A CORRECTION TO:
“SOME CHARACTERIZATIONS OF SPACE-FORMS”

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In [1], for the application to Theorem 4.3, one needs a version of Proposition 3.1 assuming that the manifold supports an inhomogeneous Poincaré-Sobolev inequality of the type (0.1) below. Indeed, an easy modification of the original argument yields the following

Proposition 3.1 bis. Let \((M, \langle \cdot, \cdot \rangle)\) be a complete manifold and assume that, for some \(0 \leq \alpha < 1\) and some non-negative function \(h\), the inhomogeneous Sobolev–Poincaré type inequality

\[
\int_M (|\nabla \phi|^2 + h \phi^2) \geq S(\alpha)^{-1} \left\{ \int_M |\phi|^{\frac{2\alpha}{1-\alpha}} \right\}^{1-\alpha}
\]

holds for every \(\phi \in C_0^\infty(M)\) with a positive constant \(S(\alpha) > 0\). Suppose that \(\psi \in \text{Lip}_{\text{loc}}(M)\) is a positive solution of

\[
\psi \Delta \psi + a(x) \psi^2 + A|\nabla \psi|^2 \geq 0 \quad \text{(weakly) on } M
\]

satisfying

\[
\int_{B_r} |\psi|^\sigma = o(r^2) \quad \text{as } r \to +\infty
\]

with \(A \in \mathbb{R}, \sigma - A - 1 > 0, \sigma \neq 0, \text{ and } a(x) \in C^0(M)\). Then

\[
\left\| a_+ (x) + \frac{4(\sigma - A - 1)}{\sigma^2} h \right\|_{L^\frac{\sigma}{\sigma-1}(M)} \geq \frac{4(\sigma - A - 1)}{\sigma^2} S(\alpha)^{-1}.
\]

Furthermore, if \(\psi\) is assumed to be non-negative and not identically zero, then (0.4) holds under the further assumption that \(\sigma > 0\).

Thus the statement of Theorem 4.3 should read

Theorem 4.3. Let \(f : (M^m, \langle \cdot, \cdot \rangle) \to (N^n, \langle \cdot, \cdot \rangle)\) be an isometric immersion of the complete manifold \(M\) of dimension \(m \geq 3\) into the Cartan-Hadamard manifold \(N\) whose sectional curvature \((\text{along } f)\) satisfies

\[
(0 \geq) \quad N \text{Sect}_{f(x)} \geq -^N R(x)
\]

for some function \(^N R \in C^0(M)\). Denote by \(H\) and \(II\), respectively, the mean curvature vector field and the second fundamental tensor of \(f\). Assume that, for
some $\varepsilon > 0$,
\[ \| \varepsilon^2 |H|^2 + (m - 1)^N R(x) + |\{ |II| + m |H| \}(x) \|_{L^\infty} < \frac{m}{m-1} S_2(m, \varepsilon)^{-1}. \]
Then $M$ has only one end.

Correspondingly, (4.12) in Remark 4.4 must read
\[ \| \varepsilon^2 |H|^2 + q(x) \|_{L^\infty} \leq S_2(m, \varepsilon)^{-1}. \]

Versions of Theorems 4.5 and 4.6 based on Proposition 3.1 bis can also be formulated. We omit the details.

Finally, the assumption that $N$ has non-positive sectional curvature must be added in the statement of Theorem 4.6.

REFERENCES