\( \Lambda(q) \) PROCESSES

RON C. BLEI

Abstract. Motivated by some classical notions in harmonic analysis, \( \Lambda(q) \) processes are introduced in the context of a study of stochastic interdependencies. An extension of a classical theorem of Salem and Zygmund regarding random Fourier series is obtained. The Littlewood exponent of \( \Lambda(q) \) processes is estimated and, in some archetypical cases, computed.

0. Introduction

In [1], we considered stochastic processes with respect to which every deterministic function on \([0, 1]\) was stochastically integrable. Such processes \( X \) were normed by

\[
\|X\| = \sup \left\{ E \left| \sum_{j=1}^{N} \epsilon_j (X(t_j) - X(t_{j-1})) \right| : N > 0, \ \epsilon_j = \pm 1, \ j = 1, \ldots, N, \right\}
\]

and were said to have finite expectation (cf. [1, §1]). One of the basic questions arising in this context is how to determine, in some precise sense, a degree of interdependencies between increments of a process with finite expectation. This problem was the motivation behind \( \alpha \)-chaos [3] as well as the subsequent computation of its Littlewood exponent [4]; the present paper is a continuation of that work. A description of the intuition underlying results of [3, 4], as well as the present paper, can be found in [5].

We first set the stage. Throughout, \((\Omega, \mathcal{A}, P)\) will denote a probability space. Let \( E = \{X_j\}_{j \in \mathbb{N}} \) be an orthonormal system of random variables in \( L^2(\Omega, P) \), and define

\[
\phi_E(x) = \sup \left\{ P \left( \left| \sum_j \sigma_j X_j \right| \geq x \middle| \sum_j \sigma_j^2 = 1 \right) : x > 0. \right\}
\]
In [3], we said that \( E \) was a sub- \( \alpha \)-system if
\[
\delta_E(\alpha) \equiv \lim_{x \to \infty} (\ln(1/\phi_E(x))/x^{2/\alpha}) > 0,
\]
and an \( \alpha \)-system if
\[
\theta_E \equiv \inf\{\gamma : \delta_E(\gamma) > 0\} = \alpha.
\]
For infinite \( E \), \( \theta_E \in [1, \infty] \). An archetypical example of an \( \alpha \)-system is produced by taking a system of independent symmetric uniformly bounded random variables \( \{X_j\}_{j \in \mathbb{N}} \), fixing an \( \alpha \)-dimensional lattice set \( F \subset \mathbb{N}^d \), and defining
\[
E = \{X_{j_1} \cdots X_{j_d}\}_{(j_1, \ldots, j_d) \in F}
\]
for which \( \theta_E = \alpha \) (e.g., [2]). In the present paper, we bring the case \( \theta_E = \infty \) into a sharper focus.

Definition 1.1. An orthonormal system \( \{X_j\}_{j \in \mathbb{N}} = E \) is a sub-\( \Lambda(q) \) system if
\[
\lambda_E(q) = \lim_{x \to \infty} (\phi_E(x)/x^q) < \infty,
\]
and a \( \Lambda(q) \) system if
\[
\sup\{p : \lambda_E(p) < \infty\} = q.
\]

An example of an infinite \( \Lambda(q) \) system is produced by taking infinitely many independent copies of a symmetric random variable with finite \( L^q \)-norm but infinite \( L^{q+\epsilon} \)-norm for all \( \epsilon > 0 \).

The notions above are naturally transported to a framework of stochastic processes. Throughout, we shall restrict attention to processes \( X \) with orthogonal increments whose variance is given by \( E|X(t) - X(s)|^2 = t - s, \ 0 \leq s < t \leq 1 \) (note: \( \|X\| \leq 1 \)). In [3], we defined \( X \) to be an \( \alpha \)-chaos when
\[
\sup\{\theta_E : E \text{ is a system of normalized increments of } X\} = \alpha.
\]
The Wiener process is an archetypical example of a 1-chaos, while Wiener’s homogeneous chaos of order \( k \), \( k \) a positive integer, is an example of a \( k \)-chaos (cf. [12], [9], [3, Remark 4.2(1)]). For noninteger \( \alpha \), examples of \( \alpha \)-chaos are produced canonically via the existence of \( \alpha \)-dimensional lattice sets [3, Theorem 4.1].

Definition 1.2. A process \( X \) with orthogonal increments and \( E|X(t) - X(s)|^2 = t - s, \ 0 \leq s < t \leq 1 \) is a sub-\( \Lambda(q) \) process if
\[
\beta_X(q) \equiv \sup\{\lambda_E(q) : E \text{ is a system of normalized increments of } X\}
\]
is finite, and a \( \Lambda(q) \) process if
\[
\sup\{p : \beta_X(p) < \infty\} = q.
\]

Clearly, an \( \alpha \)-chaos is a sub-\( \Lambda(q) \) process for all \( q < \infty \). Indeed, the \( \alpha \)-scale of [3] can be viewed as a resolution of the “right end-point (\( q = \infty \))” of the \( q \)-scale in the framework of the present paper.

Since every sub-\( \Lambda(q) \) process is a stochastic integrator in the sense of [1, §2], we easily obtain (and state without proof)
Lemma 1.3. Suppose $X$ is a process with orthogonal increments and
\[ E|X(t) - X(s)|^2 = t - s, \quad 0 \leq s < t \leq 1 \]
(in particular, $E \int_{[0,1]} f dX|^2 \leq \|f\|^2_2$ for all $f \in L^2([0, 1], dt)$). Let
\[ \tilde{X}(n) = \int_{[0,1]} e_n dX, \quad n \in \mathbb{N}, \]
be the transform of $X$ relative to $\{e_n\}_{n \in \mathbb{N}}$, a given orthonormal basis of $L^2([0, 1], dt)$ (cf. [1, §2]). Then, $X$ is a sub-$\Lambda(q)$ process if and only if $\{\tilde{X}(n)\}_{n \in \mathbb{N}}$ is a sub-$\Lambda(q)$ system.

Corollary 1.4. $\Lambda(q)$ processes exist for every $2 \leq q < \infty$.

Proof. Let $E = \{X_n\}_{n \in \mathbb{N}}$ be a $\Lambda(q)$ system. Let $U$ be the unitary map from $L^2([0, 1], dt)$ onto the $L^2(\Omega, \mathcal{F}, \mathbb{P})$-closure of the linear span of $E$ determined by
\[ Ue_n = X_n, \quad n \in \mathbb{N}, \]
where $\{e_n\}_{n \in \mathbb{N}}$ is an orthonormal basis of $L^2([0, 1], dt)$. Define
\[ X(t) = Ul_{[0,1]}, \quad 0 \leq t \leq 1. \]
It is easy to see that $X$ satisfies the hypotheses of Lemma 1.3 and that its transform relative to $\{e_n\}_{n \in \mathbb{N}}$ is given by
\[ \tilde{X}(n) = X_n, \quad n \in \mathbb{N}, \]
and so, $X$ is a $\Lambda(q)$ process. □

The main result of the next section (Theorem 2.3) is a sufficient condition for a.s. continuity of a random function represented as a Fourier series randomized by a sub-$\Lambda(q)$ system. This theorem, an analogue of [3, Theorem 2.5], is an extension of a classical theorem due to Salem and Zygmund [11]. As an easy consequence, we obtain that the sample paths of sub-$\Lambda(q)$ processes are almost surely continuous and, when $q > 2$, of unbounded variation (Corollary 2.4).

In §3 we estimate Littlewood exponents of sub-$\Lambda(q)$ and $\Lambda(q)$ processes. We recall some definitions [4]. Let $X$ be any stochastic process on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Define its “$p$th variation” by
\[ \|X\|_{(p)} = \sup \left\{ \left( \sum_{j,k} \left| \mathbf{E}1_{A_j} (X(t_k) - X(t_{k-1})) \right|^p \right)^{1/p} : \{A_j\}_{j \in \mathbb{N}} \subset \mathcal{F}, \right\}, \]
and its Littlewood exponent by
\[ l_X = \inf \{p : \|X\|_{(p)} < \infty \}. \]
A straightforward application of Littlewood's classical inequality [8] implies that if \( X \) has finite expectation then \( l_X \leq 4/3 \). An adaptation of Littlewood's example (finite Fourier transform), showing that 4/3 is best possible in the inequality of [8], establishes that there are processes \( X \) for which \( \|X\| < \infty \), and \( l_X = 4/3 \). At the other end of the scale, \( l_X = 1 \) when \( X \) is an \( \alpha \)-chaos [3, Theorem 2].\(^1\) The main result of §3 in this paper (Theorem 3.1), filling a gap between 1 and 4/3, is that

(i) \( l_X \leq (q + 2)/(q + 1) \) whenever \( X \) is a sub-\( \Lambda(q) \) process, and

(ii) for every \( q \geq 2 \), there are \( \Lambda(q) \) processes for which \( l_X = (q + 2)/(q + 1) \).

The proof of part (ii) of Theorem 3.1 is based on Bourgain's recent solution of the \( \Lambda(q) \)-set problem [6]. The question whether for every \( \Lambda(q) \) process \( X \), \( l_X = (q + 2)/(q + 1) \) is an open problem.

2. Sub-\( \Lambda(q) \) systems and random Fourier series

**Lemma 2.1.** Let \( (\mathcal{F}, \mu) \) be a probability space, and let \( \mathcal{D} \) be a linear subspace of \( L^\infty(\mathcal{F}, \mu) \) so that

\[
\rho(\mathcal{D}) = \rho = \inf \{ \mu(|f| \geq \|f\|_\infty/2) : f \in \mathcal{D} \} > 0.
\]

Suppose \{\( X_j \)\}\(_{j \in \mathbb{N}} \) = \( E \subset L^2(\Omega, \mathbb{P}) \) is a sub-\( \Lambda(q) \) system. Let \{\( f_j \)\} be a finite collection of functions in \( \mathcal{D} \) so that

\[
(2.1) \quad \left\| \sum_j |f_j|^2 \right\|_\infty \leq 1,
\]

and define the random function

\[
g = \sum_j f_j \otimes X_j.
\]

Then,

\[
P(\|g\|_\infty > x) < \left(2^q \lambda_q(x)/\rho \right)x^q,
\]

\( (\|g\|_\infty \equiv \text{ess sup}_{s \in \mathcal{F}} |\sum_j f_j(s)X_j|) \)

**Proof.** By (2.1), we have for all \( s \in \mathcal{F} \) and all \( x > 0 \),

\[
P(|g(s)| > x) = E 1_{\{ |g(s)| > x \}} \leq \lambda_q(x)/x^q.
\]

Integrating the inequality above and applying Fubini's Theorem, we obtain

\[
E \int_\mathcal{F} 1_{\{ |g(s)| > x \}} d\mu(s) \leq \lambda_q(x)/x^q,
\]

\[
E \int_{\{ s : |g(s)| \geq \|g\|_\infty/2 \}} 1_{\{ |g(s)| > x \}} d\mu(s) \leq \rho E 1_{\{ |g| > 2x \}} \leq \rho \lambda_q(x)/x^q
\]

which implies the conclusion of the lemma. \( \Box \)

\(^1\)When \( X \) is a simple process, \( l_X = 0 \). When \( l_X < 1 \), stochastic integration with respect to \( X \) reduces to usual integration over \( \Omega \times [0, 1] \). From our point of view, the interesting range of \( l_X \) is \([1, 4/3]\).
The following is an immediate consequence.

**Lemma 2.2.** Suppose \{X_j\}_{j \in \mathbb{N}} = E is a sub-\(\Lambda(q)\) system. Fix an arbitrary positive integer \(N\), let \(T_N\) denote the space of trigonometric polynomials on \([0, 1]\) of degree \(N\), and let \{\{f_j\} \subset T_N\) be finite. Then

\[
P \left( \left\| \sum_j f_j \otimes X_j \right\|_\infty \geq D x \left( \left\| \sum_j |f_j|^2 \right\|_\infty \right)^{1/2} \right) \leq 2\pi N/x^q
\]

for all \(x > 0\), where \(D > 0\) depends only on \(\lambda_E(q)\).

**Proof.** By Bernstein's theorem (e.g. [7, Exercise 1.2.12]), \(\rho(T_N) \geq 1/2\pi N\).

Now apply Lemma 2.1. \(\square\)

The following is an extension of [3, Theorem 2.5].

**Theorem 2.3.** Let \{X_j\}_{j \in \mathbb{N}} be a sub-\(\Lambda(q)\) system. Define blocks of integers

\[
B_k = \{\pm[k^{q/2}], \pm[(k^{q/2}) + 1], \ldots, \pm(\lfloor (k + 1)^{q/2} \rfloor - 1)\}, \quad k = 1, 2, \ldots
\]

(\lfloor \cdot \rfloor denotes the "closest integer" function). Let \{a_n\}_{n=-\infty}^{\infty} be a sequence of scalars so that

\[
\sum_{k=1}^{\infty} (\ln k) s_k < \infty.
\]

Then, \(\sum_{n=-\infty}^{\infty} a_n X_n e^{2\pi i n t}\) is almost surely a Fourier series of a continuous function on \([0, 1]\).

**Proof.** Define blocks of integers

\[
C_k = \{\pm 2^k, \pm(2^k + 1), \ldots, \pm(2^{k+1} - 1)\},
\]

define the corresponding random trigonometric polynomials

\[
p_k(t) = \sum_{n \in C_k} a_n X_n e^{2\pi i n t},
\]

and consider the events

\[
E_k = \left\{ \left\| p_k \right\|_\infty \geq D (2^{k/q}) k \left( \sum_{n \in C_k} |a_n|^2 \right)^{1/2} \right\}, \quad k = 1, 2, \ldots
\]

\((D > 0\) is the constant appearing in Lemma 2.2). By Lemma 2.2,

\[
P(E_k) \leq 2\pi / k^q.
\]
Therefore, by the Borel-Cantelli lemma, we obtain \( P(\lim E_k) = 0 \) which implies that

\[
(2.4) \quad (\|p_k\|_\infty)_{k=1}^{\infty} \text{ is } \mathcal{O}\left( (2^{k/q})k \left( \sum_{n \in C_k} |a_n|^2 \right)^{1/2} \right) \text{ almost surely.}
\]

Observe that \( |B_k| \approx k^{(q/2)-1} \) and that, following a partition of each \( C_k \) into \( B_n \)'s, we have

\[
\left( \sum_{n \in C_k} |a_n|^2 \right)^{1/2} \leq \left( \sum_{n \in [4^{k+1}]} |s_n|^2 \right)^{1/2}.
\]

Therefore, since \( (s_n^\infty)_{n=1} \) is a decreasing sequence, we have

\[
(2.5) \quad \left( \sum_{n \in C_k} |a_n|^2 \right)^{1/2} \leq K [2^{k/q}] s_{[4^{k+1}]}.
\]

And so, following (2.4) and (2.5), to obtain that \( \sum_{k=1}^{\infty} \|p_k\|_\infty \) is almost surely convergent and thus the theorem, we need to verify

\[
(2.6) \quad \sum_{k=1}^{\infty} [4^{k/q}] s_{[4^{k+1}]} k < \infty.
\]

Finally, observe that (2.6) is implied, via a change of index, by the assumption (2.3). \( \square \)

**Corollary 2.4.** The sample paths of every sub-\( \Lambda(q) \) process are almost surely continuous and of unbounded variation.

**Proof.** The stochastic series of \( X \) relative to the usual trigonometric system is given by

\[
(2.7) \quad X(t) - X(0) = \hat{X}(0)t + \sum_{n \neq 0} \hat{X}(n) \frac{e^{2\pi i nt}}{2\pi in} (e^{2\pi i nt} - 1).
\]

By Lemma 1.1, \( \{\hat{X}(n)\}_{n=-\infty}^{\infty} \) is a sub-\( \Lambda(q) \)-system. Therefore, since \( a_n = 1/n \) satisfies the hypothesis (2.3) in Theorem 2.3, we obtain that the stochastic series (2.7) represents almost surely a continuous function on \([0, 1]\).

Following a computation similar to the one in [3, Remark 3.8], we deduce that every sub-\( \Lambda(q) \) process is chaotic and, by [3, Proposition 3.9], that the sample paths of \( X \) are almost surely of unbounded variation. \( \square \)

### 3. The Littlewood Exponent of sub-\( \Lambda(q) \) Processes

**Theorem 3.1.** (i) Let \( X \) be a sub-\( \Lambda(q) \) process. Then

\[
1 \leq (q + 2)/(q + 1).
\]
(ii) For every \( q \geq 2 \), there exist \( \Lambda(q) \) processes \( X \) so that 
\[
l_X = (q + 2)/(q + 1).
\]

In what follows below, we fix a sub-\( \Lambda(q) \) process \( X \), a measurable partition \( \{A_j\}_{j \in \mathbb{N}} \) of \( \Omega \), and a subdivision \( 0 = t_0 < t_1 < \cdots < t_k < \cdots \leq 1 \). Denote 
\[
a_{jk} = \mathbf{E} 1_{A_j}(X(t_k) - X(t_{k-1})), \quad j, k \in \mathbb{N}.
\]
Throughout, \( K \) will denote a numerical constant.

**Lemma 3.2.** For all \( p > q/(q - 1) \),
\[
\sum_j \left( \sum_k |a_{jk}| \right)^p \leq K
\]
where \( K > 0 \) depends only on \( p \) and \( \beta_X(q) \).

**Proof.** We verify (3.1) by duality. Fix \( p > q/(q - 1) \) and \( (b_{jk}) \subset \mathbb{C} \) so that
\[
\sum_j \left( \sup_k |b_{jk}| \right)^{p'} = 1, \quad \frac{1}{p} + \frac{1}{p'} = 1.
\]
Following a rearrangement of the \( j \)'s, we can assume that
\[
\sup_k |b_{jk}|^{p'} \leq \frac{1}{j}, \quad j = 1, 2, \ldots.
\]
Write \( Y_k = (X(t_k) - X(t_{k-1}))/\sqrt{t_k - t_{k-1}} \), \( d_{jk} = b_{jk}/\sqrt{t_k - t_{k-1}} \), and obtain from (3.2),
\[
\left( \sum_k |d_{jk}|^2 \right)^{1/2} \leq 1/j^{1/p'}, \quad j = 1, 2, \ldots.
\]
Estimate
\[
\left| \sum_{j,k} a_{jk} b_{jk} \right| = \left| \sum_j \mathbf{E} 1_{A_j} \sum_k d_{jk} Y_k \right| \leq \mathbf{E} \sum_j 1_{A_j} \left| \sum_k d_{jk} Y_k \right|
\]
\[
\leq \sup_j \left| \sum_k d_{jk} Y_k \right| = \int_0^\infty \mathbf{P} \left( \bigcup_j \left\{ \sum_k d_{jk} Y_k > t \right\} \right) dt
\]
\[
\leq 1 + \int_1^\infty \sum_j \mathbf{P} \left( \left| \sum_k d_{jk} Y_k \right| > t \right) dt
\]
\[
\leq 1 + \left( \int_1^\infty t^{-q} dt \right) \beta_X(q) \sum_j 1/j^{q/p'} \equiv K < \infty
\]
(the last line above follows from (3.3) and the assumption \( \beta_X(q) < \infty \). \( \square \)
Lemma 3.3.

\[
\sum_j \left( \sum_k |a_{jk}|^2 \right)^{1/2} \leq K.
\]

Proof. (3.4) is a consequence of Littlewood's inequality for bounded bilinear forms [8]; this argument yields \( K \leq \sqrt{2} \) (= Khintchin's constant). We shall give a direct proof in our specific context, bypassing Littlewood's inequality and obtaining \( K = 1 \). We establish (3.4) by duality: suppose \((b_{jk}) \subseteq C\) satisfies

\[
\sup_j \left( \sum_k |b_{jk}|^2 \right)^{1/2} = 1,
\]

and estimate

\[
\left| \sum_{j,k} a_{jk} b_{jk} \right| = \left| \sum_{j,k} \mathbf{1}_{A_j} (X(t_k) - X(t_{k-1})) b_{jk} \right| = \left| \sum_k \mathbb{E} \left( \sum_j b_{jk} \mathbf{1}_{A_j} \right) (X(t_k) - X(t_{k-1})) \right|
\]

(without loss of generality, we assume that the sums above are performed over finitely many \(j\)'s and \(k\)'s)

\[
\leq \sum_k \left( \mathbb{E} \left| \sum_j b_{jk} \mathbf{1}_{A_j} \right| \right)^{1/2} \left( t_k - t_{k-1} \right)^{1/2} \quad \text{(by Schwarz's inequality)}
\]

\[
= \sum_k \left( \sum_j |b_{jk}|^2 \mathbb{P}(A_j) \right)^{1/2} \left( t_k - t_{k-1} \right)^{1/2}
\]

\[
\leq \left( \sum_k \sum_j |b_{jk}|^2 \mathbb{P}(A_j) \right)^{1/2} \left( \sum_k \left( t_k - t_{k-1} \right) \right)^{1/2}
\]

\[
= \left\| \sum_j \left( \sum_k |b_{jk}|^2 \right)^{1/2} \mathbf{1}_{A_j} \right\|_{L^2(\Omega, \mathbb{P})} \leq \left\| \sum_j \left( \sum_k |b_{jk}|^2 \right)^{1/2} \mathbf{1}_{A_j} \right\|_{L^\infty(\Omega, \mathbb{P})} = \sup_j \left( \sum_k |b_{jk}|^2 \right)^{1/2} = 1 \quad \text{(by (3.5))}. \quad \Box
\]
Proof of Theorem 3.1. (i) We need to show that \( \|X\|_p < \infty \) for all \( p = (b + 2)/(b + 1) > (q + 2)/(q + 1) \). To this end, we will verify
\[
\sum_{j, k} |a_{jk}|^{(b+2)/(b+1)} \leq \left( \sum_j \left( \sum_k |a_{jk}|^2 \right)^{1/2} \right)^{2/(b+1)} \cdot \left( \sum_j \left( \sum_k |a_{jk}|^{b/(b-1)} \right)^{(b-1)/(b+1)} \right),
\]
and then apply Lemmas 3.2 and 3.3. To establish (3.6), first write
\[
\sum_{j, k} |a_{jk}|^{(b+2)/(b+1)} = \sum_j \sum_k |a_{jk}|^{2/(b+1)} |a_{jk}|^{b/(b+1)},
\]
and apply Hölder’s inequality to \( \sum_k \) with exponents \( b + 1 \) and \((b + 1)/b\) to obtain
\[
\sum_{j, k} |a_{jk}|^{(b+2)/(b+1)} \leq \sum_j \left( \sum_k |a_{jk}|^2 \right)^{1/(b+1)} \left( \sum_k |a_{jk}|^{b/(b+1)} \right)^{b/(b+1)}.
\]
Now apply Hölder’s inequality to \( \sum_j \) above with exponents \((b + 1)/2\) and \((b + 1)/(b - 1)\), and deduce (3.6).

(ii) We consider the discrete abelian group \( \Gamma = \bigoplus Z_{k_j} \), where \( (k_j) \) is a sequence of integers increasing to infinity, and view its dual group \( \hat{\Gamma} = \otimes Z_{k_j} = \Omega \) as a probability space with \( P = \text{Haar measure} \). Fix \( q > 2 \), and a set of characters \( E \subset \Gamma \) which is a \( \Lambda(q) \) system: such systems \( E \) were produced in [6] (in the terminology of [6], \( E \) is a \( \Lambda(q) \) set but not \( \Lambda(q + \varepsilon) \) for any \( \varepsilon > 0 \)). Such \( E \subset \bigoplus Z_{k_j} \), by the productions in [6], can be assumed to satisfy
\[
E = \bigcup_{j=1}^{\infty} E_j, \quad E_j \subset Z_{k_j}, \quad \text{(the coordinates of } E_j \text{ are nonzero only at the } k_j\text{th entry; this means that the } E_j\text{'s are mutually independent systems of random variables on } \Omega),
\]
\[
\sup_j \lambda_{E_j}(q) < \infty, \quad \sup_j \lambda_{E_j}(q + \varepsilon) = \infty, \quad \text{for all } \varepsilon > 0,
\]
and
\[
|E_j| \approx [k_j^{2/q}].
\]
Let \( U \) be a unitary map from \( L^2([0, 1], dt) \) into \( L^2_P(\Omega, P) \), and define
\[
X(t) = U1_{[0, t]}, \quad 0 \leq t \leq 1.
\]
By (3.7) and Lemma 2.3, \( X \) is a \( \Lambda(q) \) process and therefore, by part (iii), \( l_X \leq (q + 2)/(q + 1) \). By (3.8), following an estimation similar to the one in [4, Theorem 2, part (ii)], we obtain \( \|X\|_p = \infty \) for all \( p < (q + 2)/(q + 1) \), and therefore \( l_X = (q + 2)/(q + 1) \).
REFERENCES


DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CONNECTICUT, STORRS, CONNECTICUT 06268