \) and \( T \) are trees with the same number of vertices, they also have the same number of edges, and to prove that \( G_\Lambda \) is isomorphic to \( T \), it is enough to show that if there is an edge connecting the vertices \( t_i \) and \( t_j \) in \( T \), there is an edge connecting the vertices \( S_i \) and \( S_j \) in \( G_\Lambda \).
Let us recall some useful facts about Nakayama algebras. If $\Lambda$ is a Nakayama algebra, then every indecomposable $\Lambda$-module is of the form $P_i/r^kP_i$, where $k \geq 0$ and $P_i$ is an indecomposable projective $\Lambda$-module. If $P_i/r^kP_i$ is an indecomposable nonprojective $\Lambda$-module, then it is shown in [1] that the almost split sequence with $P_i/r^kP_i$ as right-hand term has the form

$$0 \to P_{i-1}/r^kP_{i-1} \to P_{i-1}/r^{k-1}P_{i-1} \to P_i/r^{k+1}P_i \to P_i/r^kP_i \to 0.$$  

It follows from this that $\tau$-orbits preserve the length of modules, and all simples belong to the same $\tau$-orbit. We also recall that given the Kupisch series for a Nakayama algebra $\Lambda$, we always have an epimorphism $P_i \to rP_{i+1}$. If $L(P_{i+1}) = L(P_i) + 1$, this epimorphism is also an isomorphism.

Now, suppose that $t_i$ or $t_j$ is $t_1$, say $t_i = t_1$. $L(P_i) = 1$, so $P_i$ is the unique simple projective. Since $t_j$ is a neighbour of $t_i$, we see from the construction above, that $L(P_j) = 2$. But that means $rP_j$ is simple, and then either $rP_j \cong P_1$, or $rP_j$ is in the $\tau$-orbit determined by $P_1$, so $S_j$ is a neighbour of $S_1$ in $G_\Lambda$. Suppose that neither $t_i$ nor $t_j$ are $t_1$, but that there is an edge $t_i - t_j$. Let $i < j$. Then $L(P_i) = L(P_j) + 1$ by the construction above. Therefore $L(rP_j) = L(P_j)$. Since $\Lambda$ is Nakayama, $rP_j$ belongs to the $\tau$-orbit of a projective module with the same length as $P_i$. We remember that the ordering of the projectives was defined by help of the walk $V$ in $T$, and since $T$ is a tree, and every edge in $T$ appears in $V$ exactly twice, we have $L(P_k) > L(P_i)$ for every edge $k$ such that $i < k < j$. If $P_m$ is an indecomposable $\Lambda$-module, the length of the $\tau$-orbit determined by $P_m$, $l(P_m^\tau)$, is the number of nonisomorphic objects in the $\tau$-orbit. For a Nakayama algebra $\Lambda$, the following formula is easily obtained, using the form of almost split sequences indicated above: $l(P_m^\tau) = h - m + 1$, where $h$ is maximal with the property that $L(P_p) - L(P_m) > 0$ for all $p$ such that $m < p < h$. Further if $L(P_m) = q$, then the modules in this $\tau$-orbit are the modules of the form $P_p/r^qP_p$, where $m \leq p < h$. In our case, if we let $L(P_i) = q$, it follows that $P_{j-1}/r^qP_{j-1}$ is in the $\tau$-orbit of $P_j$. But since we have an epimorphism $P_{j-1} \to rP_j$, and $L(rP_j) = L(P_j) = q$, we have $rP_j \cong P_{j-1}/r^qP_{j-1}$. Therefore $rP_j$ is in the $\tau$-orbit of $P_j$, and we have an edge $S_i - S_j$ in $G_\Lambda$.

We have now proved the main result of this paper:

**Theorem.** If $T$ is a finite tree, then there is a grading $g$ such that $(T, g)$ is representation-finite, and such that the corresponding simply connected algebra $\Lambda$ is a Nakayama algebra.

**Example.** Let $T$ be the tree:

```
  8
 / \
7   6
/   |
5   4
/   |
3   2
/   |
1
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Let $V$ be the walk: $t_1 \rightarrow t_2 \rightarrow t_3 \rightarrow t_4 \rightarrow t_5 \rightarrow t_6 \rightarrow t_7 \rightarrow t_8 \rightarrow t_4 \rightarrow t_1$, which is a walk around the tree, passing through every edge in the tree exactly twice. To the tree $T$ and the walk $V$ we may associate the Kupisch series $(P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8)$ corresponding to the admissible sequence $(1, 2, 2, 2, 3, 4, 3, 3)$.

The $AR$-quiver $\Gamma_A$ of the Nakayama algebra $\Lambda$ is the following:

![Diagram of $\Gamma_A$]

We see that $P_2, P_5, P_6, P_7$ and $P_8$ all are projective injectives. All arrows pointing upward correspond to irreducible monomorphisms, all arrows pointing downward correspond to irreducible epimorphisms. If there is an irreducible monomorphism $X \rightarrow Y$, $LY = LX + 1$, and if there is an irreducible epimorphism $X \rightarrow Y$, $LX = LY + 1$. If $X \in \text{ind} \, \Lambda$, $\text{Soc} \, X$ is the unique simple module $S$ such that there is a chain of irreducible monomorphisms $S \rightarrow \cdots \rightarrow X$, $X/r \, X$ is the simple module $T$ such that there is a chain of irreducible epimorphisms $X \rightarrow \cdots \rightarrow T$. $L(X)$, the length of $X$, is equal to the shortest walk in $\Gamma_A$ from $\text{Soc} \, X$ to $X$.

If we start with a tree $T$, choose a point $t$, and a walk $V$ around the tree, and construct the corresponding Nakayama algebra $\Lambda$ in the way described above, it is possible to find the number of nonisomorphic indecomposable projective injective $\Lambda$-modules just by looking at the tree $T$.

**Proposition.** The number of projective injective $\Lambda$-modules is equal to the number of vertices in $T$, different from $t_1$, which have only one neighbour.

**Proof.** $P_i$ is projective injective if and only if $L(P_{i+1}) < L(P_i) + 1$. If $t_i \neq t_1$ is a point in $T$ having only one neighbour $t_j$, then every walk in $T$ from $t_1$ to $t_i$ must pass through $t_j$, therefore $j < i$, and $t_{i+1}$ is not a neighbour of $t_i$. But then $t_{i+1}$ is a neighbour of a point $t_k$ which does not lie farther away from $t_i$ than $t_j$, and $L(P_{i+1}) \leq L(P_j) + 1 = L(P_i) < L(P_i) + 1$, which means that $P_i$ is a projective injective module. On the other hand, if $P_i$ is a projective injective module, then $L(P_{i+1}) < L(P_i) + 1$, and $t_{i+1}$ is not a neighbour of $t_i$. But then $t_i$ can have only one neighbour (recall that the walk $V$ that defines the ordering passes through every edge exactly twice). The relation between $V$ and $\Gamma_A$ can be described in the following manner.

**Proposition.** Let $\theta$ be a chain of irreducible maps in $k(\Gamma_A)$ given by

$$\theta: P_1 = M(0, t_1) \rightarrow P_2 \rightarrow \cdots \rightarrow P_i \rightarrow \cdots \rightarrow rP_{i+1} \rightarrow \cdots \rightarrow P_n \rightarrow \cdots \rightarrow M(2(n-1), t_1)$$
which passes through all the projectives in the order given by the Kupisch series, and
which satisfies the condition that if $P_n$ is the last projective in the ordering, then
$P_n \rightarrow \cdots \rightarrow M(2(n - 1), t_1)$ is the unique path from the projective injective module $P_n$
to the simple injective $\Lambda$-module $M(2(n - 1), t_1)$. Then $V$ is the walk in $T$ constructed
by taking for each module in $\theta$ the corresponding point in $T$, and passing through the
points in the order defined by $\theta$.

Proof. This can be proven in the same way as the main theorem above.

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