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A CORRECTION TO "EQUIVARIANT COHOMOLOGY AND LOWER BOUNDS FOR CHROMATIC NUMBERS"

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While the main theorem, Theorem 2.4, of the paper remains true as stated, it turns out that Theorems 2.2 and 2.6 are proved only in the case when n is a prime number. The author apologizes for the mistake.

R. Zivaljevic of the Belgrade Mathematics Institute, following earlier work of Murad-Ozaydiu, found a counterexample to Theorem 2.6 in the case $t = 2, n = 6$: If γ is a 1-dimensional complex representation of $\mathbb{Z}/6$ associated with an embedding $\mathbb{Z}/6 \to S^1$, then a map

$$S(3\gamma) \to S(2,6) \tag{1}$$

(where $S(V)$ is the unit sphere of V) can be found by obstruction theory: Because the source is a free $\mathbb{Z}/6$-CW complex of dimension 5 and the target has no nonequivariant homotopy in the dimensions < 4 (it is S^4), the only obstruction to the existence of (1) is in

$$H^5_{\mathrm{Borel}}(S(3\gamma), \pi_4 S(2,6)) = H^5(\mathbb{Z}/6, \mathbb{Z}).$$

The obstruction can be calculated by trying to construct (1) on the level of chain complexes: The chain complex of $S(3\gamma)$ (made into a free $\mathbb{Z}/6$-CW complex in the standard way) is

$$\mathbb{Z}[\mathbb{Z}/6] \xrightarrow{T} \mathbb{Z}[\mathbb{Z}/6] \xrightarrow{N} \cdots \xrightarrow{N} \mathbb{Z}[\mathbb{Z}/6] \xrightarrow{T} \mathbb{Z}[\mathbb{Z}/6]$$

where $T = 1 - \alpha$, α is a generator of $\mathbb{Z}/6$. Let the free generator of the above complex in dimension k be ι_k. $S(2,6)$, on the other hand, is the boundary of the standard 5-simplex [012345] with $\mathbb{Z}/6$ acting by the cyclic permutation (012345). The desired chain map is given explicitly as follows:

$$\begin{aligned}
&\iota_0 \mapsto [0],\\
&\iota_1 \mapsto [0,1],\\
&\iota_2 \mapsto [012] + [023] + [034] + [045],\\
&\iota_3 \mapsto -[0123] - [0134] - [0145],\\
&\iota_4 \mapsto -N([01234]),\\
&\iota_5 \mapsto 0.
\end{aligned}$$

Thus, the obstruction is 0.

The mistake is in the proof of Theorem 3.5, namely in the assertion that $\gamma = 0$ in the first line on p. 573. In the reason given, the map γ does not factor as stated. Theorem 3.5 remains tautologically true in the case when n is a prime, and therefore

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the proofs of Theorems 2.6, 2.2 and 2.4 remain valid in this case. As it turns out, Theorem 2.4 can be proven as stated by the method of Alon, Frankl and Lovasz (Trans. Amer. Math. Soc. **298** (1986), 359–370. MR **88g:**05098).

Proof of Theorem 2.4. As remarked above, the original proof is correct in the case when n is a prime number. In the general case, the theorem will be proven by induction on n. Let $n = pq$ where $p, q < n$. Suppose

$$w(G,t) > (n-1)(t-1).$$

Let Γ be the set of all subsets E of N such that $w(G|E,q) > (q-1)(t-1)$.

Claim 1.

$$w(\Gamma,p) > (p-1)(t-1).$$

Proof. Suppose not. Then I can find p subsets $N_1,\ldots,N_p$ in N which are not in Γ and such that the complement of $N_1 \cup \cdots \cup N_p$ in N has

$$\leq (p-1)(t-1) \text{ elements.} \tag{2}$$

We have $N_i \notin \Gamma$, so

$$w(G|N_i,q) \leq (q-1)(t-1).$$

Then we get, for each i, q sets

$$M_{i1},\ldots,M_{iq} \subseteq N_i \tag{3}$$

no subset of which is in G, and such that the complement of the union of (3) in N_i has $\leq (q-1)(t-1)$ elements. Thus, the complement of the union of all (3) over all $i = 1,\ldots,p$ in N has

$$\begin{aligned} &\leq p(q-1)(t-1) + (p-1)(t-1) \\ &= (n-p)(t-1) + (p-1)(t-1) = (n-1)(t-1) \end{aligned}$$

elements, contradicting the assumption about $w(G,n)$. □

Now, by the induction hypothesis, for $E \in \Gamma$,

$$\chi(G|E,q) > t-1.$$

Thus, in coloring G by $t-1$ colours, we find q disjoint subsets

$$X_{E,1},\ldots,X_{E,q} \subset E$$

such that $X_{E,i} \in G$ are coloured by the same colour i_E. Now colour Γ by $t-1$ colours in such a way that E is coloured by i_E. Then by the induction hypothesis and the Claim, we find disjoint $E_1,\ldots,E_p \in \Gamma$ coloured by the same colour. Thus,

$$X_{E_1,1},\ldots,X_{E_1,q},\ldots,X_{E_p,1},\ldots,X_{E_p,q} \in G$$

are coloured by the same colour, as claimed. □

Igor Kriz, *Equivariant cohomology and lower bounds for chromatic numbers*, Trans. Amer. Math. Soc. **333** (1992), 567–577. MR **92m:**05085

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