ALMOST COMPLEX MANIFOLDS
AND CARTAN'S UNIQUENESS THEOREM

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ABSTRACT. We present a generalization of Cartan’s uniqueness theorem to the almost complex manifolds.

1. INTRODUCTION

The primary goal of this article is to present a generalization to the almost complex manifolds of the following celebrated theorem of H. Cartan, which is usually called Cartan’s uniqueness theorem (see p. 66, [13]).

Theorem 1.1 (H. Cartan). Let $\Omega$ be a bounded domain in $\mathbb{C}^n$. If a holomorphic mapping $f : \Omega \to \Omega$ satisfies that $f(p) = p$ and $df_p = \text{Id}$ for some $p \in \Omega$, then $f$ is the identity mapping.

In order to state the main theorem of this article, we shall introduce the necessary terminology and concepts.

A pair $(M, J)$ is called an almost complex manifold if $M$ is a $C^\infty$-smooth real manifold and $J$ is a field of endomorphisms of the tangent bundle $TM$ with $J^2 = -\text{Id}$, i.e. for each $p \in M$, $J_p : T_p M \to T_p M$ is an endomorphism with $J_p^2 = -\text{Id}$. We call $J$ an almost complex structure on $M$. Throughout this paper, by a smooth almost complex manifold we mean a manifold with a $C^\infty$-smooth almost complex structure.

Given two almost complex manifolds $(M, J)$ and $(M', J')$, a $C^1$ mapping $f$ from $M$ to $M'$ is said to be $(J, J')$-holomorphic (or simply pseudo-holomorphic, so there is no danger of confusion) if its differential $df : TM \to TM'$ satisfies

\begin{equation}
    df \circ J = J' \circ df
\end{equation}

on $TM$. If $(M, J)$ is a Riemann surface, $f$ is called a pseudo-holomorphic curve. In the case $(M, J)$ is the unit disc $\mathbb{D}$ in $\mathbb{C}$ with the standard complex structure $J_{st}$, we call $f$ a pseudo-holomorphic disc. We denote by $\mathcal{O}_{(J, J')}(M, M')$ the space of $(J, J')$-holomorphic mappings from $M$ to $M'$.

By the existence theorem of pseudo-holomorphic discs (Nijenhuis and Woolf [15]), we can define the Kobayashi pseudo-distance ([8]) and the Kobayashi-Royden pseudo-metric ([16]) for the almost complex manifolds.

Let $(M, J)$ be an almost complex manifold. Given two points $p$ and $q$ in $M$, a finite sequence of pseudo-holomorphic discs $c = \{\phi_j\}_{j=1}^k \subset \mathcal{O}_{(J_{st}, J)}(\mathbb{D}, M)$...
is called a chain of pseudo-holomorphic discs from $p$ to $q$ if there are points $p = p_0, p_1, \ldots, p_k = q$ in $M$ and $a_1, a_2, \ldots, a_k$ in $D$ such that

$$\phi_j(0) = p_{j-1} \quad \text{and} \quad \phi_j(a_j) = p_j$$

for $j = 1, \ldots, k$. For this chain, we define its length $\ell(c)$ by

$$\ell(c) = \log \frac{1 + |a_1|}{1 - |a_1|} + \ldots + \log \frac{1 + |a_k|}{1 - |a_k|}.$$ 

Note that $\log \frac{1 + |z|}{1 - |z|}$ is the Poincaré distance from 0 to $z$ in $D$. The Kobayashi pseudo-distance $d_{(M,J)}$ on $(M,J)$ is then defined by

$$d_{(M,J)}(p,q) = \inf \ell(c),$$

where the infimum is taken over all chains of pseudo-holomorphic discs from $p$ to $q$. The Kobayashi-Royden pseudo-metric $F_{(M,J)}$ is the infinitesimal version of the Kobayashi pseudo-distance defined by

$$F_{(M,J)}(p,v) = \inf \left\{ \frac{1}{|a|} : \phi \in \mathcal{O}(J_{a,v})(D,M) \text{ with } \phi(0) = p, \ d\phi(e) = av \right\},$$

where $e$ is the unit vector in $T_0D$ and $p \in M$ and $v \in T_pM$. We exploit from [10] and [11] the following properties that are exactly the same as in the integrable case ([8] and [10]):

(a) $F_{(M,J)}$ is upper semi-continuous and

$$d_{(M,J)}(p,q) = \inf \int_0^1 F_{(M,J)}(\gamma(t), \gamma'(t))dt,$$

where the infimum is taken over all piecewise smooth paths $\gamma : [0,1] \to M$ with $\gamma(0) = p$ and $\gamma(1) = q$.

(b) Let $f : (M,J) \to (M', J')$ be a pseudo-holomorphic mapping. For any points $p$ and $q$ in $M$ and tangent vector $v \in T_pM$, we have

$$d_{(M',J')}(f(p), f(q)) \leq d_{(M,J)}(p, q)$$

and

$$F_{(M',J')}(f(p), df_p(v)) \leq F_{(M,J)}(p, v).$$

(c) The Kobayashi pseudo-distance $d_{(M,J)}$ is finite and continuous on $M \times M$.

(d) If $d_{(M,J)}$ is a distance, it induces the standard topology on $M$.

We say that $(M,J)$ is (Kobayashi) hyperbolic if $d_{(M,J)}$ is a proper distance. Note that for any neighborhood $U$ of $p \in M$, there is a constant $r > 0$ such that the Kobayashi ball $B_{(M,J)}(p,r) = \{ q \in M : d_{(M,J)}(p,q) < r \}$ is contained in $U$ when $(M,J)$ is hyperbolic.

Now we state our main theorem.

**Theorem 1.2.** Let $(M,J)$ be a $C^\infty$-smooth almost complex manifold. Moreover, $M$ is connected and Kobayashi hyperbolic. Suppose that there is a pseudo-holomorphic mapping $f : M \to M$ with $f(p) = p$ and $df_p = \text{Id}$. Then $f$ is the identity mapping.

The proof of this theorem appears in Section 5. Sections 2, 3 and 4 contain a regularity theorem and derivative estimates for pseudo-holomorphic mappings which will be used in the proof of Theorem 1.2.
2. REGULARITY OF ALMOST COMPLEX-HOLOMORPHIC MAPPINGS

We now study the smoothness of pseudo-holomorphic mappings. Since the problem is local, we assume that our manifold is a domain in a Euclidean space. Let \((\Omega, J) \subset \mathbb{R}^{2n}\) and \((\Omega', J') \subset \mathbb{R}^{2m}\) be domains with almost complex structures \(J \in C^\infty(\overline{\Omega})\) and \(J' \in C^\infty(\overline{\Omega'})\). (If the underlying space of an almost complex manifold is a domain in a Euclidean space, we will call it the almost complex domain.) Assume that \(\Omega\) is bounded and has smooth boundary. Regard \(J\) and \(J'\) as matrix-valued functions on \(\Omega\) and \(\Omega'\), respectively. In this section \(j, k, l, \ldots = 1, 2, \ldots, 2n\) and \(\alpha, \beta, \gamma, \ldots = 1, 2, \ldots, 2m\).

Let \(f : \Omega \rightarrow \Omega'\) be a pseudo-holomorphic mapping of class \(C^1(\overline{\Omega})\). Then \(J' = J' \circ f\) is \(2m \times 2m\) matrix-valued function defined on \(\Omega\) of class \(C^1(\overline{\Omega})\). We will fix \(f\) and simply denote \(J'\) by \(J\) for the rest of this section. Let \(J = (a^b_j)\) and \(J' = (b^\alpha_j)\), where \(a^b_j \in C^\infty(\overline{\Omega})\) and \(b^\alpha_j \in C^1(\overline{\Omega})\).

Denote by \(L^2(\Omega, \mathbb{R}^{2m})\) (resp. \(L^2(\Omega, M_{2m \times 2n}(\mathbb{R}))\)) the space of \(\mathbb{R}^{2m}\)-valued (resp. \(2m \times 2n\) matrix-valued) square integrable functions. For \(g \in L^2(\Omega, \mathbb{R}^{2m})\) and \(\varphi \in L^2(\Omega, M_{2m \times 2n}(\mathbb{R}))\), we write \(g = (g_\alpha)\) and \(\varphi = (\varphi^j_\gamma)\). Define the inner products of \(L^2(\Omega, \mathbb{R}^{2m})\) and \(L^2(\Omega, M_{2m \times 2n}(\mathbb{R}))\) by

\[
(g, h) = \int_\Omega (\sum_\alpha g_\alpha h_\alpha),
\]

\[
(\varphi, \psi) = \int_\Omega \text{trace}(\varphi^\alpha \psi + (J')^\alpha \varphi^\alpha)J' \psi
= \int_\Omega (\sum_\alpha \varphi^\alpha_j \psi_\alpha + \sum_\alpha \beta, \gamma, j \varphi^\alpha_j b^{\beta}_j b^{\alpha}_j \psi^j_\gamma),
\]

where \(g, h \in L^2(\Omega, \mathbb{R}^{2m})\) and \(\varphi, \psi \in L^2(\Omega, M_{2m \times 2n}(\mathbb{R}))\).

For fixed \(f\), we can define the densely defined linear differential operator \(\overline{\partial} : L^2(\Omega, \mathbb{R}^{2m}) \rightarrow L^2(\Omega, M_{2m \times 2n}(\mathbb{R}))\) by

\[
\overline{\partial} g = dg + J' dg J,
\]

where \(dg\) denotes the Jacobian matrix of \(g\). Since \(f\) satisfies equation (1.1), it follows that \(\overline{\partial} f = 0\). The \((\alpha, j)\)-th entry of \(\overline{\partial} g\) can be expressed by

\[
(\overline{\partial} g)^\alpha_j = \frac{\partial g_\alpha}{\partial x_j} + \sum_\beta, k \tilde{b}^{\beta}_{j} \frac{\partial g_\beta}{\partial x_k} a^k_j.
\]

We consider the following linear differential operator \(\vartheta : L^2(\Omega, M_{2m \times 2n}(\mathbb{R})) \rightarrow L^2(\Omega, \mathbb{R}^{2m})\) by

\[
(\vartheta \varphi)_\alpha = -\sum_j \frac{\partial \varphi^{\alpha}_j}{\partial x_j} + \sum_\beta, j, k \tilde{b}^{\alpha}_j a^k_j \frac{\partial \varphi^{\beta}_j}{\partial x_k}.
\]

In fact, the principal part of the formal adjoint operator of \(\overline{\partial}\) is of the form \((I + J'' J') \vartheta\). Replacing \(\varphi\) by \(\overline{\partial} g\), we have

\[
((\vartheta \overline{\partial} g)_\alpha = -\sum_j \frac{\partial \overline{\partial} g_\alpha}{\partial x_j} + \sum_\beta, j, k \tilde{b}^{\alpha}_j a^k_j \frac{\partial \overline{\partial} g_\beta}{\partial x_k} (\overline{\partial} g)^\beta_j.
\]
Applying equation \([2.1]\), we have that
\[
(\partial^2 \overline{g})_\alpha = -\sum_j \frac{\partial^2 g_\alpha}{\partial x_j \partial x_j} - \sum_{\beta,j,k} b_\beta^a a_j^k \left( \frac{\partial^2 g_\beta}{\partial x_j \partial x_k} - \frac{\partial^2 g_\beta}{\partial x_k \partial x_j} \right) + \sum_{\beta,\gamma,j,k,l} b_\beta^a b_\gamma^b a_j^k a_l^j \frac{\partial^2 g_\gamma}{\partial x_k \partial x_l}
\]
where \((Cg)_\alpha\) is part of \((\partial \overline{g})_\alpha\) of lower order given by
\[
(Cg)_\alpha = -\sum_{\beta,j,k} \frac{\partial g_\beta}{\partial x_k} \frac{\partial}{\partial x_j} (b_\beta^a a_j^k)
\]
\[
+ \sum_{\beta,\gamma,j,k,l} b_\beta^a b_\gamma^b a_j^k \frac{\partial g_\gamma}{\partial x_k} \frac{\partial}{\partial x_j} (b_\gamma^b a_j^k)
\]

Remark 2.1. Since \(a_j^k, b_\beta^a\) and its first derivatives are continuous on \(\overline{\Omega}\), it follows that \((Cg)_\alpha \in L^2(\Omega)\) if \(g \in W^{1,2}(\Omega, \mathbb{R}^{2m}) = \bigoplus_{j,k=1}^{2m} W^{1,2}(\Omega)\). In particular, \((Cf)_\alpha \in L^p(\Omega)\) for any \(p \geq 1\).

Let \(p > 2n\). For any positive integer \(k\), we have \(kp > 2n\); hence by Theorem 5.23 in [1], \(W^{k,p}(\Omega)\) is a Banach algebra, i.e. \(uv \in W^{k,p}(\Omega)\) for any \(u\) and \(v\) in \(W^{k,p}(\Omega)\). Additionally, using the chain rule, \(b_\beta^a \in W^{k,p}(\Omega)\) whenever \(f_\alpha \in W^{k,p}(\Omega)\) for each \(\alpha\). Moreover, \((Cf)_\alpha \in W^{k-1,p}(\Omega)\).

For convenience, we let \(A^k_l = \sum \alpha a^k_j a^l_j \in C^\infty(\overline{\Omega})\). In fact, \(A^k_l\) is the \((k,l)\)-th entry of the matrix \(JJ^l\). Since \(\sum_{\beta} b_\beta^a b_\gamma^b = -\delta_{\alpha,\gamma}\), it follows that
\[
(\partial \overline{g})_\alpha = -\sum_j \frac{\partial}{\partial x_j} (\overline{g})_j^\alpha + \sum_{\beta,j,k} b_\beta^a a_j^k \frac{\partial}{\partial x_k} (\overline{g})_j^\beta
\]
\[
= -\sum_j \frac{\partial^2 g_\alpha}{\partial x_j \partial x_j} - \sum_{k,l} A^k_l \frac{\partial^2 g_\alpha}{\partial x_k \partial x_l}
\]
\[
+(Cg)_\alpha
\]
when each \(g_\alpha\) is of class \(C^\infty\). For any \(h \in C^1_0(\Omega)\), we obtain
\[
\int_\Omega (\partial \overline{g})_\alpha h = \sum_j \int_\Omega (\overline{g})_j^\alpha \frac{\partial h}{\partial x_j} - \sum_{\beta,j,k} \int_\Omega (\overline{g})_j^\beta \frac{\partial}{\partial x_k} (b_\beta^a a_j^k h)
\]
\[
= \sum_j \int_\Omega \frac{\partial g_\alpha}{\partial x_j} \frac{\partial h}{\partial x_j} + \sum_{k,l} \int_\Omega \frac{\partial g_\alpha}{\partial x_k} \frac{\partial}{\partial x_l} (A^k_l h)
\]
\[
\int_\Omega (Cg)_\alpha h.
\]

Since \(C^\infty(\Omega)\) is dense in \(W^{1,2}(\Omega)\), we take a sequence \(f^\nu\) in \(C^\infty(\Omega, \mathbb{R}^{2m})\) which converges to \(f\) in \(W^{1,2}(\Omega, \mathbb{R}^{2m})\). Then \((\partial f^\nu)_\alpha\), \((Cf^\nu)_\alpha\) and all the remaining first derivatives of \(f^\nu\) converge to those of \(f\) in \(L^2(\Omega)\). Since \((\partial f)_\alpha = 0\), the sequence of
equations \((2.2)\) for \(f^\nu\) converges to
\[
- \sum_j \int_\Omega \frac{\partial f_\alpha}{\partial x_j} \frac{\partial h}{\partial x_j} - \sum_{k,l} \int_\Omega \frac{\partial f_\alpha}{\partial x_k} \frac{\partial (A^k_l h)}{\partial x_l} = \int_\Omega (C f)_\alpha h
\]
for any \(h \in C_0^1(\Omega)\).

Take the linear partial differential operator \(H = \sum_j \frac{\partial^2}{\partial x_j \partial x_j} + \sum_{k,l} A^k_l \frac{\partial^2}{\partial x_k \partial x_l}\). The symbol of \(H\) is \(% \sum_j \xi_j^2 + \sum_{k,l} A^k_l \xi_k \xi_l = |\xi|^2 + |J \xi|^2%. So \(H\) is strictly elliptic on \(\Omega\) with smooth coefficients. Equation \((2.3)\) means that
\[
H f_\alpha = (C f)_\alpha
\]
in the weak sense.

By our assumption, it follows that \((C f)_\alpha \in L^2(\Omega)\) for each \(\alpha\). By the elliptic regularity theorem (Theorem 8.8 in \([5]\)), we have \(f_\alpha \in W^{2,2}_loc(\Omega)\) for each \(\alpha\).

Let \(p > 2n\). Since \((C f)_\alpha \in L^p(\Omega)\), by the uniqueness of solutions of the Dirichlet problem for the elliptic equation (Corollary 9.18 in \([5]\)), it follows that \(f_\alpha \in W^{2,p}_{loc}(\Omega) \cap C^0(\Omega)\) for each \(\alpha\). From Remark 2.1 we have \((C f)_\alpha \in W^{1,p}_{loc}(\Omega)\); hence Theorem 9.19 in \([5]\) implies that \(f_\alpha \in W^{3,p}_{loc}(\Omega)\) for each \(\alpha\). Simultaneously, \((C f)_\alpha \in W^{2,p}_{loc}(\Omega)\). Repeating our argument, we show that \(f_\alpha \in W^{k,p}_{loc}(\Omega)\) for each positive integer \(k\). By the Sobolev imbedding theorem, we have

**Proposition 2.2.** Let \((M^{2n}, J)\) and \((M^{2m}, J')\) be \(C^\infty\)-smooth almost complex manifolds. Any \(C^1\) pseudo-holomorphic mapping from \(M\) to \(M'\) is of class \(C^\infty\).

For the regularity of pseudo-holomorphic curves \((n = 1)\), see Theorem 3.2.2 in \([12]\) and Theorem 2.2.1 in \([17]\).

### 3. First Order Estimate of Pseudo-Holomorphic Mappings

In this section, we derive the Cauchy estimate for pseudo-holomorphic mappings. For the first order estimate, it suffices to treat the case of pseudo-holomorphic discs.

**Proposition 3.1** (Sikorav \([17]\)). Fix \(r, \eta \in (0, 1)\). Let \(W\) be a bounded domain in \(\mathbb{C}^n\). Then there exist positive constants \(\varepsilon\) and \(C\) with the following property:

If \(\phi : D \rightarrow W\) is a differentiable mapping such that
\[
\frac{\partial \phi}{\partial \bar{z}} + q(\phi) \frac{\partial \phi}{\partial z} = 0,
\]
where \(q : W \rightarrow End_{\mathbb{R}}(\mathbb{C}^n)\) is of class \(C^r\) and \(\|q\|_{C^r} \leq \varepsilon\), then \(\phi\) is of class \(C^{1+r}\) on \(D(1-\eta)\). Moreover,
\[
\|\phi\|_{C^{1+r}(D(1-\eta))} \leq C \|\phi\|_{L^\infty}.
\]

The \(C^0\) and \(C^k\) norms for a \(C^k\) mapping \(f : U \subset \mathbb{R}^n \rightarrow \mathbb{R}^m\) is usually defined by \(\|f\|_{C^0(U)} = \sum_{j=1}^m \sup_{x \in U} |f_j(x)|\) and \(\|f\|_{C^k(U)} = \sum_{j=1}^m \sum_{|\alpha| \leq k} \|D^\alpha f_j\|_{C^0(U)}\), where \(|\cdot|\) is a standard Euclidean norm. For \(0 < r < 1\), the \(C^{k+r}\) (Holder) norm is defined by
\[
\|f\|_{C^{k+r}(U)} = \|f\|_{C^k(U)} + \sum_{j=1}^m \sup_{|\alpha| = k} \sup_{x \neq y \in U} \frac{|D^\alpha f_j(x) - D^\alpha f_j(y)|}{|x - y|^r}.
\]
Note that for a $C^1$ mapping $f : U \subset \mathbb{R}^n \to \mathbb{R}^m$, $\|f\|_{C^1(U)}$ is equivalent to
\[
\|f\|_{C^0(U)} + \sup_{v \in \partial U} |df(v)|.
\]

Now we present:

**Theorem 3.2.** Let $(\Omega, J) \subset \mathbb{R}^{2n}$ and $(\Omega', J') \subset \mathbb{R}^{2m}$ be almost complex domains. For each point $p \in \Omega'$, there is a bounded neighborhood $U$ of $p$ in $\Omega'$ such that \{$\|f\|_{C^1(K)} : f \in \mathcal{O}(J,J')(\Omega,U)$\} is uniformly bounded for any compact subset $K$ of $\Omega$.

**Proof.** First, let us study the pseudo-holomorphic discs in $\Omega'$. Applying a linear change of coordinates and a translation of $\mathbb{R}^{2m}$, we may assume that $p = 0$ and $J'$ coincides with the canonical complex structure at $0$, i.e. $J_0 = J_{st}$. Take a neighborhood $V$ of $0$ such that $J' + J_{st}$ is invertible on $V$.

Suppose that $\phi : D \to V \subset \Omega'$ is a pseudo-holomorphic disc. Then the following equation holds:
\[
\frac{\partial \phi}{\partial x} = J'_\phi \frac{\partial \phi}{\partial \bar{x}}.
\]
Since $\frac{\partial \phi}{\partial \bar{x}} = \frac{\partial \phi}{\partial z} + \frac{\partial \phi}{\partial \bar{z}}$ and $\frac{\partial \phi}{\partial y} = J_{st} \left( \frac{\partial \phi}{\partial z} - \frac{\partial \phi}{\partial \bar{z}} \right)$, we have
\[
(J'_\phi + J_{st}) \frac{\partial \phi}{\partial z} = -(J'_\phi - J_{st}) \frac{\partial \phi}{\partial \bar{z}}.
\]
Defining the mapping $q : V \to \text{End}_{\mathbb{R}}(\mathbb{C}^m)$ by $q(a) = (J'_a + J_{st})^{-1}(J'_a - J_{st})$, we see that (3.1) can be written as
\[
\frac{\partial \phi}{\partial \bar{z}} + q(\phi) \frac{\partial \phi}{\partial z} = 0.
\]
Since $V$ is relatively compact in $\Omega'$, $q$ has the same (Hölder) regularity as that of $J'$ on $V$.

Define the renormalization $q_\beta$ of $q$ by $q_\beta : \beta^{-1}V = \{\beta^{-1}a : a \in V\} \to \text{End}_{\mathbb{R}}(\mathbb{C}^m)$ and $q_\beta(a) = q(\beta a)$ for an arbitrary real number $\beta > 0$. Take a sufficiently small $\beta$ such that $B(0,1) \subset \beta^{-1}V$, equivalently $B(0,\beta) \subset V$. Then for fixed $0 < r < 1$, we have
\[
\|q_\beta\|_{C^r(B(0,1))} = \|q_\beta\|_{C^0(B(0,1))} + \sup_{x,y \in B(0,1)} \frac{|q_\beta(x) - q_\beta(y)|}{|x - y|^r}
\]
\[
= \|q\|_{C^0(B(0,\beta))} + \sup_{x,y \in B(0,1)} \frac{|q(\beta x) - q(\beta y)|}{|\beta x - \beta y|^r} \beta^r
\]
\[
\leq \|q\|_{C^0(B(0,\beta))} + \sup_{x,y \in V} |q(x) - q(y)| \frac{1}{|x - y|^r} \beta^r.
\]
Since $q(0) = 0$, it follows that $\|q\|_{C^0(B(0,\beta))} \to 0$ as $\beta \to 0$. For a sufficiently small $\beta$, we have that $\|q_\beta\|_{C^r(B(0,1))} < \varepsilon$, where $\varepsilon$ is in Proposition 3.1 for the case $W = B(0,1)$. Now a new mapping $\phi_\beta = \beta^{-1}\phi$ satisfies
\[
\frac{\partial \phi_\beta}{\partial \bar{z}} + q_\beta(\phi_\beta) \frac{\partial \phi_\beta}{\partial z} = 0.
\]
Let \( U = B(0, \beta) \). By Proposition 3.1, we can deduce that
\[
\| \phi \|_{C^1(D(1-\eta))} \leq \beta \| \phi \|_{C^{1+r}(D(1-\eta))} \\
\leq C \| \phi \|_{L^\infty} \\
\leq C \| \phi \|_{L^\infty}
\]
for any \( \phi \in \mathcal{O}(J_{\beta} J_{r})(D, U) \).

By 5.4a in [15], there is a constant \( R > 0 \) such that for any vector \( v \in T\Omega \) based on \( K \) with \( |v| \leq R \), there is a pseudo-holomorphic disc \( \phi : D \to \Omega \) such that \( d\phi(e) = v \), where \( e \) is an unit vector in \( T_0D \). For any \( f \in \mathcal{O}(J_{\beta} J_{r})(\Omega, U) \), \( f \circ \phi : D \to U \) is pseudo-holomorphic; hence it follows that \( |df(v)| = |d(f \circ \phi)(e)| \leq \|d(f \circ \phi)\| \leq \|f \circ \phi\|_{C^{1}(D(1-\eta))} \leq C\|f \circ \phi\|_{L^\infty} \leq C\|f\|_{C^{0}} \). Therefore we have
\[
\|f\|_{C^{1}(K)} \sim \|f\|_{C^{0}(K)} + \sup_{x \in K} \sup_{v \in T_{x}U; |v| \leq R} \frac{1}{R}|df(x)| \\
\leq \|f\|_{C^{0}(\Omega)} + \frac{C}{R}\|f\|_{C^{0}(\Omega)} \\
\leq (1 + \frac{C}{R})\|f\|_{C^{0}(\Omega)} .
\]
This proves the theorem. \( \square \)

4. Pseudo-holomorphic jet bundles

In order to prove Theorem 1.2, we need some information about the \( \infty \)-jet of a certain family of pseudo-holomorphic mappings at a given point. These can be obtained by jet bundles.

Gauduchon ([1]) has shown that there is a natural almost complex structure in a pseudo-holomorphic 1-jet bundle such that the lifting of the pseudo-holomorphic mapping is also pseudo-holomorphic. In the first two subsections, we follow Gauduchon’s work (see chapter 4 in [2] and [3]).

4.1. Horizontal distribution. Let \( \pi : E \to M \) be a vector bundle with a linear connection \( \nabla \). For any point \( u \in E_x = \pi^{-1}(x) \), the vertical tangent space \( T^v_u E \) at \( u \) is a subspace of \( T_u E \) whose elements are tangent to \( E_x \). Let \( T^v E = \bigcup_{u \in E} T^v_u E \).

Fix any section \( \xi \in \Gamma(E) \) with \( \xi(x) = u \). For each vector \( X \in T_x M \), we define a lifting \( \tilde{X}_u \) in \( T_u E \) by
\[
\tilde{X}_u = d\xi_x(X) - \nabla_X \xi,
\]
where \( \nabla_X \xi \in E_x \) is considered as an element of \( T^v_u E \). This definition of \( \tilde{X}_u \) is independent of the choices for \( \xi \). Therefore, the horizontal subspace \( H^v_u \) at \( u \) can be uniquely defined as a lifting subspace of \( T_u M \) in \( T_u E \) up to the linear connection \( \nabla \). We call \( H^v = \bigcup_{u \in E} H^v_u \) the horizontal distribution. It is easy to check that \( H^v \) is a smooth distribution and that the following properties hold:

(a) \( T_u E = H^v_u \oplus T^v_u E \) at each \( u \in E \).

(b) Let \( v^\nabla : H^v \oplus T^v E \to T^v E \) be a natural projection (vertical projection). If \( Y \in T_u E \) with \( d\xi_x(X) = Y \) for some section \( \xi \), then \( v^\nabla(Y) = \nabla_X \xi \).

(c) The vertical projection \( v^\nabla \) is also smooth. This means that for any smooth vector field \( X \) of \( TE \), \( v^\nabla(X) \) is a smooth vector field of \( T^v E \).
4.2. Pseudo-holomorphic 1-jet bundle and its almost complex structure.

Given two smooth \((C^∞)\) almost complex manifolds \((M^{2n}, J)\) and \((M'^{2m}, J')\), a \((J,J')\)-holomorphic (or pseudo-holomorphic) 1-jet bundle over \(M \times M'\) is defined by

\[
\mathcal{J}^1_{(J,J')}(M, M') = \bigcup_{(x,y)\in M \times M'} \text{Hom}(J_x, J'_y)(T_xM, T_yM'),
\]

where \(\text{Hom}(J_x, J'_y)(T_xM, T_yM')\) is the space of \((J_x, J'_y)\)-linear transformations from \(T_xM\) to \(T_yM'\). Now \(\pi = \pi_1 \times \pi_2 : \mathcal{J}^1_{(J,J')}(M, M') \to M \times M'\) is a vector bundle of rank \(2mn\). We will frequently use the notation \(\mathcal{J}^1(M, M')\) instead of \(\mathcal{J}^1_{(J,J')}(M, M')\) for simplicity.

Choose any linear connection \(\nabla\) on \(\mathcal{J}^1(M, M')\). We have the canonical identifications

\[
T_u\mathcal{J}^1(M, M') \cong T_{\pi_1(u)}M \times T_{\pi_2(u)}M' \times T_u M \mathcal{J}^1(M, M') \\
\cong T_{\pi_1(u)}M \times T_{\pi_2(u)}M' \times \text{Hom}(J_{\pi_1(u)}M, J_{\pi_2(u)}M').
\]

By this, any tangent vector \(Y \in T_u \mathcal{J}^1(M, M')\) can be decomposed into

\[Y = (X_1, X_2, v^\nabla(Y)),\]

where:

i) \(X_1\) and \(X_2\) are images of the natural projection of \(Y\) into \(T_{\pi_1(u)}M\) and \(T_{\pi_2(u)}M'\), respectively,

ii) \(v^\nabla(Y)\) is considered as an element in \(\text{Hom}(J_{\pi_1(u)}M, T_{\pi_2(u)}M')\).

Now we can define an almost complex structure \(J^\nabla\) on \(\mathcal{J}^1(M, M')\) depending on \(\nabla\) by

\[
J^\nabla(Y) = (J_{\pi_1(u)}X_1, J'_{\pi_2(u)}X_2, J'_{\pi_2(u)} \circ v^\nabla(Y)).
\]

(4.1)

It is easy to see \(v^\nabla(J^\nabla(Y)) = J'_{\pi_2(u)} \circ v^\nabla(Y)\); hence \(J^\nabla\) is well defined. Furthermore, \(J^\nabla\) is a smooth almost complex structure. Hence \((\mathcal{J}^1(M, M'), J^\nabla)\) is also a smooth almost complex manifold.

**Theorem 4.1** (Gauduchon [4]). There is a linear connection \(\nabla\) on \(\mathcal{J}^1(M, M')\) with following property:

For any pseudo-holomorphic mapping \(f : M \to M'\), its lifting \(L(f) : (M, J) \to (\mathcal{J}^1(M, M'), J^\nabla)\) is also pseudo-holomorphic.

4.3. Higher order jet bundles. We can define the \(k\)-jet bundles over \(M \times M'\) inductively. But we need only the local information, so we shall consider the Euclidean case.

Let \((\Omega, J) \subset \mathbb{R}^{2n}\) and \((\Omega', J') \subset \mathbb{R}^{2m}\) be smooth almost complex domains. Let \((x_1, \ldots, x_{2n})\) and \((w_1, \ldots, w_{2m})\) be the standard coordinate systems for \(\mathbb{R}^{2n}\) and \(\mathbb{R}^{2m}\), respectively. Assume that

\[
(*) \quad \{\partial/\partial x_1, \ldots, \partial/\partial x_n\}
\]

is a complex basis of \(T_x \Omega\) for each \(x \in \Omega\).

Condition \((*)\) means that \(\{\partial/\partial x_1, \ldots, \partial/\partial x_n\}\) and its images under \(J_x\) form a real basis of \(T_x \Omega\).
By (\ast) a \((J,J')\)-linear mapping from \(T_2\Omega\) to \(T_y\Omega'\) is completely determined by the images of \(\{\partial/\partial x_1, \ldots, \partial/\partial x_n\}\); hence \(\mathcal{J}^1(\Omega, \Omega')\) is a trivial bundle. From now on, we consider \(\mathcal{J}^1(\Omega, \Omega')\) as an open set \(\Omega \times \Omega' \times \mathbb{R}^{2nm}\) in \(\mathbb{R}^{2(n+m+nm)}\). More precisely, a coordinate mapping is given by

\[
\tau = \left( \pi_1(\tau), \pi_2(\tau), \left[ dw_\alpha \left( \frac{\partial}{\partial x_j} \right) \right]_{\alpha=1,\ldots,2m} \right).
\]

The lifting \(L(f)\) of a pseudo-holomorphic mapping \(f\) is parameterized by

\[
L(f)(x) = \left( x_1, \ldots, x_{2m}, f_1(x), \ldots, f_{2m}(x), \left[ \frac{\partial f_\alpha}{\partial x_j} (x) \right]_{\alpha=1,\ldots,2m, j=1,\ldots,n} \right).
\]

To compare \(\|f\|_{C^l}\) with \(\|L(f)\|_{C^{l-1}}\), we have to consider the partial derivatives of \(f\) that are missing in the above expression of \(L(f)(x)\). Solving the system of linear equations \(J'_f \circ df = df \circ J\) with respect to \(\{\partial f_\alpha/\partial x_j\}_{j>n}\), we have

\[
\frac{\partial f_\alpha}{\partial x_j}(x) = \sum_{\beta=1}^{2m} \sum_{k=1}^{n} A^{\alpha\beta}_{jk}(x, f(x)) \frac{\partial f_\beta}{\partial x_k}(x)
\]

for \(j > n\), where \(A^{\alpha\beta}_{jk}\) is a globally defined \(C^\infty\)-smooth function on \(\Omega \times \Omega'\). Therefore, for each compact subset \(K\) in \(\Omega\) and any positive integer \(l\), there is a suitable constant \(M_l\) depending on \(K\) with

\[
\left\| \frac{\partial f_\alpha}{\partial x_j} \right\|_{C^l(K)} \leq M_l \sum_{\beta=1}^{2m} \sum_{k=1}^{n} \left\| \frac{\partial f_\beta}{\partial x_k} \right\|_{C^{l-1}(K)}
\]

for \(j > n\). We may deduce that

\[
\|f\|_{C^l(K)} \lesssim \|L(f)\|_{C^{l-1}(K)}
\]

uniformly for \(f \in \mathcal{O}(\nu, \nu')(\Omega, \Omega')\).

By the expression \(\|f\|_{C^l(K)} \lesssim \|L(f)\|_{C^{l-1}(K)}\), we also obtain

**Proposition 4.2.** Let \(f, g \in \mathcal{O}(\nu, \nu')(\Omega, \Omega')\) and \(\nu \geq 1\). If \(f\) and \(g\) share the same \(\nu\)-jet at \(p \in \Omega\), then \(L(f)\) and \(L(g)\) share the same \((\nu - 1)\)-jet at \(p\).

We now go to the 2-jet.

Take any linear connection \(\nabla_1\) on \(\mathcal{J}^1(\Omega, \Omega')\). From our assumption (\ast) about \(\Omega\), the pseudo-holomorphic 2-jet bundle over \(\Omega \times \Omega'\) defined by

\[
\mathcal{J}^2(\Omega, \Omega') = \mathcal{J}^1(\nu, \nu')(\Omega, \mathcal{J}^1(\Omega, \Omega'))
\]

is also trivial. Choosing \(\nabla_\nu\), inductively, we can define a pseudo-holomorphic \((\nu+1)\)-jet bundle by

\[
\mathcal{J}^{\nu+1}(\Omega, \Omega') = \mathcal{J}^1(\nu, \nu')(\Omega, \mathcal{J}^{\nu}(\Omega, \Omega')).
\]

For any choice of \(\nabla_\nu\) at each step, \(\mathcal{J}^{\nu}(\Omega, \Omega')\) is always trivial.

From now on, we fix a suitable linear connection \(\nabla_\nu\) as in Theorem 4.1 at each step. Then for a pseudo-holomorphic mapping \(f : \Omega \to \Omega'\), its lifting \(L'(f) = L(L^{\nu-1}(f)) \circ \Omega 	o \mathcal{J}^{\nu}(\Omega, \Omega')\) is always \((\nu, \nu)^{(\nu)}\)-holomorphic.

Given \(f \in \mathcal{O}(\nu, \nu')(\Omega, \Omega')\) and \(p \in \Omega\), a family of mappings defined by

\[
\mathcal{F}^{\nu}_p(f; \Omega, \Omega') = \{ g \in \mathcal{O}(\nu, \nu')(\Omega, \Omega') : g\text{ has the same }\nu\text{-jet with }f\text{ at }p \}
\]

has the following property.
Theorem 4.3. Let \((\Omega, J) \subset \mathbb{R}^{2n}\) and \((\Omega', J') \subset \mathbb{R}^{2m}\) be hyperbolic almost complex domains. Assume that \(\Omega\) satisfies condition (*). For any \(f \in O_{(J,J')}((\Omega, \Omega'))\), there is a neighborhood \(V_0\) of \(p\) such that \(\{L^\nu(g) : g \in F_p^{\nu-1}(f; \Omega, \Omega')\}\) is uniformly bounded on \(V_0\). Moreover, we can find \(V_\nu\) such that \(V_{\nu+1} \subset \subset V_\nu\) for each \(\nu = 1, 2, \ldots, \).

Proof. Choose \(r > 0\) such that the Kobayashi ball \(U = B_{(\Omega, J)}((f(p), r)\) is a bounded neighborhood of \(f(p)\) as in Theorem 3.1. Denote \(V = B_{(\Omega', J')}((f(p), r)\). Since \(F_p^{\nu}(f; \Omega, \Omega') = \{g \in O_{(J,J')}((\Omega, \Omega') : g(p) = f(p)\}\), we have \(g(V) \subset U\) for any \(g \in F_p^{\nu}(f; \Omega, \Omega')\). Take any relatively compact neighborhood \(U_1\) of \(p\) in \(V\).

By Theorem 3.1 \(\{||g||_{C^1(V)} : g \in F_p^{\nu}(f; \Omega, \Omega')\}\) is uniformly bounded so that \(\{L(g) : g \in F_p^{\nu}(f; \Omega, \Omega')\}\) is uniformly bounded on \(V_1\). This proves the case \(\nu = 1\).

Since \((V, J)\) and \((U, J')\) are also Kobayashi hyperbolic, Theorem 3 in [11] implies that every bounded domain in \(J_1^{(J,J')}(V, U)\) is hyperbolic with respect to \(J^{V_1}\). Therefore, we may assume that

\[
\bigcup_{g \in F_p^{\nu}(f; \Omega, \Omega')} L(g)(V_1) \subset \Omega_1,
\]

where \(\Omega_1\) is a hyperbolic neighborhood of \(L(f)(p)\) in \(J_1^{(J,J')}(V, U)\).

Suppose that our theorem holds for the case \(\nu \leq \lambda\). Since the pair \((V_1, J)\) and \((\Omega_1, J^{V_1})\) satisfy the assumption of the theorem, there are neighborhoods \(V'_1, \ldots, V'_\lambda\) of \(p\) in \(V_1\) such that \(\{L^\nu(h) : h \in F_p^{\nu-1}(L(f); V_1, \Omega_1)\}\) is uniformly bounded on \(V'_\nu\) for \(\nu = 1, \ldots, \lambda\), and such that \(V'_1 \subset \subset V'_2 \subset \subset \cdots \subset \subset V'_\lambda\). By Proposition 4.2 we have

\[
L(f_p^{\nu}(f; \Omega, \Omega')) \subset F_p^{\nu-1}(L(f); V_1, \Omega_1)
\]

for any \(\nu\). Therefore \(L^{\nu+1}(g) = L^\nu(L(g))\) is uniformly bounded on \(V_{\nu+1} = V'_\nu\) for \(g \in F_p^{\nu}(f; \Omega, \Omega')\) and for \(\nu = 1, \ldots, \lambda\). This proves the theorem by the induction hypothesis. \(\square\)

For this sequence \(\{V_\nu\}\) of nested neighborhoods of \(p\), we have

Corollary 4.4. \(\{||g||_{C^\nu(V_\nu)} : g \in F_p^{\nu-1}(f; \Omega, \Omega')\}\) is uniformly bounded.

Proof. From (4.3), we have

\[
||g||_{C^\nu(V_\nu)} \lesssim ||L(g)||_{C^{\nu-1}(V_\nu)} \lesssim \cdots \lesssim ||L^\nu(g)||_{C^0(V_\nu)}
\]

uniformly for \(g \in O_{(J,J')}((\Omega, \Omega')\). When \(g \in F_p^{\nu-1}(f; \Omega, \Omega')\), the last term of this inequality is bounded by Theorem 4.3 \(\square\)

5. Proof of Theorem 1.2

Let \((M, J)\) be a connected hyperbolic almost complex manifold of class \(C^\infty\). Suppose that there is a pseudo-holomorphic self-mapping \(f : M \to M\) with \(f(p) = p\) and \(df_p = I\) for some \(p \in M\). From Proposition 2.2, \(f\) is of class \(C^\infty\) and we can compare all partial derivatives of \(f\) with those of the identity mapping. To prove that \(f\) is the identity, we need the unique continuation property for pseudo-holomorphic mappings.

Proposition 5.1. Let \((M, J)\) and \((M', J')\) be smooth almost complex manifolds. Moreover \(M\) is connected. Suppose that two pseudo-holomorphic mappings \(f, g : M \to M'\) share the same \(\infty\)-jet at some point in \(M\). Then \(f \equiv g\) on \(M\).
Proof. It is sufficient to prove that \( A = \{ p \in M : f \text{ and } g \text{ share the same } \infty \text{-jet at } p \} \) is open. Then our assertion follows, since \( A \) is open, closed and nonempty set.

Suppose that \( p \in A \). There is a neighborhood \( U_p \) of \( p \) such that any point \( q \) in \( U_p \) can be joined to \( p \) by a single pseudo-holomorphic disc \((6) \text{ and } (10)\). Take any \( q \) in \( U_p \) and suppose that there is a pseudo-holomorphic disc \( \phi : D \to M \) with \( \phi(0) = p \) and \( \phi(1/2) = q \). Since \( p \in A \), the two pseudo-holomorphic discs \( f \circ \phi, g \circ \phi : D \to M' \) share the same \( \infty \text{-jet at } 0 \). By the unique continuation property of pseudo-holomorphic curves (see [3] and [12]), it holds that \( f \circ \phi \equiv g \circ \phi \). Furthermore \( f(q) = g(q) \). Since \( q \) is an arbitrary point in \( U_p \), we have \( f|_{U_p} \equiv g|_{U_p} \). Hence \( p \in U_p \subset A \), and \( A \) is open. This proves the proposition. \( \square \)

By Proposition 5.1, it is sufficient to prove that \( D^\alpha f_j(p) = 0 \) for any \( j \) and any multi-indices \( |\alpha| \geq 2 \). Then \( f \) has the same \( \infty \text{-jet with the identity mapping.} \) Therefore \( f \) is the identity mapping.

Choose a local coordinate system \( \varphi : (V, 0) \to (M, p) \) about \( p \) with \( \varphi(V) \subset M \). Since the Kobayashi distance function \( d_{(M,J)} \) is continuous, we can take a positive real number \( r < \min_{q \in \partial \varphi(V)} d_{(M,J)}(p,q) \). Then the Kobayashi ball \( B_{(M,J)}(p,r) \) is contained in \( \varphi(V) \). By the distance-decreasing property of the Kobayashi distance, we have \( f(B_{(M,J)}(p,r)) \subset B_{(M,J)}(p,r) \) for all \( r \). Now we identify \( p = 0 \), \( \varphi(V) = V \) is a bounded domain in \( \mathbb{R}^{2n} \) and \( J = \varphi^*J = (d\varphi)^{-1} \circ J \circ d\varphi \) is an induced almost complex structure on \( V \). For sufficiently small \( r \) we may assume that \( U = \varphi^{-1}(B_{(M,J)}(p,r), J) \) satisfies condition \( (\ast) \) in Section 4.

Consider an iterated family \( \{ f^m = f \circ f^{m-1} \}_{m=1,2,\ldots} \) of \( f \). Note that \( f|_{U} \) is in \( \mathcal{O}_{(J,J)}(U, U) \), so is \( f^m|_{U} \). Now we have

**Proposition 5.2.** \( (D^\alpha (f^m)_j)(0) = m(D^\alpha f_j)(0) \) for \( |\alpha| = 2 \).

**Proof.** Since \( d(f^m)_0 = (df_0)^m = \text{Id} \), we have

\[
(\mathbf{5.1}) \quad \frac{\partial (f^m)_j}{\partial x_k}(0) = \delta_{j,k}
\]

for \( m = 1, 2, \ldots \).

Let \( D^\alpha = \frac{\partial^2}{\partial x_{\alpha_1} \partial x_{\alpha_2}} \). Since \( (f^m)_j = f_j \circ f^{m-1} \), we have

\[
\frac{\partial^2}{\partial x_{\alpha_1} \partial x_{\alpha_2}} (f^m)_j(0) = \frac{\partial}{\partial x_{\alpha_1}} \left( \sum_{k=1}^{2n} \frac{\partial f_j}{\partial x_k} (f^{m-1}(x)) \frac{\partial (f^{m-1})_k}{\partial x_{\alpha_2}}(x) \right)(0)
\]

\[
= \sum_{k,l=1}^{2n} \frac{\partial^2 f_j}{\partial x_l \partial x_k} (f^{m-1}(0)) \frac{\partial (f^{m-1})_l}{\partial x_{\alpha_1}}(0) \frac{\partial (f^{m-1})_k}{\partial x_{\alpha_2}}(0)
\]

\[
+ \sum_{k=1}^{2n} \frac{\partial^2 f_j}{\partial x_k^2} (f^{m-1}(0)) \frac{\partial^2 (f^{m-1})_k}{\partial x_{\alpha_1} \partial x_{\alpha_2}}(0)
\]

\[
= \frac{\partial^2 f_j}{\partial x_{\alpha_1} \partial x_{\alpha_2}}(0) + \frac{\partial^2 (f^{m-1})_j}{\partial x_{\alpha_1} \partial x_{\alpha_2}}(0),
\]

where the last equality follows by (5.1). This equation proves the case of \( |\alpha| = 2 \) by induction.
Suppose that $D^\alpha f_j(0) = 0$ for any $2 \leq |\alpha| < \nu$ and $j = 1, \ldots, 2n$. Let $|\beta| = \nu$ and $D^\beta = \partial^\nu / \partial x_{\beta_1} \cdots \partial x_{\beta_{\nu}}$. From (5.2), we obtain

$$D^\beta (f^m)_j(0) = \frac{2^n}{\gamma_1 \cdots \gamma_{\nu} = 1} \frac{\partial^\nu f_j}{\partial x_{\gamma_1} \cdots \partial x_{\gamma_{\nu}}} (f^{m-1}(0)) \frac{\partial (f^{m-1})_{\gamma_1}}{\partial x_{\beta_1}} (0) \cdots \frac{\partial (f^{m-1})_{\gamma_{\nu}}}{\partial x_{\beta_{\nu}}} (0)$$

+ (terms which contain $D^\alpha f_j$ for $2 \leq |\alpha| < \nu$)

$$+ \frac{\partial f_j}{\partial x_k} (f^{m-1}(0)) \frac{\partial^\nu (f^{m-1})_{\nu}}{\partial x_{\beta_1} \cdots \partial x_{\beta_{\nu}}} (0)$$

$$= \frac{\partial^\nu f_j}{\partial x_{\beta_1} \cdots \partial x_{\beta_{\nu}}} (0) + \frac{\partial^\nu (f^{m-1})_j}{\partial x_{\beta_1} \cdots \partial x_{\beta_{\nu}}} (0)$$

$$= D^\beta f_j(0) + D^\beta (f^{m-1})_j(0).$$

This proves the proposition. \(\square\)

We are now ready to complete the proof of Theorem 1.2.

Suppose that $D^\alpha f_j(0) \neq 0$ for some multi-index $\alpha$ with $|\alpha| = 2$ and some $j$. By Proposition 5.2, we have $|(D^\alpha (f^m)_j)(0)| = m|(D^\alpha f_j)(0)| \to \infty$ as $m \to \infty$. Since $f^m(0) = f(0) = 0$ and $d(f^m)_0 = df_0 = I_d$, we have $f^m \in F_f(f;U,U)$ for each $m$. Corollary 4.4 implies that $\{|(D^\alpha (f^m)_j)(0)|\}_{m=1,2,\ldots}$ must be bounded. Therefore it follows that $D^\alpha f_j(0) = 0$ for each $|\alpha| = 2$ and $j$.

Inductively let us assume that $D^\beta f_j(0) \neq 0$ and $D^\alpha f_k(0) = 0$ for $2 \leq |\alpha| < |\beta| = \nu$ and $k = 1, \ldots, 2n$. Proposition 5.2 implies that $|(D^\alpha (f^m)_k)(0)| = m|(D^\alpha f_k)(0)| = 0$ for $2 \leq |\alpha| < \nu$ and $k = 1, \ldots, 2n$. Hence it follows that $f^m \in F_{f_0}^{\nu-1}(f;U,U)$. But Proposition 5.2 also means that $|(D^\beta (f^m)_j)(0)| = m|(D^\beta f_j)(0)| \to \infty$ as $m \to \infty$. It is a contradiction to Corollary 4.4. Therefore we have $D^\alpha f_j(0) = 0$ for any $|\alpha| \geq 2$.

Consequently $f$ has same $\infty$-jet with the identity mapping at 0. This proves Theorem 1.2. \(\square\)

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