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A CLASS OF PROCESSES ON THE PATH SPACE OVER A COMPACT RIEMANNIAN MANIFOLD WITH UNBOUNDED DIFFUSION

JÖRG-UWE LÖBUS

Abstract. A class of diffusion processes on the path space over a compact Riemannian manifold is constructed. The diffusion of such a process is governed by an unbounded operator. A representation of the associated generator is derived and the existence of a certain local second moment is shown.

1. Introduction and basic notation

Infinite dimensional diffusion processes have been studied from several points of view. For example, S. Kusuoka [10], introduced diffusion type Dirichlet forms on Banach spaces. The existence of associated processes is then obtained by using regularity arguments. On the other hand, M. Röckner and T.S. Zhang [16] and A. Eberle [6] used finite dimensional approximation methods to treat infinite dimensional diffusion processes. In these papers, the diffusion is governed by bounded operators.

In contrast, we show the existence of a class of processes with unbounded diffusion operators. For this, we use methods and results of modern Dirichlet form theory (N. Bouleau and F. Hirsch [3], B.K. Driver and M. Röckner [5], M. Fukushima, Y. Oshima, and M. Takeda [8], Z.M. Ma and M. Röckner [13]). The basic structure of a diffusion form we deal with is

$$\mathcal{E}(F,F) := \int \langle \mathbf{D}F, \mathbf{A}\mathbf{D}F\rangle_{\mathbb{H}}\, d\nu\,, \quad F \in D(\mathcal{E})\,,$$

where $\mathbb{H}$ is the Cameron-Martin space, $\mathbf{D}$ denotes the corresponding gradient operator, and ν is the Wiener measure on the space $\mathbf{P}_{m_0}(M)$ of all Brownian paths γ on the compact Riemannian manifold M with $\gamma(0) = m_0 \in M$. In our setting, the diffusion operator $\mathbf{A} : L^2(\mathbf{P}_{m_0}(M) \to \mathbb{H}, \nu) \supseteq D(\mathbf{A}) \to L^2(\mathbf{P}_{m_0}(M) \to \mathbb{H}, \nu)$ is unbounded. Let us, however, mention that there are authors speaking in quite different situations of unbounded diffusion coefficients, namely when omitting the operator $\mathbf{A}$ and replacing the measure $d\nu$ with $Cd\nu$ where C is a possibly unbounded densitiy function (see, e.g., S. Aida [1]).

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We verify closability (Section 2) as well as quasi-regularity which implies the existence of an associated right process $\mathbf{M}=(\Omega,\mathcal{F},(X_t)_{t\ge 0},(P_\gamma)_{\gamma\in\mathbf{P}_{m_0}(M)})$ (Section 3). Furthermore, we provide a representation of the associated generator A (Section 4). In particular, the fact that a certain subspace of the space of the cylindrical functions over $\mathbf{P}_{m_0}(M)$ is a subset of the domain of A is used to determine the following local second moment,

$$\lim_{t\to 0}\frac{1}{t}\int\sum_{v=1}^{N}\|x^v(\gamma(\cdot))-x^v(\tau(\cdot))\|^2_{L^2([0,1],ds)}\,P_\tau(X_t\in d\gamma)$$
$$=\sum_{v=1}^{N}\int_{s\in[0,1]}\Gamma(x^v(\tau(s)),x^v(\tau(s)))\,ds\,,$$

weakly in $L^1(\mathbf{P}_{m_0}(M),\nu)$ (Section 5). Here, M is considered as isometrically embedded in some $\mathbb{R}^N$ and $x^v(p)$, $v\in\{1,\dots,N\}$, denote the standard coordinates of $p\in M$ embedded in $\mathbb{R}^N$; finally, Γ is the carré du champ operator corresponding to $(\mathcal{E},D(\mathcal{E}))$.

Let M be a compact connected Riemannian manifold of dimension d without boundary, isometrically embedded in some $\mathbb{R}^N$. Let T_m denote the tangent space to M at $m(\in M)$ and let $\langle\,.,.\,\rangle_{T_m}$ denote the inner product on T_m. We fix a covariant derivative ∇ compatible with the underlying Riemannian metric and assume that ∇ is torsion skew symmetric, which means that, if T is the torsion tensor of ∇, then $\langle T\langle\xi,\eta\rangle,\eta\rangle\equiv 0$ for all vector fields ξ and η on M. This convention guarantees compatibility with the works of B.K. Driver [4], B.K. Driver and M. Röckner [5], and E.P. Hsu [9].

Let $O(M)$ denote the orthonormal frame bundle with respect to M. Furthermore, we denote the canonical projection $O(M)\to M$ by π and the canonical horizontal vector fields by $H_1,\dots,H_d$. Let X be the space of all Brownian trajectories on $[0,1]$ with values in $\mathbb{R}^d$. For fixed $m_0\in M$, we introduce the path space $\mathbf{P}_{m_0}(M)$ by

$$\mathbf{P}_{m_0}(M):=\{\gamma\in C([0,1]\to M):\,\gamma(0)=m_0\}$$

and equip it with the topology of uniform convergence. Let μ denote the Wiener measure on X and let $r_0\in O(M)$ such that $\pi(r_0)=m_0$. According to J. Eells and D. Elworthy [7] and P. Malliavin [12], the solution r_x to the Stratonovich SDE

$$\begin{cases}\partial r_x(t) &= \sum\limits_{i=1}^{d} H_i\left(r_x(t)\right)\partial x_i(t)\,,\quad t\in[0,1],\\ r_x(0) &= r_0,\end{cases}$$

$x=(x_1,\dots,x_d)\in X$, defines (μ-a.e.) a mapping $I:X\to\mathbf{P}_{m_0}(M)$ by $I(x)(t):=\pi\left(r_x(t)\right)$, $x\in X$, $t\in[0,1]$. Considering, simultaneously, x as a d-dimensional standard Brownian motion, $I(x)$ becomes a Brownian motion on M whose law on $\mathbf{P}_{m_0}(M)$ (the Wiener measure on $\mathbf{P}_{m_0}(M)$) is denoted by ν.

Finally, as discussed in [9], Section 4, there is an inverse map L of I in the sense that $L\circ I=$ identity μ-a.e. on X and $I\circ L=$ identity ν-a.e. on $\mathbf{P}_{m_0}(M)$. Note that, for $x\in X$ and $\gamma\in\mathbf{P}_{m_0}(M)$ with $\gamma=I(x)$ and $x=L(\gamma)$, the path r_x in $O(M)$ is well defined and that, for $a\in\mathbb{R}^d$ and $0\le s,t\le 1$,

$$\langle r_x(s)a,r_x(s)a\rangle_{T_{\gamma(s)}}=\langle r_x(t)a,r_x(t)a\rangle_{T_{\gamma(t)}}=|a|^2_{\mathbb{R}^d}\,.$$

The parallel transport from $T_{\gamma(s)}$ to $T_{\gamma(t)}$ along $\gamma \in \mathbf{P}_{m_0}(M)$ is ν-a.e. defined as follows. For $x \in X$ and $\gamma \in \mathbf{P}_{m_0}(M)$ with $\gamma = I(x)$ and $x = L(\gamma)$, set

$$\mathcal{T}^{\gamma}_{t\leftarrow s} := r_x(t) r_x^{-1}(s)\,, \quad 0 \le s, t \le 1.$$

Introduce the abbreviation $L^p(\nu)$ for $L^p(\mathbf{P}_{m_0}(M), \nu)$, $1 \le p \le \infty$, and define the set of all cylindrical functions on $\mathbf{P}_{m_0}(M)$,

$$Z := \big\{ F(\gamma) = f(\gamma(s_1); \ldots; \gamma(s_k)), \gamma \in \mathbf{P}_{m_0}(M) : \\ 0 < s_1 < \ldots < s_k = 1,\ f \in C^\infty(M^k),\ k \in \mathbb{N} \big\}\ .$$

Set

$$Y := \big\{ F(\gamma) = f(\gamma(s_1); \ldots; \gamma(s_k)), \gamma \in \mathbf{P}_{m_0}(M) : \\ F \in Z,\ s_1, \ldots, s_k \in \{\tfrac{l}{2^n} : l \in \{1, \ldots, 2^n\},\ n \in \mathbb{N}\} \big\}\ . \tag{1.1}$$

As Z is dense in $L^2(\nu)$ (see [5]), Y is also dense in $L^2(\nu)$. Let $(e_j)_{j=1,\ldots,d}$ be a standard basis in $\mathbb{R}^d$ and let

$$H_1(t) = 1\,, \quad t \in [0,1],$$

$$H_{2^m+k}(t) = \begin{cases} 2^{\frac{m}{2}} & \text{if } t \in [\frac{k-1}{2^m}, \frac{2k-1}{2^{m+1}}), \\ -2^{\frac{m}{2}} & \text{if } t \in [\frac{2k-1}{2^{m+1}}, \frac{k}{2^m}), \\ 0 & \text{otherwise}, \end{cases} \qquad k = 1, \ldots, 2^m\,,\ m = 0, 1, \ldots\ ,$$

denote the system of the Haar functions on $[0,1]$. Furthermore, define

$$g_{d(r-1)+j} := H_r \cdot e_j\,, \quad r \in \mathbb{N}\,,\ j \in \{1, \ldots, d\}.$$

As the system of the Haar functions $(H_n)_{n\in\mathbb{N}}$ is complete in $L^2([0,1] \to \mathbb{R}, ds)$, the system $(g_n)_{n\in\mathbb{N}}$ is complete in $L^2([0,1] \to \mathbb{R}^d, ds)$. Therefore, $(S_n)_{n\in\mathbb{N}}$, defined by

$$S_n(s) := \int_0^s g_n(u)\, du\,, \quad s \in [0,1],\ n \in \mathbb{N},$$

is complete in the Cameron-Martin space $\mathbb{H}$, the space of all $\mathbb{R}^d$-valued absolutely continuous functions h on $[0,1]$ with $h(0) = 0$ endowed with the norm

$$|h|_{\mathbb{H}} := \left(\int_0^1 |h'(s)|^2_{\mathbb{R}^d}\, ds \right)^{\frac{1}{2}}\ .$$

2. Definition of the form and closability

For $F \in Y$ and ν-a.e. $\gamma \in \mathbf{P}_{m_0}(M)$, define

$$D_s F(\gamma) := \sum_{i=1}^{k} \chi_{[0,s_i]}(s) \{ \mathcal{T}^{\gamma}_{s\leftarrow s_i} (\nabla_{s_i} f)(\underline{\gamma}) \}\,, \quad s \in [0,1], \tag{2.1}$$

where $(\nabla_{s_i} f)(\underline{\gamma}) \equiv (\nabla_{s_i} f)(\gamma(s_1); \ldots; \gamma(s_k)) \in T_{\gamma(s_i)}$ denotes the gradient of the function f relative to the i-th variable while holding the remaining variables fixed.

Here, f and the $s_1, \dots, s_k$ are as in the definition of Z. Furthermore, for $F \in Z$ and

$$\begin{aligned} \mathbf{D}_s F(\gamma) &:= \int_0^s r^{-1}_{L(\gamma)}(s') D_{s'} F(\gamma)\, ds' \\ &= \sum_{i=1}^k s \wedge s_i \cdot r^{-1}_{L(\gamma)}(s_i)(\nabla_{s_i} f)(\underline{\gamma})\,, \quad s \in [0,1],\ \gamma \in \mathbf{P}_{m_0}(M), \end{aligned} \tag{2.2}$$

we have $\mathbf{D}F \in \mathbb{H}$ ν-a.e. See also (2.1).

Any $F \in Y$ has the representation $F(\gamma) = f(\gamma(s_1); \dots; \gamma(s_k))$ where $s_1 = \frac{l_1}{2^{n'}}, \dots, s_k = \frac{l_k}{2^{n'}}$, for some $k \in \mathbb{N}$, $n' \in \mathbb{N}$, $l_1, \dots, l_k \in \{1, \dots, 2^{n'}\}$, and $f \in C^\infty(M^k)$. As $H_{2^m+l}(t) = 0$ on $[0,1] \setminus [\frac{l-1}{2^m}, \frac{l}{2^m}]$, $\int_{(l-1)/2^m}^{l/2^m} H_{2^m+l}(t)\, dt = 0$, and either $(\frac{l-1}{2^m}, \frac{l}{2^m}) \subseteq [0, s_i]$ or $(\frac{l-1}{2^m}, \frac{l}{2^m}) \subseteq (s_i, 1]$ if $m \geq n'$, $l \in \{1, \dots, 2^m\}$, and $i \in \{1, \dots, k\}$, from (2.2), we obtain the following lemma which is crucial for the technical procedure.

Lemma 2.1. *Let $F \in Y$. There exists $n_0 \in \mathbb{N}$ such that, for ν-a.e. $\gamma \in \mathbf{P}_{m_0}(M)$,*

$$\langle S_n, \mathbf{D}F(\gamma)\rangle_{\mathbb{H}} = 0\,, \quad n > n_0.$$

Let us define the diffusion operator we are dealing with in this paper. Choose an increasing sequence $(\lambda_n)_{n\in\mathbb{N}}$ of positive real numbers and define the operator

$$\begin{aligned} D(\mathbf{A}) &:= \left\{ \Phi \in L^2(\mathbf{P}_{m_0}(M) \to \mathbb{H}, \nu) : \int \sum_{i=1}^\infty \lambda_i^2 \langle S_i, \Phi\rangle_{\mathbb{H}}^2\, d\nu < \infty \right\}, \\ \mathbf{A}\Phi(\gamma) &:= \sum_{i=1}^\infty \lambda_i \langle S_i, \Phi(\gamma)\rangle_{\mathbb{H}} S_i\,,\ \gamma \in \mathbf{P}_{m_0}(M)\,, \quad \Phi \in D(\mathbf{A}), \end{aligned}$$

mapping $L^2(\mathbf{P}_{m_0}(M) \to \mathbb{H}, \nu) \supseteq D(\mathbf{A}) \to L^2(\mathbf{P}_{m_0}(M) \to \mathbb{H}, \nu)$. For $F \in Y$, we have $\int \langle S_i, \mathbf{D}F\rangle_{\mathbb{H}}^2\, d\nu < \infty$, $i \in \mathbb{N}$. By Lemma 2.1, for $F \in Y$, there is $n_0 \in \mathbb{N}$ such that, for all $i > n_0$, it holds that $\int \langle S_i, \mathbf{D}F\rangle_{\mathbb{H}}^2\, d\nu = 0$. Therefore, we obtain $\{\mathbf{D}F : F \in Y\} \subseteq D(\mathbf{A})$. Furthermore, for all $F \in Y$, we get

$$\begin{aligned} \int \langle \mathbf{D}F, \mathbf{A}\mathbf{D}F\rangle_{\mathbb{H}}\, d\nu &= \int \left\langle \mathbf{D}F, \sum_{i=1}^\infty \lambda_i \langle S_i, \mathbf{D}F\rangle_{\mathbb{H}} S_i \right\rangle_{\mathbb{H}} d\nu \\ &= \sum_{i=1}^\infty \lambda_i \int \langle S_i, \mathbf{D}F\rangle_{\mathbb{H}}^2\, d\nu \\ &< \infty\,. \end{aligned} \tag{2.3}$$

Consequently, the nonnegative symmetric bilinear form

$$\begin{aligned} \mathcal{E}(F,F) &:= \int \langle \mathbf{D}F, \mathbf{A}\mathbf{D}F\rangle_{\mathbb{H}}\, d\nu \\ &= \int \left| \mathbf{A}^{1/2}\mathbf{D}F \right|_{\mathbb{H}}^2 d\nu\,, \quad F \in Y, \end{aligned} \tag{2.4}$$

is well defined.

Remarks. (1) It is known from [5], Lemma 3, or [9], Proposition 5.3, that the operator $\mathbf{D} : Z \to L^2(\mathbf{P}_{m_0}(M) \to \mathbb{H}, \nu)$ is closable on $L^2(\nu)$. Let $(\mathbf{D}, D(\mathbf{D}))$ denote this closure. Equivalently, the Ornstein-Uhlenbeck form

$$\mathcal{E}^{OU}(F,F) := \int |\mathbf{D}F|^2_{\mathbb{H}}\, d\nu\,, \quad F \in Z,$$

is closable on $L^2(\nu)$. For its closure $(\mathcal{E}^{OU}, D(\mathcal{E}^{OU}))$, we have $D(\mathbf{D}) = D(\mathcal{E}^{OU})$.

(2) Fix $\gamma \in \mathbf{P}_{m_0}(M)$ and consider, for this remark, $\mathbf{A}$ as an operator mapping $D(\mathbf{A}) \supseteq \mathbb{H} \to \mathbb{H}$. S. Albeverio and M. Röckner [2], for example, suggest in a similar situation to reduce a form of type (2.4) to a more simple one by choosing another Hilbert space $(H, \langle\cdot,\cdot\rangle_H) := (D(\mathbf{A}^{1/2}), \langle \mathbf{A}^{1/2}\cdot, \mathbf{A}^{1/2}\cdot\rangle_{\mathbb{H}})$. The price one has to pay is that the classical relation between directional derivative ∂_h and gradient $\mathbf{D}$, namely $\partial_h F = \langle \mathbf{D}F, h\rangle_H$, is in general not satisfied anymore if $\partial_h F = \langle \mathbf{D}F, h\rangle_{\mathbb{H}}$.

Theorem 2.2. *The bilinear form $(\mathcal{E}, Y)$ is closable on $L^2(\nu)$.*

Proof. Suppose $F_n \in Y$, $n \in \mathbb{N}$, with $F_n \underset{n\to\infty}{\longrightarrow} 0$ in $L^2(\nu)$ and $\mathcal{E}(F_n - F_m, F_n - F_m) \underset{m,n\to\infty}{\longrightarrow} 0$. In particular, (2.4) implies

$$\mathbf{A}^{1/2}\mathbf{D}F_n \underset{n\to\infty}{\longrightarrow} \Psi \quad \text{in} \quad L^2(\mathbf{P}_{m_0}(M) \to \mathbb{H}, \nu) \tag{2.5}$$

for some $\Psi \in L^2(\mathbf{P}_{m_0}(M) \to \mathbb{H}, \nu)$. Define

$$\mathbf{J}F := \sum_{i=1}^{\infty} \lambda_i^{-1/2} \langle S_i, F\rangle_{\mathbb{H}} S_i\,, \quad F \in L^2(\mathbf{P}_{m_0}(M) \to \mathbb{H}, \nu)\,. \tag{2.6}$$

Since $\mathbf{J}$ is a bounded operator on $L^2(\mathbf{P}_{m_0}(M) \to \mathbb{H}, \nu)$, we verify

$$\mathbf{D}F_n = \mathbf{J}\mathbf{A}^{1/2}\mathbf{D}F_n \underset{n\to\infty}{\longrightarrow} \mathbf{J}\Psi \quad \text{in} \quad L^2(\mathbf{P}_{m_0}(M) \to \mathbb{H}, \nu)$$

from (2.5). As $(\mathbf{D}, Z)$ is closable on $L^2(\nu)$, we obtain $\mathbf{J}\Psi = 0$. It follows from (2.6) and $\lambda_i > 0$, $i \in \mathbb{N}$, that $\Psi = 0$. Thus, relation (2.5) leads to $\mathbf{A}^{1/2}\mathbf{D}F_n \underset{n\to\infty}{\longrightarrow} 0$ in $L^2(\mathbf{P}_{m_0}(M) \to \mathbb{H}, \nu)$ which implies $\mathcal{E}(F_n, F_n) = \int \left|\mathbf{A}^{1/2}\mathbf{D}F_n\right|^2_{\mathbb{H}} d\nu \underset{n\to\infty}{\longrightarrow} 0$. □

Let $(\mathcal{E}, D(\mathcal{E}))$ denote the closure of $(\mathcal{E}, Y)$ on $L^2(\nu)$.

Remark. (3) Let $F \in D(\mathcal{E})$ and let $F_n \in Y$, $n \in \mathbb{N}$, be a sequence converging to F in $\mathcal{E}_1^{1/2} = (\|\cdot\|^2_{L^2(\nu)} + \mathcal{E}(\cdot,\cdot))^{1/2}$-norm. Since $\lambda_i > 0$, $i \in \mathbb{N}$, is an increasing sequence of positive real numbers and $(\mathcal{E}, Y) = (\mathcal{E}^{OU}, Y)$ if $\lambda_i = 1$, $i \in \mathbb{N}$, from (2.3) it follows that F_n, $n \in \mathbb{N}$, is a Cauchy sequence in $(\mathcal{E}^{OU})_1^{1/2} = (\|\cdot\|^2_{L^2(\nu)} + \mathcal{E}^{OU}(\cdot,\cdot))^{1/2}$-norm. Therefore, $F_n \underset{n\to\infty}{\longrightarrow} F$ in $(\mathcal{E}^{OU})_1^{1/2}$-norm. Thus, we have $D(\mathcal{E}) \subseteq D(\mathcal{E}^{OU}) = D(\mathbf{D})$. Since, by self-adjointness, $\mathbf{A}^{1/2}$ is a closed operator, it holds that $\{\mathbf{D}F : F \in D(\mathcal{E})\} \subseteq D(\mathbf{A}^{1/2})$ and relations (2.3) and (2.4) yield

$$\begin{aligned} \mathcal{E}(F,F) &= \sum_{i=1}^{\infty} \lambda_i \int \langle S_i, \mathbf{D}F\rangle^2_{\mathbb{H}}\, d\nu \\ &= \int \left|\mathbf{A}^{1/2}\mathbf{D}F\right|^2_{\mathbb{H}} d\nu\,, \quad F \in D(\mathcal{E}). \end{aligned} \tag{2.7}$$

3. Quasi-regularity and associated process

Let $h \in \mathbb{H}$, $t \in \mathbb{R}$, $s \in [0,1]$, and $\gamma \in \mathbf{P}_{m_0}(M)$ and let σ denote the solution to the geometric flow equation

$$\begin{cases} \dot{\sigma}^h(t,s)(\gamma) & = & \mathcal{T}_{s\leftarrow 0}^{\sigma^h(t,\cdot)(\gamma)} r_0 h(s), \\ \sigma^h(0,s)(\gamma) & = & \gamma(s). \end{cases}$$

Note that " $\dot{}$ " stands for differentiation with respect to t. In particular, we have $\sigma^h(\,\cdot\,,s)(\gamma) \in C^1(\mathbb{R} \to M)$, $\sigma^h(t)(\gamma) \equiv \sigma^h(t,\,\cdot\,)(\gamma) \in \mathbf{P}_{m_0}(M)$. For $h \in \mathbb{H}$ and ν-a.e. $\gamma \in \mathbf{P}_{m_0}(M)$, there exists a unique solution (see [4] and [9]). For $F \in Y$ given as in (1.1), the directional derivative along the direction $h \in \mathbb{H}$ satisfies

$$\begin{aligned} \partial_h F &:= \lim_{t\to 0} \frac{F(\sigma^h(t)) - F}{t} \\ &= \sum_{i=1}^{k} \langle \nabla_{s_i} f, \dot{\sigma}^h(0,s_i)\rangle_{T_{\cdot}(s_i)} \\ &= \sum_{i=1}^{k} \langle \nabla_{s_i} f, \mathcal{T}^{\cdot}_{s_i\leftarrow 0} r_0 h(s_i)\rangle_{T_{\cdot}(s_i)} \\ &= \sum_{i=1}^{k} \langle \mathcal{T}^{\cdot}_{0\leftarrow s_i}(\nabla_{s_i} f), r_0 h(s_i)\rangle_{T_{m_0}} \\ &= \langle \mathbf{D}F, h\rangle_{\mathbb{H}} \quad \nu\text{-a.e.} \end{aligned} \tag{3.1}$$

See also (2.2).

Remark. (4) For every $h \in \mathbb{H}$, the operator $\partial_h : Z \to L^2(\nu)$ is closable on $L^2(\nu)$. Let $(\partial_h, D(\partial_h))$ denote the corresponding closure. It holds that $D(\mathcal{E}) \subseteq D(\mathbf{D}) \subseteq D(\partial_h)$, $h \in \mathbb{H}$, and $\partial_h F = \langle \mathbf{D}F, h\rangle_{\mathbb{H}}$, $F \in D(\mathbf{D})$; cf. [9], Theorem 5.2 and Proposition 5.3. Therefore,

$$\mathcal{E}(F,F) = \sum_{i=1}^{\infty} \lambda_i \int (\partial_{S_i} F)^2 \, d\nu\,, \quad F \in D(\mathcal{E}).$$

Proposition 3.1. *The form $(\mathcal{E}, D(\mathcal{E}))$ is a Dirichlet form on $L^2(\nu)$.*

Proof. We have

$$\begin{aligned} \mathcal{E}(F,F) &= \sum_{i=1}^{\infty} \lambda_i \int (\partial_{S_i} F)^2 \, d\nu \\ &= \sum_{i=1}^{\infty} \lambda_i \int \left(\left.\frac{d}{dt}\right|_0 F(\sigma^{S_i}(t)) \right)^2 d\nu\,, \quad F \in Y. \end{aligned} \tag{3.2}$$

It follows directly from [13], Proposition I, 4.10, and the chain rule that $(\mathcal{E}, D(\mathcal{E}))$ is a Dirichlet form. □

An important tool for the subsequent technical procedure will be the following assertion; cf. [13], Chapter IV, Lemma 4.1. Note that, for $u, v \in D(\mathcal{E})$, we have $u \vee v \in D(\mathcal{E}) \subseteq D(\mathbf{D}) \subseteq D(\partial_{S_i})$, $i \in \mathbb{N}$ (see Remarks (3) and (4)).

Lemma 3.2. *Let $u, v \in D(\mathcal{E})$. For all $i \in \mathbb{N}$, we have*

$$\partial_{S_i}(u \vee v) = \chi_{\{u>v\}}\partial_{S_i}u + \chi_{\{u<v\}}\partial_{S_i}v + \frac{1}{2}\chi_{\{u=v\}}(\partial_{S_i}u + \partial_{S_i}v) \quad \nu\text{-a.e.}$$

Proof. Having representation (3.2) of $(\mathcal{E}, Y)$ in mind, the proof can be obtained from that of [13], Chapter IV, Lemma 4.1 by replacing therein $\frac{\partial}{\partial k}$ with ∂_{S_i} and $\mathcal{F}C_b^\infty$ with Y. □

Proposition 3.3. *Suppose*

$$\lambda_n \leq c\, n^{1-\varepsilon}, \quad n \in \mathbb{N}, \quad \text{for some } c > 0 \text{ and } \varepsilon \in (0,1). \tag{3.3}$$

Then the Dirichlet form $(\mathcal{E}, D(\mathcal{E}))$ is quasi-regular.

Proof. In steps 1-3 below, we show that there is an $\mathcal{E}$-nest consisting of compact sets.

Step 1. For $r \in \mathbb{N}$, $l \in \{0, \ldots, 2^{r-1}-1\}$, and $k = 2^{r-1}+l$, set $s_k := (2l+1)2^{-r}$. Let $x^v(p)$ denote the standard coordinates of $p \in M$ embedded in $\mathbb{R}^N$, $v \in \{1, \ldots, N\}$. Fix $\tau \in \mathbf{P}_{m_0}(M)$, $k = 2^{r-1}+l$, and $v \in \{1, \ldots, N\}$. Consider the functions $f_{v,k,\tau}(p) := x^v(p) - x^v(\tau(s_k))$, $p \in M$, and

$$F_{v,k,\tau}(\gamma) := f_{v,k,\tau}(\gamma(s_k)) = x^v(\gamma(s_k)) - x^v(\tau(s_k)), \quad \gamma \in \mathbf{P}_{m_0}(M); \tag{3.4}$$

obviously, $F_{v,k,\tau}$ belongs to Y. Furthermore, let either $i = j$ or $i = d(2^m+u-1)+j$ for some $m \in \{0, 1, \ldots\}$, $u \in \{1, \ldots, 2^m\}$ and $j \in \{1, \ldots, d\}$. We have

$$\begin{aligned} |\langle S_i, \mathbf{D}F_{v,k,\tau}(\gamma)\rangle_{\mathbb{H}}| &= \left|\left\langle \mathcal{T}^\gamma_{0\leftarrow s_k}\nabla_{s_k}x^v(\gamma(s_k)), r_0 S_i(s_k)\right\rangle_{T_{m_0}}\right| \\ &\leq \left\|\mathcal{T}^\gamma_{0\leftarrow s_k}\nabla_{s_k}x^v(\gamma(s_k))\right\|_{T_{m_0}} \|r_0 S_i(s_k)\|_{T_{m_0}} \\ &= \|\nabla_{s_k}x^v(\gamma(s_k))\|_{T_{\gamma(s_k)}} |S_i(s_k)|_{\mathbb{R}^d} \\ &= \|\nabla_{s_k}x^v(\gamma(s_k))\|_{T_{\gamma(s_k)}} \left|(S_i(s_k))^j\right| \end{aligned} \tag{3.5}$$

for ν-a.e. $\gamma \in \mathbf{P}_{m_0}(M)$. As mentioned in [5], proof of Proposition 5, $\|\nabla x^v(p)\|_{T_p}$, $p \in M$, is bounded by some constant K since M is compact. Furthermore, the definitions of s_k and S_i yield $|(S_i(s_k))^j| \leq 1$ if $i = j$ for some $j \in \{1, \ldots, d\}$. Moreover, $|(S_i(s_k))^j| \leq 2^{-(m/2+1)}$ if $i = d(2^m+u-1)+j$ for some $m \in \{0, 1, \ldots\}$, $u \in \{1, \ldots, 2^m\}$, $j \in \{1, \ldots, d\}$, and $m < r$ as well as $s_k = (2l+1)2^{-r} \in (\frac{u-1}{2^m}, \frac{u}{2^m})$. Otherwise, we have $S_i(s_k) = 0$. Therefore, (3.5) implies

$$\begin{aligned} \sum_{i=1}^\infty \lambda_i \left(\partial_{S_i}F_{v,k,\tau}(\gamma)\right)^2 &= \sum_{i=1}^\infty \lambda_i \langle S_i, \mathbf{D}F_{v,k,\tau}(\gamma)\rangle^2_{\mathbb{H}} \\ &= \sum_{j=1}^d \lambda_j \langle S_j, \mathbf{D}F_{v,k,\tau}(\gamma)\rangle^2_{\mathbb{H}} \\ &\quad + \sum_{m=0}^\infty \sum_{u=1}^{2^m} \sum_{j=1}^d \lambda_{d(2^m+u-1)+j} \langle S_{d(2^m+u-1)+j}, \mathbf{D}F_{v,k,\tau}(\gamma)\rangle^2_{\mathbb{H}} \\ &\leq K^2 d\lambda_d + K^2 d \sum_{m=0}^\infty \lambda_{d\,2^{m+1}} 2^{-(m+2)} \end{aligned} \tag{3.6}$$

for ν-a.e. $\gamma \in \mathbf{P}_{m_0}(M)$. Finally, from (3.3), we obtain

$$\sum_{i=1}^{\infty} \lambda_i \left(\partial_{S_i} F_{v,k,\tau}(\gamma)\right)^2 \le K^2 c d^{2-\varepsilon} \frac{2^{1+\varepsilon}-1}{2^{1+\varepsilon}-2} =: C_1 \tag{3.7}$$

for ν-a.e. $\gamma \in \mathbf{P}_{m_0}(M)$ and

$$\mathcal{E}(F_{v,k,\tau}, F_{v,k,\tau}) \le C_1\,, \tag{3.8}$$

where the right-hand side is independent of k (resp. s_k), $v \in \{1,\dots,N\}$, and τ.

Step 2. We apply a method introduced in [5] and [15]. Set

$$G_{n,\tau} := \sup_{\substack{k\in\{1,\dots,n\}\\ v\in\{1,\dots,N\}}} |F_{v,k,\tau}|\,, \quad n \in \mathbb{N}.$$

It follows now from (3.7), relation

$$\mathcal{E}(G_{n,\tau}, G_{n,\tau}) = \int \sum_{i=1}^{\infty} \lambda_i \left(\partial_{S_i} G_{n,\tau}(\gamma)\right)^2 \, d\nu\,,$$

and Lemma 3.2 that

$$\mathcal{E}(G_{n,\tau}, G_{n,\tau}) \le C_1\,, \quad n \in \mathbb{N}.$$

Since M is compact, there exists $C_2 > 0$, such that

$$|x^v(p)| \le \frac{1}{2}\sqrt{C_2}\,, \quad p \in M,\ v \in \{1,\dots,N\}.$$

From (3.4) and the definition of $G_{n,\tau}$ it follows that $\|G_{n,\tau}\|^2_{L^2(\nu)} \le C_2$ and, thus,

$$\mathcal{E}_1(G_{n,\tau}, G_{n,\tau}) \le C_1 + C_2 =: C_3\,, \quad n \in \mathbb{N}. \tag{3.9}$$

Step 3. In this step, we proceed as in [5] and [15]. In particular, we apply the Banach-Saks property of the Hilbert space $(D(\mathcal{E}), \mathcal{E}_1^{1/2})$, which states that every bounded sequence in $(D(\mathcal{E}), \mathcal{E}_1^{1/2})$ has a subsequence whose Cesaro means converge strongly (see, for example, [14]). Accordingly, relation (3.9) and the fact that the sequence $(G_{n,\tau})_{n\in\mathbb{N}}$ satisfies $G_{n,\tau} \le G_{n+1,\tau}$, $n \in \mathbb{N}$, imply that the function

$$H_\tau(\gamma) := \sup_{\substack{s\in[0,1]\\ v\in\{1,\dots,N\}}} \left|x^v(\gamma(s)) - x^v(\tau(s))\right|\,, \quad \gamma \in \mathbf{P}_{m_0}(M),$$

belongs to $D(\mathcal{E})$ and that

$$\mathcal{E}_1(H_\tau, H_\tau) \le C_3\,.$$

Let $\{\tau_k : k \in \mathbb{N}\}$ be a dense set in $\mathbf{P}_{m_0}(M)$. Set

$$K_n := \inf_{1\le k\le n} H_{\tau_k}\,, \quad n \in \mathbb{N}.$$

We have $K_n \in D(\mathcal{E})$. Again, recalling the Banach-Saks property of $(D(\mathcal{E}), \mathcal{E}_1^{1/2})$, the last relation implies that $\mathcal{E}_1(K_n, K_n) \underset{n\to\infty}{\longrightarrow} 0$. According to [13], Chapter III, Proposition 3.5, there exists a subsequence K_{n_k}, $k \in \mathbb{N}$, and an $\mathcal{E}$-nest F_m, $m \in \mathbb{N}$, such that K_{n_k} converges uniformly to zero (as $k \to \infty$) on each F_m. Consult also [3], Section I.8. As in [5], proof of Proposition 5, it follows now from the definition of K_n, $n \in \mathbb{N}$, that each F_m is totally bounded. Thus, F_m, $m \in \mathbb{N}$, form an $\mathcal{E}$-nest consisting of compact sets.

Step 4. For fixed $\tau \in \mathbf{P}_{m_0}(M)$, the system of functions $F_{v,k,\tau}$, $v \in \{1,\dots,N\}$, $k \in \mathbb{N}$, introduced in (3.4) separates the points in $\mathbf{P}_{m_0}(M)$.

Together with Theorem 2.2 and the result of Step 3, quasi-regularity follows now from its definition (see [13], Chapter IV, Definition 3.1). □

Proposition 3.4. *The form $(\mathcal{E}, D(\mathcal{E}))$ is local.*

Proof. We follow [5], proof of Proposition 5 (ii), and [13], Example V.1.12. Let $F, G \in D(\mathcal{E}) \cap L^\infty(\nu)$ with $\text{supp}[F] \cap \text{supp}[G] = \emptyset$. According to [13], Propositions I.4.17 (i) and V.1.2 (ii), we have to verify $\mathcal{E}(F,G) = 0$. Since $\mathcal{E}(F,G) = \int \langle \mathbf{A}^{1/2}\mathbf{D}F, \mathbf{A}^{1/2}\mathbf{D}G\rangle_{\mathbb{H}}\, d\nu$ (cf. (2.7)), it is sufficient to show that

$$\mathbf{D}F = 0 \quad \nu\text{-a.e. on } \mathbf{P}_{m_0}(M) \setminus \text{supp}[F]\,. \tag{3.10}$$

From $D(\mathcal{E}) \subseteq D(\mathcal{E}^{OU})$ and [5], equation (11), we obtain

$$\mathbf{D}(U \cdot V) = U \cdot \mathbf{D}(V) + V \cdot \mathbf{D}(U)\,, \quad U, V \in D(\mathcal{E}) \cap L^\infty(\nu)\,. \tag{3.11}$$

See also [13], Example V.1.12. Furthermore, from [13], Proposition V.1.7, we get the existence of $V \in D(\mathcal{E}) \cap L^\infty(\nu)$ with $0 \leq V \leq \chi_{\mathbf{P}_{m_0}(M)\setminus\text{supp}[F]}$ and $V > 0$ ν-a.e. on $\mathbf{P}_{m_0}(M) \setminus \text{supp}[F]$; here χ denotes the indicator function. Now, relation (3.11) implies

$$0 = F \cdot \mathbf{D}(V) + V \cdot \mathbf{D}(F) \quad \nu\text{-a.e.}$$

This yields (3.10). □

As a consequence of Propositions 3.3 and 3.4, we get with [13], Theorems IV.3.5 and V.1.11:

Theorem 3.5. *There exists a diffusion process* $\mathbf{M}$ *associated with* $(\mathcal{E}, D(\mathcal{E}))$.

4. Generator

We start with a technical lemma.

Lemma 4.1. *Let $F \in Y$ and $n \in \mathbb{N}$. Then the derivatives $\frac{d}{dt}\big|_0 \partial_{S_n} F(\sigma^{S_n}(-t))$, $\frac{d}{dt}\big|_0 \frac{d\nu\circ\sigma^{S_n}(-t)}{d\nu}$, and $\frac{d}{dt}\big|_0 \left\{\partial_{S_n} F(\sigma^{S_n}(-t))\frac{d\nu\circ\sigma^{S_n}(-t)}{d\nu}\right\}$ exist in $L^2(\nu)$ and we have*

$$\begin{aligned} &\frac{d}{dt}\bigg|_0 \left\{\partial_{S_n} F(\sigma^{S_n}(-t))\frac{d\nu\circ\sigma^{S_n}(-t)}{d\nu}\right\} \\ &\qquad = -\partial_{S_n}\partial_{S_n} F + \frac{d}{dt}\bigg|_0 \frac{d\nu\circ\sigma^{S_n}(-t)}{d\nu} \cdot \partial_{S_n} F \quad \nu\text{-a.e.} \end{aligned} \tag{4.1}$$

Proof. Step 1. Introduce

$$\varphi_t(\gamma) := \frac{d\nu\circ\sigma^{S_n}(-t)(\gamma)}{d\nu}(\gamma)\,, \quad \gamma \in \mathbf{P}_{m_0}(M),\ t \in \mathbb{R}.$$

The existence of $\frac{d}{dt}\big|_0 \varphi_t$ in $L^2(\nu)$ is shown in [4], Theorem 8.5 and in the proof of Theorem 9.1. Note that the result for $h \in C^1([0,1] \to \mathbb{R}^d)$ presented in [4] can be extended to general $h \in \mathbb{H}$ by [9], Theorems 3.5 and 4.1 and the proof of Theorem 5.1.

Step 2. Let $n \in \mathbb{N}$ and let $F \in Y$ be given as in (1.1). According to (3.1), we have, for ν-a.e. $\gamma \in \mathbf{P}_{m_0}(M)$,

$$\begin{aligned} \psi_t(\gamma) &:= \partial_{S_n} F(\sigma^{S_n}(-t)(\gamma)) \\ &= \sum_{i=1}^k \left\langle \mathcal{T}_{0\leftarrow s_i}^{\sigma^{S_n}(-t)(\gamma)}(\nabla_{s_i} f)(\underline{\sigma^{S_n}(-t)(\gamma)}), r_0 S_n(s_i)\right\rangle_{T_{m_0}}, \quad t \in \mathbb{R}. \end{aligned}$$

From $|S_n(s)|_{\mathbb{R}^d} \le 1$, $s \in [0,1]$, it follows that, for ν-a.e. $\gamma \in \mathbf{P}_{m_0}(M)$,

$$\begin{aligned} |\psi_t(\gamma)| &\le \sum_{i=1}^k \left\| \mathcal{T}_{0\leftarrow s_i}^{\sigma^{S_n}(-t)(\gamma)} (\nabla_{s_i} f)(\underline{\sigma^{S_n}(-t)(\gamma)}) \right\|_{T_{m_0}} \|r_0 S_n(s_i)\|_{T_{m_0}} \\ &\le \sum_{i=1}^k \left\| (\nabla_{s_i} f)(\underline{\sigma^{S_n}(-t)(\gamma)}) \right\|_{T_{\sigma^{S_n}(-t,s_i)(\gamma)}}, \quad t \in \mathbb{R}. \end{aligned}$$

Since $f \in C^\infty(M^k)$, and M is compact, there exists $C_4 > 0$ such that

$$|\psi_t(\gamma)| \le C_4, \quad \nu\text{-a.e. } \gamma \in \mathbf{P}_{m_0}(M), \ t \in \mathbb{R}. \tag{4.2}$$

Furthermore, in virtue of [9], Theorem 4.1 (iii),

$$\psi_t \underset{t\to 0}{\longrightarrow} \psi_0 \quad \nu\text{-a.e.} \tag{4.3}$$

Step 3. The aim of this step is to verify the existence of $\frac{d}{dt}\big|_0 \psi_t$ in $L^2(\nu)$. To this end, fix $i \in \{1, \dots, k\}$. Since $(\nabla_{s_i} f)^v$ is then a smooth function on M^k, from [9], Section 5, and [4], Lemma 9.1, it follows that

$$\frac{d}{dt}\bigg|_0 \left((\nabla_{s_i} f)(\underline{\sigma^{S_n}(-t)(\gamma)}) \right)^v \text{ exists in } L^2(\nu), \quad v \in \{1, \dots, N\}. \tag{4.4}$$

Furthermore, there is a $C_5 > 0$ such that

$$|(\nabla_{s_i} f)^v| \le C_5, \quad v \in \{1, \dots, N\}. \tag{4.5}$$

By [4], Corollary 4.2 and inequalities (i) as well as (ii) of Lemma 4.1 of the same reference, for all $v \in \{1, \dots, N\}$,

$$\begin{aligned} &\frac{d}{dt}\bigg|_0 \left(\mathcal{T}_{s_i\leftarrow 0}^{\sigma^{S_n}(-t)(\gamma)} r_0 S_n(s_i) \right)^v \\ &\qquad = \frac{d}{dt}\bigg|_0 \left(r_{L(\gamma(-t,j,i))}(s_i) S_n(s_i) \right)^v \text{ exists in } L^2(\nu) \end{aligned} \tag{4.6}$$

if $n = j \in \{1, \dots, d\}$. Even though in [4] the geometric flow is generated by a C^1-function, for $n = d(2^m + l - 1) + j$ with $m \in \{0, 1, \dots\}$, $l \in \{1, \dots, 2^m\}$, $j \in \{1, \dots, d\}$, we may obtain (4.6) from the above reference by decomposing $S_n = S_n^1 + S_n^2 + S_n^3$ where

$$\begin{aligned} S_n^1 &= 2^{\frac{m}{2}} \chi_{[\frac{l-1}{2^m},1]}(s)(s - \tfrac{l-1}{2^m}), \\ S_n^2 &= -2^{\frac{m}{2}+1} \chi_{[\frac{2l-1}{2^{m+1}},1]}(s)(s - \tfrac{2l-1}{2^{m+1}}), \\ S_n^3 &= 2^{\frac{m}{2}} \chi_{[\frac{l}{2^m},1]}(s)(s - \tfrac{l}{2^m}). \end{aligned}$$

Moreover, by isometric embedding of M into $\mathbb{R}^N$ we verify

$$\begin{aligned} \left| \left(\mathcal{T}_{s_i\leftarrow 0}^{\cdot} r_0 S_n(s_i) \right)^v \right| &\le \left| \mathcal{T}_{s_i\leftarrow 0}^{\cdot} r_0 S_n(s_i) \right|_{\mathbb{R}^N} \\ &= \left\| \mathcal{T}_{s_i\leftarrow 0}^{\cdot} r_0 S_n(s_i) \right\|_{T_{\cdot(s_i)}} \\ &= |S_n(s_i)|_{\mathbb{R}^d} \\ &\le 1, \quad v \in \{1, \dots, N\}. \end{aligned} \tag{4.7}$$

Introduce

$$\alpha_t^v := \left((\nabla_{s_i} f)(\underline{\sigma^{S_n}(-t)(\gamma)}) \right)^v, \quad \beta_t^v := \left(\mathcal{T}_{s_i\leftarrow 0}^{\sigma^{S_n}(-t)(\gamma)} r_0 S_n(s_i) \right)^v,$$

$t \in \mathbb{R},\ v \in \{1, \dots, N\}$. The existence of

$$\frac{d}{dt}\bigg|_0 \sum_{v=1}^{N} ((\nabla_{s_i} f)(\sigma^{S_n}(-t)(\gamma)))^v (\mathcal{T}_{s_i \leftarrow 0}^{\sigma^{S_n}(-t)(\gamma)} r_0 S_n(s_i))^v = \frac{d}{dt}\bigg|_0 \sum_{v=1}^{N} \alpha_t^v \beta_t^v$$

in $L^2(\nu)$ follows from (4.4)-(4.7), $\alpha_t^v \underset{t\to 0}{\longrightarrow} \alpha_0^v$ ν-a.e., and the inequality

$$\begin{aligned}
&\left\| \frac{\sum_{v=1}^N \alpha_t^v \beta_t^v - \sum_{v=1}^N \alpha_0^v \beta_0^v}{t} - \sum_{v=1}^{N} (\dot{\alpha_0^v} \beta_0^v + \alpha_0^v \dot{\beta_0^v}) \right\|_{L^2(\nu)} \\
&\quad \le \sum_{v=1}^{N} \left\| \left(\frac{\alpha_t^v - \alpha_0^v}{t} - \dot{\alpha_0^v} \right) \beta_0^v \right\|_{L^2(\nu)} + \sum_{v=1}^{N} \left\| \left(\frac{\beta_t^v - \beta_0^v}{t} - \dot{\beta_0^v} \right) \alpha_t^v \right\|_{L^2(\nu)} \\
&\qquad + \sum_{v=1}^{N} \left\| (\alpha_t^v - \alpha_0^v) \dot{\beta_0^v} \right\|_{L^2(\nu)} \\
&\quad \le \sum_{v=1}^{N} \left\| \frac{\alpha_t^v - \alpha_0^v}{t} - \dot{\alpha_0^v} \right\|_{L^2(\nu)} \|\beta_0^v\|_{L^\infty(\nu)} + C_5 \sum_{v=1}^{N} \left\| \frac{\beta_t^v - \beta_0^v}{t} - \dot{\beta_0^v} \right\|_{L^2(\nu)} \\
&\qquad + \sum_{v=1}^{N} \left\| ((\alpha_t^v - \alpha_0^v) \dot{\beta_0^v})^2 \right\|_{L^1(\nu)}^{1/2}, \quad v \in \{1, \dots, N\};
\end{aligned}$$

note that, by (4.5), $((\alpha_t^v - \alpha_0^v)\dot{\beta_0^v})^2 \in L^1(\nu)$ is dominated by $4C_5^2(\dot{\beta_0^v})^2 \in L^1(\nu)$, $v \in \{1, \dots, N\}$. Finally, by isometry

$$\frac{d}{dt}\bigg|_0 \psi_t(\gamma) = \sum_{i=1}^{k} \frac{d}{dt}\bigg|_0 \left\langle (\nabla_{s_i} f)(\underline{\sigma^{S_n}(-t)(\gamma)}), \mathcal{T}_{s_i \leftarrow 0}^{\sigma^{S_n}(-t)(\gamma)} r_0 S_n(s_i) \right\rangle_{T_{\sigma^{S_n}(-t,s_i)(\gamma)}}$$

exists $L^2(\nu)$.

Step 4. Having in mind that $\varphi_0 \equiv 1$ and that $((\psi_t - \psi_0)\dot{\varphi}_0)^2 \in L^1(\nu)$ is dominated by $4C_4^2(\dot{\varphi}_0)^2 \in L^1(\nu)$, the existence of $\frac{d}{dt}\big|_0 \varphi_t$ and $\frac{d}{dt}\big|_0 \psi_t$ in $L^2(\nu)$ (cf. Steps 1 and 3), relations (4.2), (4.3), and

(4.8)
$$\begin{aligned}
&\left\| \frac{\varphi_t \psi_t - \varphi_0 \psi_0}{t} - \dot{\varphi}_0 \psi_0 - \varphi_0 \dot{\psi}_0 \right\|_{L^2(\nu)} \\
&\quad \le C_4 \left\| \frac{\varphi_t - \varphi_0}{t} - \dot{\varphi}_0 \right\|_{L^2(\nu)} + \left\| \frac{\psi_t - \psi_0}{t} - \dot{\psi}_0 \right\|_{L^2(\nu)} + \left\| ((\psi_t - \psi_0)\dot{\varphi}_0)^2 \right\|_{L^1(\nu)}^{1/2}
\end{aligned}$$

imply the existence of $\frac{d}{dt}\big|_0 \left\{ \partial_{S_n} F(\sigma^{S_n}(-t)) \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu} \right\}$ in $L^2(\nu)$.

Step 5. As shown, for example in [11], 4.1, it holds that $\partial_{S_n} \eta = \frac{d}{dt}\big|_0 \eta(\sigma^{S_n}(-t))$, whenever $\eta \in D(\mathbf{D}) \subseteq D(\partial_{S_n})$ and the limit $\lim_{t\to 0}(\eta(\sigma^{S_n}(t)) - \eta)$ exists in $L^2(\nu)$. Since $F \in Y$ belongs to $D(\mathbf{D}^2)$ (see [9], Section 6), we have $\partial_{S_n} F = (\mathbf{D}_1 F)^j \in D(\mathbf{D})$ if $n = j \in \{1, \dots, d\}$ and we have $\partial_{S_n} F = 2^{m/2}(-\mathbf{D}_{\frac{l-1}{2^m}} F + 2\mathbf{D}_{\frac{2l-1}{2^{m+1}}} F - \mathbf{D}_{\frac{l}{2^m}} F)^j \in D(\mathbf{D})$ if $n = d(2^m + l - 1) + j$ for $m \in \{0, 1, \dots\}$, $l \in \{1, \dots, 2^m\}$, $j \in \{1, \dots, d\}$. Therefore, relation (4.1) is a consequence of Step 4, in particular, of (4.8). □

Let $(A, D(A))$ denote the generator of $(\mathcal{E}, D(\mathcal{E}))$. Fix a version $\bar{H}$ of the map $\gamma \to \mathcal{T}_{\cdot \leftarrow 0}^{\gamma}$. Using B.K. Driver's geometrical notation (see [4], Definition 6.2 and

Theorem 9.1), we introduce

$$z_n(\gamma) = \frac{1}{2}\int_0^1 \Big(Ric_{\bar{H}\gamma}\langle S_n\rangle + \hat{\Theta}_{\bar{H}\gamma}\langle S_n\rangle \Big) \cdot dx + \int_0^1 S_n' \cdot dx\,, \quad n \in \mathbb{N},$$

where $\gamma = I(x) \in \mathbf{P}_{m_0}(M)$ for μ-a.e. $x \in X$.

Theorem 4.2. *Let $F \in Y$. We have $F \in D(A)$ and*

$$AF = \sum_{n=1}^{\infty} \lambda_n \left\{ \partial_{S_n}\partial_{S_n} F + z_n \partial_{S_n} F \right\} \quad \nu\text{-a.e.}$$

Proof. Let $G \in Y$. From (3.2), we obtain

$$\mathcal{E}(F,G) = \int \sum_{n=1}^{\infty} \lambda_n \left.\frac{d}{dt}\right|_0 G(\sigma^{S_n}(t))\partial_{S_n} F\, d\nu\,. \tag{4.9}$$

Taking into consideration that $\left.\frac{d}{dt}\right|_0 G(\sigma^{S_n}(t))$ exists in $L^2(\nu)$, that $\partial_{S_n}F = \langle S_n, \mathbf{D}F\rangle_{\mathbb{H}} \in L^2(\nu)$, and that the sum in (4.9) is, by Lemma 2.1, actually a finite one, we obtain

$$\mathcal{E}(F,G) = \left.\frac{d}{dt}\right|_0 \int \sum_{n=1}^{\infty} \lambda_n G(\sigma^{S_n}(t))\partial_{S_n} F\, d\nu\,.$$

Under the substitution $\gamma \to \sigma^{S_n}(-t)(\gamma)$, we get

$$\begin{aligned} \mathcal{E}(F,G) &= \left.\frac{d}{dt}\right|_0 \int G \cdot \sum_{n=1}^{\infty} \lambda_n \partial_{S_n} F(\sigma^{S_n}(-t))\, d\nu \circ \sigma^{S_n}(-t) \\ &= \left.\frac{d}{dt}\right|_0 \int G \cdot \sum_{n=1}^{\infty} \lambda_n \partial_{S_n} F(\sigma^{S_n}(-t)) \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu}\, d\nu\,. \end{aligned}$$

From Lemma 4.1, it can be concluded that

$$\mathcal{E}(F,G) = \int G \cdot \sum_{n=1}^{\infty} \lambda_n \left.\frac{d}{dt}\right|_0 \left\{ \partial_{S_n} F(\sigma^{S_n}(-t)) \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu} \right\} d\nu\,; \tag{4.10}$$

again, take into consideration that the above sum is actually a finite one. Let G^* be an arbitrary function belonging to $D(\mathcal{E})$. As Y is by Theorem 2.2 dense in $D(\mathcal{E})$ with respect to $\mathcal{E}_1^{1/2}$-norm, we find a sequence $G_m \in Y$, $m \in \mathbb{N}$, with $G_m \underset{m\to\infty}{\longrightarrow} G^*$ in $\mathcal{E}_1^{1/2}$-norm. In particular, $G_m \underset{m\to\infty}{\longrightarrow} G^*$ in $L^2(\nu)$. On account of $\left|\mathcal{E}(G_m - G^*, F) + (G_m - G^*, F)_{L^2(\nu)}\right| = |\mathcal{E}_1(G_m - G^*, F)| \underset{m\to\infty}{\longrightarrow} 0$, we verify $\mathcal{E}(G_m, F) \underset{m\to\infty}{\longrightarrow} \mathcal{E}(G^*, F)$. Now, relation (4.10) and Lemma 4.1 yield

$$\mathcal{E}(F,G^*) = \int G^* \cdot \sum_{n=1}^{\infty} \lambda_n \left.\frac{d}{dt}\right|_0 \left\{ \partial_{S_n} F(\sigma^{S_n}(-t)) \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu} \right\} d\nu\,.$$

Therefore, we have $F \in D(A)$ and

$$AF = -\sum_{n=1}^{\infty} \lambda_n \left.\frac{d}{dt}\right|_0 \left\{ \partial_{S_n} F(\sigma^{S_n}(-t)) \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu} \right\} \quad \nu\text{-a.e.}$$

From Lemma 4.1, it can be deduced that

$$AF = \sum_{n=1}^{\infty} \lambda_n \left\{ \partial_{S_n}\partial_{S_n}F - \frac{d}{dt}\bigg|_0 \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu} \cdot \partial_{S_n}F \right\} \quad \nu\text{-a.e.} \tag{4.11}$$

According to [4], Lemma 9.2, or [9], proof of Theorem 5.1, it holds that

$$\frac{d}{dt}\bigg|_0 \frac{d\nu \circ \sigma^{S_n}(-t)}{d\nu} = -z_n \quad \nu\text{-a.e.}$$

From (4.11), it follows that

$$AF = \sum_{n=1}^{\infty} \lambda_n \left\{ \partial_{S_n}\partial_{S_n}F + z_n \partial_{S_n}F \right\} \quad \nu\text{-a.e.}$$

□

Remark. (5) Keeping in mind that, for $F \in Y$, the sum $\sum_{n=1}^{\infty} \lambda_n z_n \partial_{S_n} F$ is actually a finite sum (cf. Lemma 2.1), we can verify the following identity:

$$\begin{aligned}
&\sum_{n=1}^{\infty} \lambda_n z_n \partial_{S_n} F(\gamma) \\
&\quad = \sum_{n=1}^{\infty} \lambda_n \left\{ \frac{1}{2} \int_0^1 \left(Ric_{\bar{H}\gamma}\langle S_n \rangle + \hat{\Theta}_{\bar{H}\gamma}\langle S_n \rangle \right) \cdot dx + \int_0^1 S_n' \cdot dx \right\} \partial_{S_n} F(\gamma) \\
&\quad = \frac{1}{2} \int_0^1 \left(Ric_{\bar{H}\gamma}\langle \textstyle\sum \lambda_n \, \partial_{S_n} F \, S_n \rangle + \hat{\Theta}_{\bar{H}\gamma}\langle \textstyle\sum \lambda_n \, \partial_{S_n} F \, S_n \rangle \right) \cdot dx \\
&\qquad + \int_0^1 (\textstyle\sum \lambda_n \, \partial_{S_n} F \, S_n') \cdot dx \\
&\quad = \frac{1}{2} \int_0^1 \left(Ric_{\bar{H}\gamma}\langle (\mathbf{AD}F)(s)(\gamma) \rangle + \hat{\Theta}_{\bar{H}\gamma}\langle (\mathbf{AD}F)(s)(\gamma) \rangle \right) \cdot dx_s \\
&\qquad + \int_0^1 (\mathbf{AD}F)'(s)(\gamma) \cdot dx_s
\end{aligned}$$

ν-a.e., where $\gamma = I(x) \in \mathbf{P}_{m_0}(M)$ for μ-a.e. $x \in X$.

5. Local second moment

We begin this section with a general lemma. For a moment, we introduce a new setting more general than the situation in Sections 1-4. Let E be a Hausdorff topological space and let $\mathcal{B}(E)$ denote its Borel σ-algebra. Suppose, furthermore, that $\mathcal{B}(E) = \sigma(C(E))$ where $C(E)$ denotes the set of all continuous functions on E. Let ν be a probability measure on $(E, \mathcal{B})$ and $(\mathcal{E}, D(\mathcal{E}))$ a quasi-regular Dirichlet form on $L^2(E, \nu)$. Let $(A, D(A))$ denote the generator of $(\mathcal{E}, D(\mathcal{E}))$ and assume that there exists a subspace $\mathbb{G} \subseteq D(A)$, dense in $(D(\mathcal{E}), \mathcal{E}_1^{1/2})$, such that $g \in \mathbb{G}$ implies that $g^2 \in D(A)$. Then, according to [3], Proposition I.4.1.3 and Corollary I.4.2.3, there exists a unique carré du champ operator $\Gamma : D(\mathcal{E}) \times D(\mathcal{E}) \to L^1(E, \nu)$. In particular, for all $f, g \in D(\mathcal{E}) \cap L^\infty(E, \nu)$, it holds that

$$\int g\Gamma(f,f)\, d\nu = -\mathcal{E}(g, f^2) + 2\mathcal{E}(fg, f)\,. \tag{5.1}$$

Let $\mathbf{M} = (\Omega, \mathcal{F}, (X_t)_{t\ge 0}, (P_x)_{x\in E})$ denote the associated right process, set $\mathbf{1}(x) = 1$, $x \in E$, and define

$$T_t f(x) := \int f(y)\, P_x(X_t \in dy)\,, \quad t \ge 0,\ x \in E,\ f \in L^2(E,\nu).$$

Proposition 5.1. *Suppose $\mathbf{1} \in \mathbb{G}$, $T_t\mathbf{1} = \mathbf{1}$ for all $t \ge 0$, and $fg \in D(\mathcal{E}) \cap L^\infty(E,\nu)$ if $f \in D(\mathcal{E}) \cap L^\infty(E,\nu)$ and $g \in \mathbb{G}$. Then we have*

$$\begin{aligned}&\lim_{t\to 0} \frac{1}{t} \int (f(y) - f(\cdot))^2\, P_\cdot(X_t \in dy)\\ &\quad = \Gamma(f,f) \quad \textit{weakly in } L^1(E,\nu)\,,\ f \in D(\mathcal{E}) \cap L^\infty(E,\nu).\end{aligned}$$

Proof. Step 1. Let Id denote the identity in $L^2(E,\nu)$ and let $f \in D(\mathcal{E}) \cap L^\infty(E,\nu)$ and $g \in \mathbb{G}$. Since $g \in D(A) \subseteq L^2(E,\nu)$, $f^2 \in L^\infty(E,\nu) \subseteq L^2(E,\nu)$, and $T_t - Id$ is a selfadjoint operator in $L^2(E,\nu)$, $t > 0$, we can conclude

$$\begin{aligned}-\mathcal{E}(g,f^2) &= \big(Ag\,,f^2\big)_{L^2(E,\nu)}\\ &= \left(\lim_{t\to 0}\frac{(T_t - Id)g}{t}\,,f^2\right)_{L^2(E,\nu)}\\ &= \lim_{t\to 0}\left(\frac{(T_t - Id)g}{t}\,,f^2\right)_{L^2(E,\nu)}\\ &= \lim_{t\to 0}\left(g\,,\frac{(T_t - Id)f^2}{t}\right)_{L^2(E,\nu)}.\end{aligned} \tag{5.2}$$

Let $(E_\lambda)_{\lambda\ge 0}$ denote the (right continuous) resolution of the identity with respect to $-A$, i.e.,

$$-Af = \int_{[0,\infty)} \lambda\, dE_\lambda f\,, \quad f \in D(A),$$

recall that A is a nonpositive definite selfadjoint operator in $L^2(E,\nu)$. The closed form $(\mathcal{E}, D(\mathcal{E}))$ has, therefore, a representation

$$\mathcal{E}(f,f) = \int_{[0,\infty)} \lambda\, d\|E_\lambda f\|^2_{L^2(E,\nu)}\,, \quad f \in D(\mathcal{E}).$$

Taking into consideration $-\lambda \le \frac{e^{-\lambda t}-1}{t} \le 0$ if $\lambda \ge 0$ and $t > 0$, we obtain

$$\begin{aligned}\mathcal{E}(fg,f) &= \int_{[0,\infty)} \lambda\, d(E_\lambda(fg), E_\lambda f)_{L^2(E,\nu)}\\ &= -\lim_{t\to 0}\int_{[0,\infty)} \frac{e^{-\lambda t}-1}{t}\, d(E_\lambda(fg), E_\lambda f)_{L^2(E,\nu)}\\ &= -\lim_{t\to 0}\left(\int_{[0,\infty)} dE_\lambda(fg)\,, \int_{[0,\infty)} \frac{e^{-\lambda t}-1}{t}\, dE_\lambda f\right)_{L^2(E,\nu)}\\ &= -\lim_{t\to 0}\left(fg\,, \frac{T_t f - f}{t}\right)_{L^2(E,\nu)}.\end{aligned} \tag{5.3}$$

Combining $T_t\mathbf{1} = \mathbf{1}$, $t \geq 0$, (5.1), (5.2), and (5.3), we verify

$$\begin{aligned}\int g\Gamma(f,f)\,d\nu &= -\mathcal{E}(g,f^2) + 2\mathcal{E}(fg,f)\\ &= \lim_{t\to 0}\int g\cdot\left(\frac{T_tf^2-f^2}{t} - 2f\frac{T_tf-f}{t}\right)d\nu\\ &= \lim_{t\to 0}\int g(x)\cdot\frac{1}{t}\int (f(y)-f(x))^2\,P_x(X_t\in dy)\,\nu(dx)\,. \qquad (5.4)\end{aligned}$$

Step 2. Again, let $f \in D(\mathcal{E}) \cap L^\infty(E,\nu)$. For $g = \mathbf{1}$, relation (5.4) reduces to

$$2\mathcal{E}(f,f) = \lim_{t\to 0}\left\|\frac{1}{t}\int (f(y)-f(\cdot))^2\,P_{\cdot}(X_t\in dy)\right\|_{L^1(E,\nu)}. \qquad (5.5)$$

Setting

$$\varphi_t := \begin{cases}\Gamma(f,f) & \text{if } \ t=0,\\ \dfrac{1}{t}\displaystyle\int (f(y)-f(\cdot))^2\,P_{\cdot}(X_t\in dy) & \text{if } \ t>0\end{cases}$$

from $\varphi_t = \frac{T_tf^2-f^2}{t} - 2f\frac{T_tf-f}{t}$, $t > 0$, and (5.5) it follows that $\|\varphi_t\|_{L^1(E,\nu)}$ is continuous on $[0,\infty)$ and $\lim_{t\to\infty}\|\varphi_t\|_{L^1(E,\nu)} = 0$. Thus, the family $(\varphi_t)_{t\geq 0}$ is uniformly bounded in $L^1(E,\nu)$. Now the statement of the lemma is a consequence of relation (5.4). □

In the remainder of this section, we follow the setting of Sections 1-4. In particular, let $\mathbf{M} = (\Omega, \mathcal{F}, (X_t)_{t\geq 0}, (P_\gamma)_{\gamma\in\mathbf{P}_{m_0}(M)})$ denote the right process associated with $(\mathcal{E}, D(\mathcal{E}))$; cf. Theorem 3.5. Furthermore, recall that $x^v(p)$, $v \in \{1,\ldots,N\}$, denote the standard coordinates of $p \in M$ embedded in $\mathbb{R}^N$. For fixed $s \in [0,1]$ and $v \in \{1,\ldots,N\}$, introduce the function x_s^v by $x_s^v(\gamma) := x^v(\gamma(s))$, $\gamma \in \mathbf{P}_{m_0}(M)$.

Lemma 5.2. *Suppose the validity of relation* (3.3)*, i.e.,*

$$\lambda_i \leq c\, i^{1-\varepsilon}, \quad i\in\mathbb{N}, \quad \textit{for some } c>0 \textit{ and } \varepsilon\in(0,1)\,.$$

Then $x_s^v \in D(\mathcal{E}) \cap L^\infty(\nu)$ *and there exists* $C_6 > 0$ *such that*

$$\mathcal{E}(x_s^v, x_s^v) \leq C_6 \qquad (5.6)$$

for all $s \in [0,1]$ *and* $v \in \{1,\ldots,N\}$.

Proof. Let $v \in \{1,\ldots,N\}$, $s \in [0,1]$, and $s_n \in \{\frac{l}{2^m} : l \in \{1,\ldots,2^m\},\ m \in \mathbb{N}\}$, $n \in \mathbb{N}$, be a sequence with $s_n \underset{n\to\infty}{\longrightarrow} s$. Then

$$x_{s_n}^v \underset{n\to\infty}{\longrightarrow} x_s^v \quad \nu\text{-a.e.} \qquad (5.7)$$

Furthermore, as M is compact, there is a constant $C_7 > 0$ such that

$$\|x_{s_n}^v\|_{L^2(\nu)}^2 \leq C_7\,, \qquad (5.8)$$

independent of $n \in \mathbb{N}$. As in (3.5)-(3.8), it follows from (3.3) that

$$\mathcal{E}(x_{s_n}^v, x_{s_n}^v) \leq C_1\,, \qquad (5.9)$$

independent of $n \in \mathbb{N}$, where C_1 is the constant introduced in (3.7). Now, the above mentioned Banach-Saks property of the Hilbert space $(D(\mathcal{E}), \mathcal{E}_1^{1/2})$ and (5.7)-(5.9) imply $x_s^v \in D(\mathcal{E}) \cap L^\infty(\nu)$ and from the closedness of $(\mathcal{E}, D(\mathcal{E}))$ on $L^2(\nu)$, relation (5.6) with $C_6 := C_1 + C_7$ can be derived. □

Set $\mathbb{G} := Y$. According to Theorems 2.2 and 4.2, $\mathbb{G}$ is dense in $(D(\mathcal{E}), \mathcal{E}_1^{1/2})$ and we have $\mathbb{G} \subseteq D(A)$. Obviously, $g \in \mathbb{G}$ implies $g^2 \in \mathbb{G}$. Thus, there exists a carré du champ operator Γ and we have (5.1). In order to formulate the following theorem, we notice that, for $h \in L^\infty(\nu)$,

$$\begin{aligned} \int |h| \cdot \Gamma(x_s^v, x_s^v)\, d\nu &\leq 2\|h\|_{L^\infty(\nu)} \cdot \mathcal{E}(x_s^v, x_s^v) \\ &\leq 2C_6 \|h\|_{L^\infty(\nu)}, \end{aligned}$$

independent of $s \in [0,1]$ and $v \in \{1, \ldots, N\}$ (cf. (5.6)), which implies

$$\int_{\gamma \in \mathbf{P}_{m_0}} |h| \cdot \sum_{v=1}^N \int_{s\in[0,1]} \Gamma(x_s^v(\gamma), x_s^v(\gamma))\, ds\, \nu(d\gamma) \leq 2NC_6 \|h\|_{L^\infty(\nu)}. \tag{5.10}$$

Theorem 5.3. *Suppose that relation* (3.3) *is valid, i.e.,*

$$\lambda_i \leq c\, i^{1-\varepsilon}, \quad i \in \mathbb{N}, \quad \textit{for some } c > 0 \textit{ and } \varepsilon \in (0,1).$$

For all $h \in L^\infty(\nu)$, we have

$$\begin{aligned} &\lim_{t\to 0} \int h(\tau) \cdot \frac{1}{t} \int \sum_{v=1}^N \|x^v(\gamma) - x^v(\tau)\|^2_{L^2([0,1],ds)}\, P_\tau(X_t \in d\gamma)\, \nu(d\tau) \\ &= \int h(\tau) \cdot \sum_{v=1}^N \int_{s\in[0,1]} \Gamma(x_s^v, x_s^v)\, ds\, \nu(d\tau). \end{aligned} \tag{5.11}$$

Proof. In order to apply Proposition 5.1, we note that $\mathbf{1} \in \mathbb{G}$, $P_\tau(X_t \in \mathbf{P}_{m_0}(M)) = 1$, $t \geq 0$, $\tau \in \mathbf{P}_{m_0}(M)$, and that because of $\mathbb{G} = Y \subseteq D(\mathcal{E}) \cap L^\infty(\nu)$, we have $fg \in D(\mathcal{E}) \cap L^\infty(\nu)$ if $f \in D(\mathcal{E}) \cap L^\infty(\nu)$ and $g \in \mathbb{G}$.

By virtue of Lemma 5.2 and Proposition 5.1, for all $s \in [0,1]$, $v \in \{1, \ldots, N\}$, and $h \in L^\infty(\nu)$, it holds that

$$\lim_{t\to 0} \int h \cdot \frac{1}{t} \int (x_s^v(\gamma) - x_s^v)^2\, P_\cdot(X_t \in d\gamma)\, d\nu = \int h\Gamma(x_s^v, x_s^v)\, d\nu.$$

Since $\|\frac{1}{t}\int (x_s^v(\gamma) - x_s^v)^2\, P_\cdot(X_t \in d\gamma)\|_{L^1(\nu)}$ is bounded for $t > 0$ (cf. proof of Proposition 5.1), it can be concluded from dominated convergence that, for all $v \in \{1, \ldots, N\}$,

$$\begin{aligned} &\lim_{t\to 0} \int_{s\in[0,1]} \int h \cdot \frac{1}{t} \int (x_s^v(\gamma) - x_s^v)^2\, P_\cdot(X_t \in d\gamma)\, d\nu\, ds \\ &= \int_{s\in[0,1]} \int h\Gamma(x_s^v, x_s^v)\, d\nu\, ds, \quad h \in L^\infty(\nu). \end{aligned}$$

Relation (5.11) is now a direct consequence of (5.10) and Fubini's theorem. □

Remark. (6) Condition (3.3) in Proposition 3.3, Lemma 5.2, and Theorem 5.3 can be weakened. Recalling (3.6) it turns out that it is sufficient to require

$$\sum_{m=1}^\infty \lambda_{d\,2^m} 2^{-m} < \infty$$

instead of (3.3).

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Department of Mathematics and Computer Science, University of Jena, D-07740 Jena, Germany

Current address: Department of Mathematical Sciences, University of Delaware, 501 Ewing Hall, Newark, Delaware 19716-2553

E-mail address: loebus@math.udel.edu