

**CORRECTION TO "SOME APPLICATIONS OF  
 NEVANLINNA THEORY TO MATHEMATICAL LOGIC:  
 IDENTITIES OF EXPONENTIAL FUNCTIONS"**

BY

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In Corollary 5.3 of [2], which gives a completeness theorem for  $\sin$  and  $\cos$  over the complex numbers, we inadvertently left out the axiom  $\sin(-1 \cdot x) = -1 \cdot \sin(x)$ . This is needed in the proof which is sketched out, at the place where one tries to express  $\sin$  and  $\cos$  in terms of EXP, as is needed to prove the identity  $t = (t^\#)^*$  for each term  $t$ . The correct result should read:

**COROLLARY 5.3.** *If  $s$  and  $t$  are any two terms in  $\Sigma^*$  and  $t \equiv s$ , then the identity  $t = s$  is provable from the axioms*

$$\begin{array}{ll} x + (y + z) = (x + y) + z, & x(yz) = (xy)z, \\ x + y = y + x, & xy = yx, \\ x + 0 = x, & 1 \cdot x = x, \\ x(y + z) = xy + xz, & 0 \cdot x = 0, \\ \sin(x + y) = \sin(x) \cos(y) + \cos(x) \sin(y), & \\ \sin(-1 \cdot x) = -1 \cdot \sin(x) & \end{array}$$

*together with all axioms giving the facts of addition, multiplication,  $\sin$  and  $\cos$  for constants from  $C$ .*

The need for this added identity was discovered and clarified in discussions with L. van den Dries, who has proved the corresponding completeness theorem for  $\sin$  and  $\cos$  over the real numbers [1].

REFERENCES

1. Lou van den Dries, *A completeness theorem for trigonometric identities and various results on exponential functions*, Proc. Amer. Math. Soc. **96** (1986), 345-352.
2. C. Ward Henson and Lee A. Rubel, *Some applications of Nevanlinna theory to mathematical logic: Identities of exponential functions*, Trans. Amer. Math. Soc. **282** (1984), 1-32.

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