Let $Q_0S^0$ be the basepoint component of $QS^0 = \lim_n \Omega^n S^n$. A spherical class in $Q_0S^0$ is an element belonging to the image of the Hurewicz homomorphism:

$$H : \pi_\ast(S^0) \cong \pi_\ast(Q_0S^0) \to H_\ast(Q_0S^0).$$

Here and throughout this note, homology is taken with coefficients in $\mathbb{F}_2$, the field of two elements. The long-standing conjecture on spherical classes reads as follows.

**Conjecture 1.1.** There are no spherical classes in $Q_0S^0$, except the elements of Hopf invariant 1 and those of Kervaire invariant 1.

An algebraic version of this problem goes as follows. Let $V_k$ be a $k$-dimensional vector space over $\mathbb{F}_2$. Then, the polynomial algebra in $k$ variables $P_k = H^* (BV_k)$ is a module over both the Steenrod algebra $\mathcal{A}$ and the general linear group $GL_k = GL(k, \mathbb{F}_2)$. J. E. Lannes and S. Zarati constructed homomorphisms

$$\varphi_k : \text{Ext}_\mathcal{A}^{k,k+i}(\mathbb{F}_2, \mathbb{F}_2) \to (\mathbb{F}_2 \otimes P_k^{GL_k})^\ast_i$$

(see [6]) and have shown that these maps correspond to an associated graded of the Hurewicz homomorphism. The proof of this assertion is unpublished, but it is sketched by J. E. Lannes [5] and by P. G. Goerss [1]. The Hopf invariant 1 and the Kervaire invariant 1 classes are respectively represented by certain permanent cycles in $\text{Ext}_\mathcal{A}^{1,1}(\mathbb{F}_2, \mathbb{F}_2)$ and $\text{Ext}_\mathcal{A}^{2,2}(\mathbb{F}_2, \mathbb{F}_2)$, on which $\varphi_1$ and $\varphi_2$ are non-zero. Therefore, we are led to the following conjecture.

**Conjecture 1.2.** $\varphi_k = 0$ in any positive stem $i$ for $k > 2$.

In the introduction of the article [2] we are mistaken in asserting that Lannes and Zarati’s work shows that Conjecture 1.2 implies Conjecture 1.1. This comes from the usual problem with spectral sequences: if an element maps to an element of higher filtration, then in the associated graded it will map to 0. Thus $\varphi_k$ could be 0 even if $H$ is not. Of course, if Conjecture 1.2 were false on a permanent cycle, then Conjecture 1.1 would also be false.

Apart from this, all of our results and proofs in the article [2] are correct. This correction also applies to our papers [4] (joint with F. P. Peterson) and [3]. The author is grateful to Nick Kuhn for pointing out the above misunderstanding.
References


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