

ERRATUM TO  
“SPHERICAL CLASSES AND THE ALGEBRAIC TRANSFER”

NGUYỄN H. V. HUNG

Let  $Q_0S^0$  be the basepoint component of  $QS^0 = \lim_n \Omega^n S^n$ . A spherical class in  $Q_0S^0$  is an element belonging to the image of the Hurewicz homomorphism:

$$H : \pi_*^s(S^0) \cong \pi_*(Q_0S^0) \rightarrow H_*(Q_0S^0).$$

Here and throughout this note, homology is taken with coefficients in  $\mathbb{F}_2$ , the field of two elements. The long-standing conjecture on spherical classes reads as follows.

**Conjecture 1.1.** There are no spherical classes in  $Q_0S^0$ , except the elements of Hopf invariant 1 and those of Kervaire invariant 1.

An algebraic version of this problem goes as follows. Let  $V_k$  be a  $k$ -dimensional vector space over  $\mathbb{F}_2$ . Then, the polynomial algebra in  $k$  variables  $P_k = H^*(BV_k)$  is a module over both the Steenrod algebra  $\mathcal{A}$  and the general linear group  $GL_k = GL(k, \mathbb{F}_2)$ . J. E. Lannes and S. Zarati constructed homomorphisms

$$\varphi_k : Ext_{\mathcal{A}}^{k, k+i}(\mathbb{F}_2, \mathbb{F}_2) \rightarrow (\mathbb{F}_2 \otimes_{\mathcal{A}} P_k^{GL_k})_i^*$$

(see [6]) and have shown that these maps correspond to an associated graded of the Hurewicz homomorphism. The proof of this assertion is unpublished, but it is sketched by J. E. Lannes [5] and by P. G. Goerss [1]. The Hopf invariant 1 and the Kervaire invariant 1 classes are respectively represented by certain permanent cycles in  $Ext_{\mathcal{A}}^{1,*}(\mathbb{F}_2, \mathbb{F}_2)$  and  $Ext_{\mathcal{A}}^{2,*}(\mathbb{F}_2, \mathbb{F}_2)$ , on which  $\varphi_1$  and  $\varphi_2$  are non-zero. Therefore, we are led to the following conjecture.

**Conjecture 1.2.**  $\varphi_k = 0$  in any positive stem  $i$  for  $k > 2$ .

In the introduction of the article [2] we are mistaken in asserting that Lannes and Zarati’s work shows that Conjecture 1.2 implies Conjecture 1.1. This comes from the usual problem with spectral sequences: if an element maps to an element of higher filtration, then in the associated graded it will map to 0. Thus  $\varphi_k$  could be 0 even if  $H$  is not. Of course, if Conjecture 1.2 were false on a permanent cycle, then Conjecture 1.1 would also be false.

Apart from this, all of our results and proofs in the article [2] are correct.

This correction also applies to our papers [4] (joint with F. P. Peterson) and [3].

The author is grateful to Nick Kuhn for pointing out the above misunderstanding.

---

Received by the editors February 4, 2003.

2000 *Mathematics Subject Classification.* Primary 55P47, 55Q45, 55S10, 55T15.

©2003 American Mathematical Society

## REFERENCES

1. P. G. Goerss, *Unstable projectives and stable Ext: with applications*, Proc. London Math. Soc. **53** (1986), 539–561. MR **88d**:55011
2. N. H. V. Hung, *Spherical classes and the algebraic transfer*, Trans. Amer. Math. Soc. **349** (1997), 3893–3910. MR **98e**:55020
3. N. H. V. Hung, *The weak conjecture on spherical classes*, Math. Zeit. **231** (1999), 727–743. MR **2000g**:55019
4. N. H. V. Hung and F. P. Peterson, *Spherical classes and the Dickson algebra*, Math. Proc. Camb. Phil. Soc. **124** (1998), 253–264. MR **99i**:55021
5. J. Lannes, *Sur le  $n$ -dual du  $n$ -ème spectre de Brown-Gitler*, Math. Zeit. **199** (1988), 29–42. MR **89h**:55020
6. J. Lannes and S. Zarati, *Sur les foncteurs dérivés de la déstabilisation*, Math. Zeit. **194** (1987), 25–59. MR **88j**:55014

DEPARTMENT OF MATHEMATICS, VIETNAM NATIONAL UNIVERSITY, HANOI, 334 NGUYỄN TRÃI STREET, HANOI, VIETNAM

*E-mail address*: nhvhung@vnu.edu.vn