CORRIGENDUM TO “WEST’S PROBLEM ON EQUIVARIANT HYPERSPACES AND BANACH-MAZUR COMPACTA”

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In our article [1], on p. 3389, the definition of the weak topology of the \( G \)-nerve \( \mathcal{N}(\mathcal{U}) \) contains a gap. Namely, it is claimed there that the topology on \( \mathcal{N}(\mathcal{U}) \) induced from \( \mathcal{J} \) is the weak one, which is false. The author apologizes for this mistake.

Nevertheless, in the proofs of Lemmas 4.2, 4.4 and 5.2, where the topology of \( \mathcal{N}(\mathcal{U}) \) is essential, in fact the right weak topology of \( \mathcal{N}(\mathcal{U}) \) was applied. Thus, all of the proofs given in [1] are correct and complete.

Using the notation and references adopted in [1], the above-mentioned gap in the definition of the topology of the \( G \)-nerve \( \mathcal{N}(\mathcal{U}) \) may be filled by replacing the text on p. 3389 starting in line 26 and ending in line 35, by the following.

“For every simplex \( L = \langle \mu_0, \ldots, \mu_n \rangle \subset \tilde{\mathcal{N}}(\mathcal{U}) \), set

\[
\Delta(L) = \bigcup \{ \Delta(S, F_S) \mid S \text{ is a subsimplex of } L \}.
\]

Clearly, \( \Delta(L) \) is an invariant subset of the finite join \( G/H_{\mu_0} \ast \cdots \ast G/H_{\mu_n} \). We always will consider the induced topology and \( G \)-action on \( \Delta(L) \). Observe that, if \( N \) is a subsimplex of \( L \), then \( \Delta(N) \) is a closed invariant subset of \( \Delta(L) \). Indeed, let \( \xi : G/H_{\mu_0} \ast \cdots \ast G/H_{\mu_n} \to L \) be the continuous map sending the point \( \sum_{i=0}^{n} t_{\mu_i}g_{H_{\mu_i}} \in G/H_{\mu_0} \ast \cdots \ast G/H_{\mu_n} \) to the point \( \sum_{i=0}^{n} t_{\mu_i} \mu_i \in L \). Since \( P_{L, N}(F_L) \subset F_N \), where \( P_{L, N} : \prod_{\mu \in L} G/H_{\mu} \to \prod_{\mu \in N} G/H_{\mu} \) is the Cartesian projection, we see that the preimage \( \xi^{-1}(N) \) is just \( \Delta(N) \). Since \( N \) is closed in \( L \), this yields that \( \Delta(N) \) is closed in \( \Delta(L) \), as required. Invariance of \( \Delta(N) \) is evident.

It is clear that, if \( K \subset \tilde{\mathcal{N}}(\mathcal{U}) \) is yet another simplex, then \( \Delta(L) \cap \Delta(K) = \Delta(L \cap K) \). Consequently, \( \Delta(L) \cap \Delta(K) \) is closed in both \( \Delta(L) \) and \( \Delta(K) \).

Consider the following invariant subset of \( \mathcal{J} \):

\[
\mathcal{N}(\mathcal{U}) = \bigcup \{ \Delta(L) \mid L \in \tilde{\mathcal{N}}(\mathcal{U}) \}.
\]

We consider the weak topology on \( \mathcal{N}(\mathcal{U}) \) determined by the family

\[
\{ \Delta(L) \mid L \in \tilde{\mathcal{N}}(\mathcal{U}) \}.
\]

Namely, a set \( U \subset \mathcal{N}(\mathcal{U}) \) is, by definition, open in \( \mathcal{N}(\mathcal{U}) \) if and only if \( U \cap \Delta(L) \) is open in \( \Delta(L) \) for every simplex \( L \subset \tilde{\mathcal{N}}(\mathcal{U}) \).

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The $G$-action on $\mathcal{N}(U)$, defined by the following formula, makes $\mathcal{N}(U)$ a $G$-space, called the $G$-nerve of $U$:

$$g \ast \left( \sum_{\mu \in M} t_{\mu}g_{\mu}^tH_{\mu} \right) = \sum_{\mu \in M} t_{\mu}gg_{\mu}H_{\mu}, \quad g \in G.$$ 

Since the intersection $\Delta(L) \cap \Delta(K)$ is closed in both $\Delta(L)$ and $\Delta(K)$, we see that each space $\Delta(L)$ retains its original topology and is a closed invariant subset of $\mathcal{N}(U)$ (see, e.g., [2] Ch. VI, §8). We call $\Delta(L)$ a $G$-n-simplex over the n-simplex $L$.

In the proofs of Lemmas 4.4 and 5.2 the following well-known and easily proved property of the weak topology is used: a map $f : \mathcal{N}(U) \to Z$ is continuous if and only if each restriction $f_{|_{\Delta(L)}}$ is continuous."

As is defined on page 3389, lines 9–10, the elements of a $G$-normal cover $U$ are tubular slice-sets $gS_\mu$ with companion groups $H_\mu$. However, in order to emphasize the role of $H_\mu$, we have used the denotation $(gS_\mu, H_\mu)$ instead of $gS_\mu$, which in some occasions may cause confusion. Thus, in Lemmas 4.1, 4.2 and 5.2 the denotation $U = \{(gS_\mu, H_\mu) \mid g \in G, \mu \in M\}$ should be replaced by $U = \{gS_\mu \mid g \in G, \mu \in M\}$, where $S_\mu$ is an $H_\mu$-slice.

For the proof of Lemma 5.2 it is important to formulate Lemma 4.1 in the following more precise form.

**Lemma 4.1.** Let $X$ be a paracompact $G$-space and $\mathcal{V}$ an open cover of $X$. Then $X$ admits a $G$-normal cover $U = \{gS_\lambda \mid g \in G, \lambda \in \Lambda\}$ with the companion groups $\{H_\lambda\}_{\lambda \in \Lambda}$ such that each $H_\lambda$ is the stabilizer of a point $x_\lambda \in S_\lambda$ and $U$ is a star-refinement of $\mathcal{V}$.

In Lemma 5.2 under the term “$\varepsilon$-cover” we mean the family of all open balls in $L_0(n)$ which have radius $\varepsilon$.

In the formulation and in the proof of Lemma 5.2, always $G = O(n)$.

Also, one should correct the following misprints:

1. page 3389, line 14: “$O \in U_1$” should be “$O \in U_1$”.
2. page 3389, line 20: “of $U$” should be “of $\mathcal{U}$”.
3. page 3390, line 27: “$(gS_x, G_x)$” should be “$gS_x$”.
4. page 3390, line 30: “an open $G$-normal cover” should be “a $G$-normal cover”.
5. page 3391, line 28: “$\Delta(L, F_L)$” should be “$\Delta(L)$”.
6. page 3392, line 2: “$R(x) \in F_{3n-1}(s)$” should be “$R(x) \in F_{3n-1}(s^1)$, where $s^1$ is the 1-dimensional skeleton of $s$”.
7. page 3392, line 7: “$\Delta(L, F_L)$” should be “$\Delta(L)$”.
8. page 3392, line 13: “$F_{3n-1}(s)$” should be “$F_{3n-1}(s^1)$”.
9. page 3394, line 30: “$q'(g_1A_\lambda) = q'(g_0A_\lambda)$” should be “$q'(g_1H_\mu) = q''(g_0H_\lambda)$”.
10. page 3394, line 36: “of $g_0A_\lambda \cup g_1A_\mu$”.
11. page 3394, line 41: “of $g_1A_\mu$” should be “of $g_0A_\lambda \cup g_1A_\mu$”.
12. page 3398, line 22: “domain. Since” should be “domain $V$. Since”. 
References


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