Mathematical Research . . . it's KNOT what you think!

AMS
AMERICAN MATHEMATICAL SOCIETY
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KNOTS, MOLECULES AND STICK NUMBERS

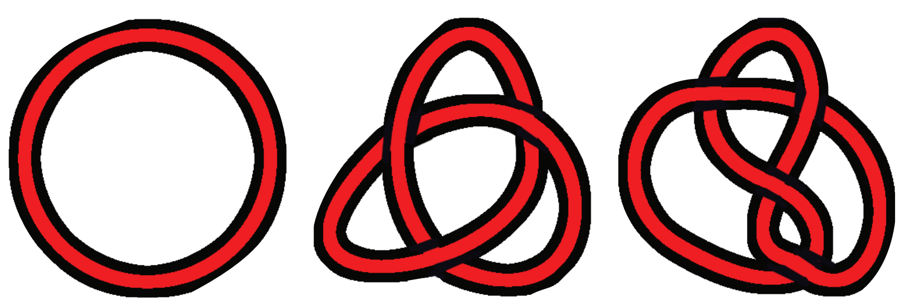
TAKE A PIECE OF STRING.

Tie a knot in it, and then glue the two loose ends of the string together. You have now trapped the knot on the string. No matter how long you attempt to disentangle the string, you will never succeed without cutting the string open.

Thanks to COLIN ADAMS: plus.maths.org

INEQUIVALENT KNOTS

THE TRIVIAL KNOT, TREFOIL KNOT, AND FIGURE OF EIGHT KNOT

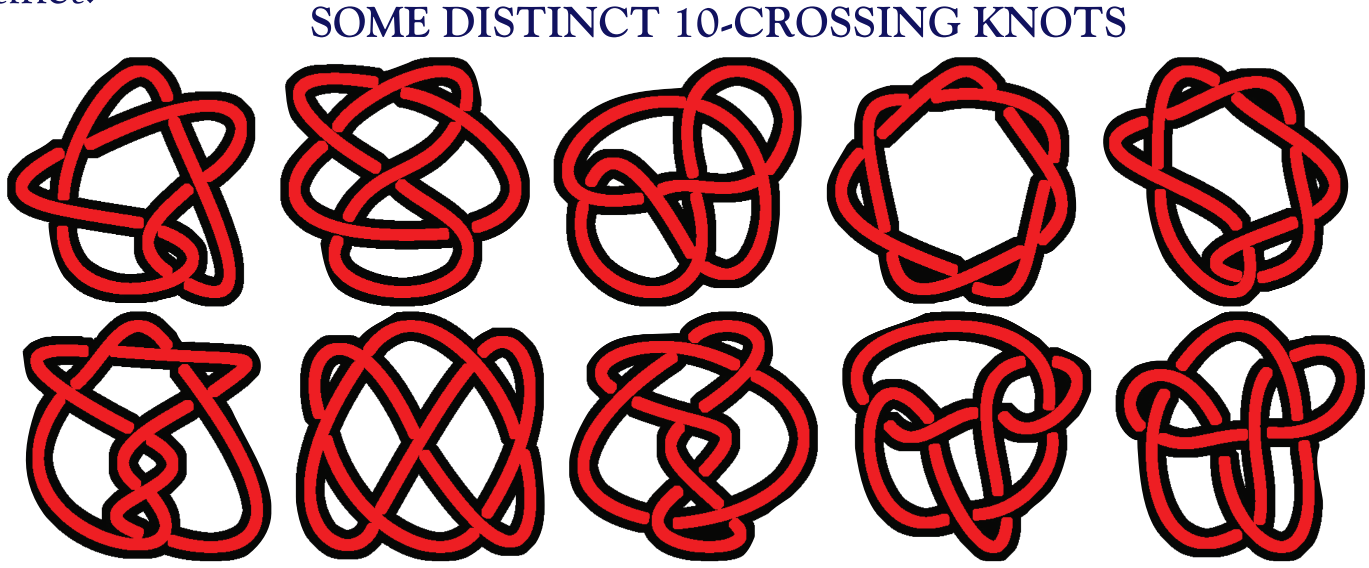


Here are some famous knots, all known to be **INEQUIVALENT**. In other words, none of these three can be rearranged to look like the others. However, proving this fact is difficult. This is where the math comes in. The simple loop of string on the top is the **UNKNOT**, also known as the trivial knot. The **TREFOIL KNOT** and the **FIGURE-EIGHT KNOT** are the two simplest nontrivial knots, the first having three crossings and the second, four. ***No other nontrivial knots can be drawn with so few crossings.***

DISTINCT KNOTS

Over the years, mathematicians have created tables of knots, all known to be distinct.

SOME DISTINCT 10-CROSSING KNOTS



So far, over **1.7 million inequivalent knots** with pictures of 16 or fewer crossings have been identified.

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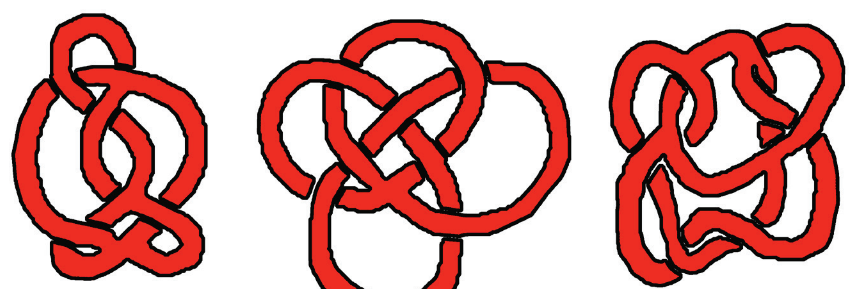
Knots, Molecules and Stick Numbers

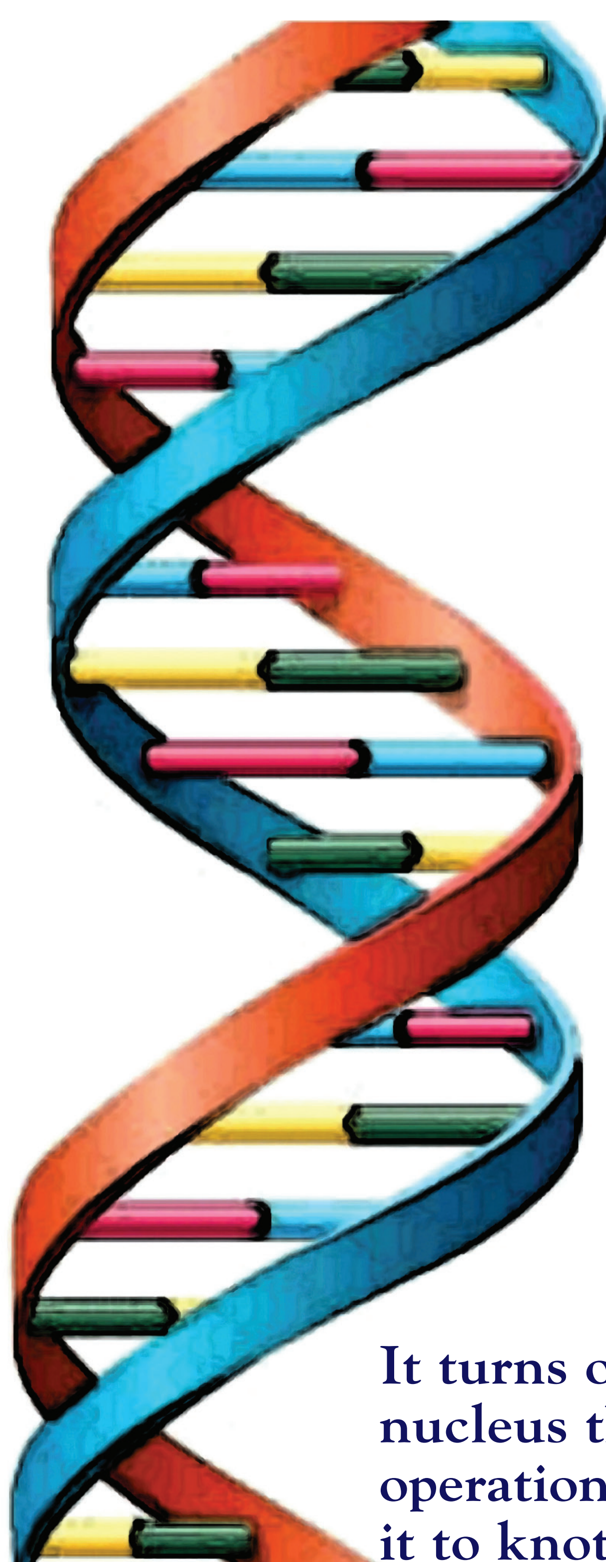
THE MATHEMATICAL THEORY OF KNOTS WAS BORN OUT OF ATTEMPTS TO MODEL THE ATOM.

Near the end of the nineteenth century, Lord Kelvin suggested that different atoms were actually different knots tied in the ether that was believed to permeate all of space. Physicists and mathematicians set to work making a table of distinct knots, believing they were making a table of the elements. **DO DIFFERENT KNOTS CORRESPOND TO DIFFERENT ELEMENTS?**



Pb? **Na?** **Fe?**

In 1887, the Michelson-Morley experiment demonstrated there was no such ether, and the physicists and chemists lost interest in knots for the next century. But for mathematicians, it was too late. They were hooked on the intrinsic beauty of the field and the mathematics that it engenders, and the theoretical aspects of knots continued to be developed. In the 1980s, chemists and biochemists again became intrigued with the theory of knots. One of the important applications is to the molecule deoxyribonucleic acid (DNA).



DNA consists of millions of atoms that together form a blueprint for life. Although it is in the shape of a double helix, we can think of it as a very long thin string packed into the nucleus of a cell, like 200 kilometers of fishing line tangled up inside a soccer ball. Yet it must be able to perform various functions such as recombination, transcription and replication. The tangling can prevent these operations from being able to occur. Biochemists and mathematicians are collaborating on attempts to figure out how various enzymes act.

It turns out that there are enzymes in the nucleus that will perform knot theoretic operations on the DNA molecule that allow it to knot and unknot.

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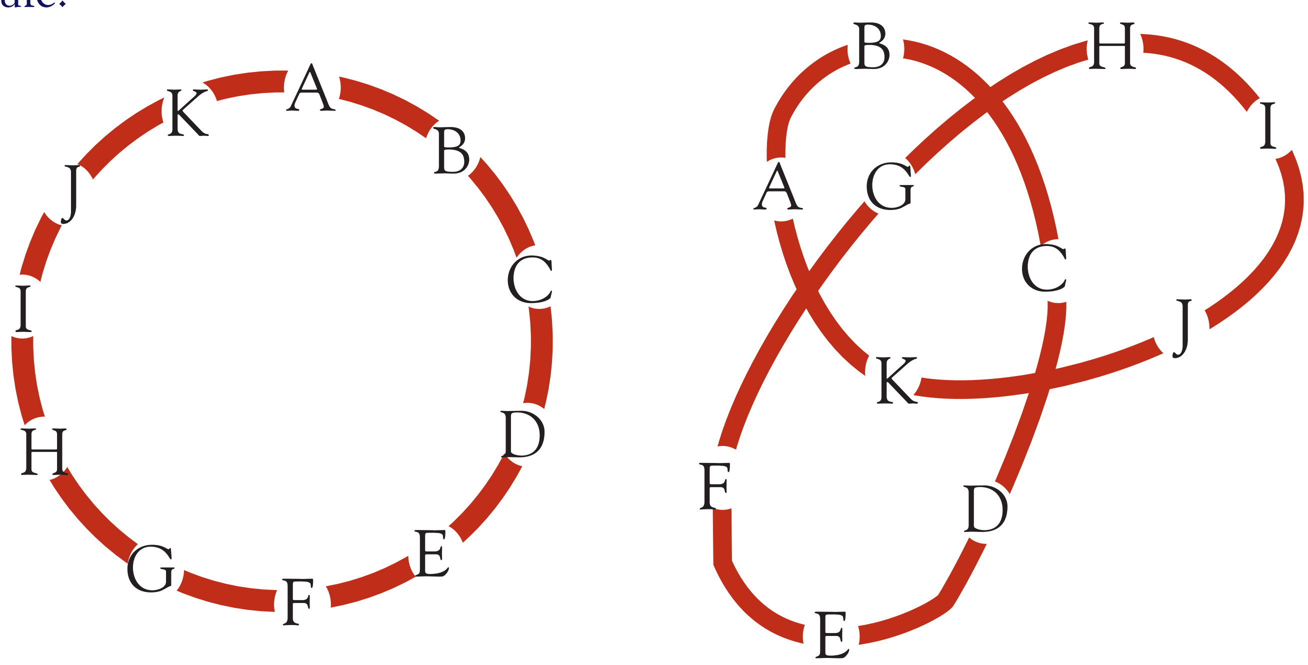
Knots, Molecules and Stick Numbers

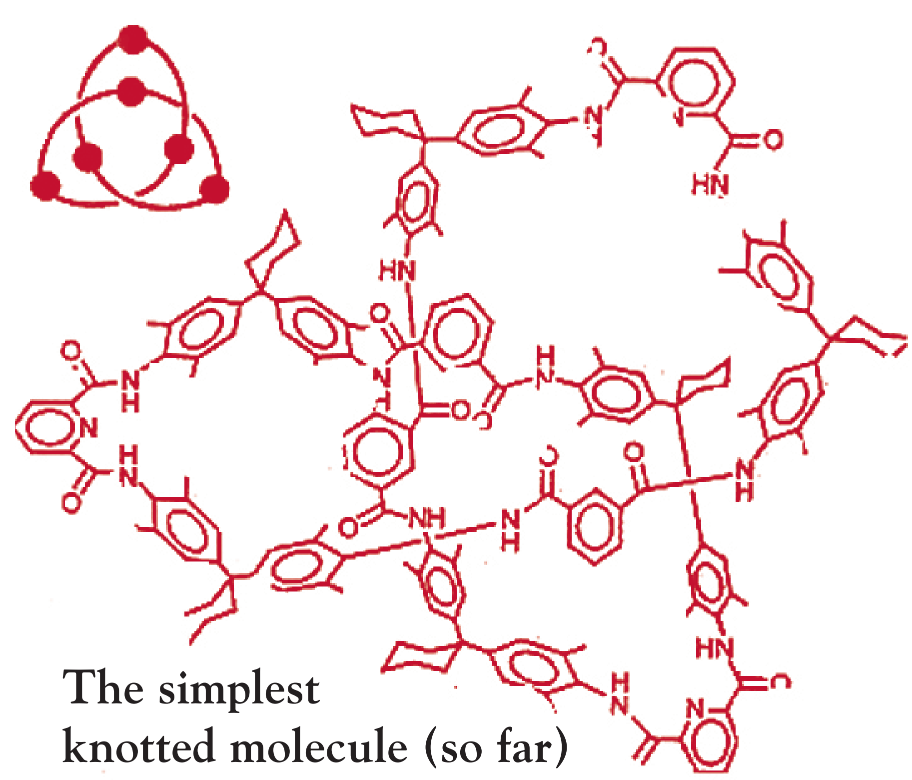
CHEMISTS ARE ALSO VERY INTERESTED IN CREATING KNOTTED MOLECULES. One can easily picture a so-called "cyclic" molecule. This is a molecule formed by a sequence of atoms bonded end-to-end. Then the last atom bonds with the first, forming a looped molecule as on the left in the figure below. Benzene is a good example of such a molecule.



A cyclic molecule (left) and a knotted molecule (right) with the same constituent atoms.

Now imagine constructing a molecule in exactly the same manner, with the same set of constituent atoms bonded in exactly the same order, only this time, before bonding the last atom to the first, a knot is tied in the molecular strand. After the ends are bonded together, we have a knotted molecule, as on the right of the figure above.

Whereas the unknotted version may behave like an oil, the knotted version may behave like a gel. It may have completely different properties, properties never before seen in a molecule. In fact, for each of the distinct knots in the tables of knots, we obtain a distinct substance from this same chain of atoms.



The simplest knotted molecule (so far)

Since there are infinitely many possible distinct knots, it appears that a single sequence of atoms bonded in a certain order can generate an infinite number of different molecules, one for each of the different knots in the table of knots. Of course, that's not quite the case. If we have a sequence of 10 atoms, we will not be able to create a knot of 50 crossings. There will be some upper limit on the complexity of the knots constructible with this many atoms.

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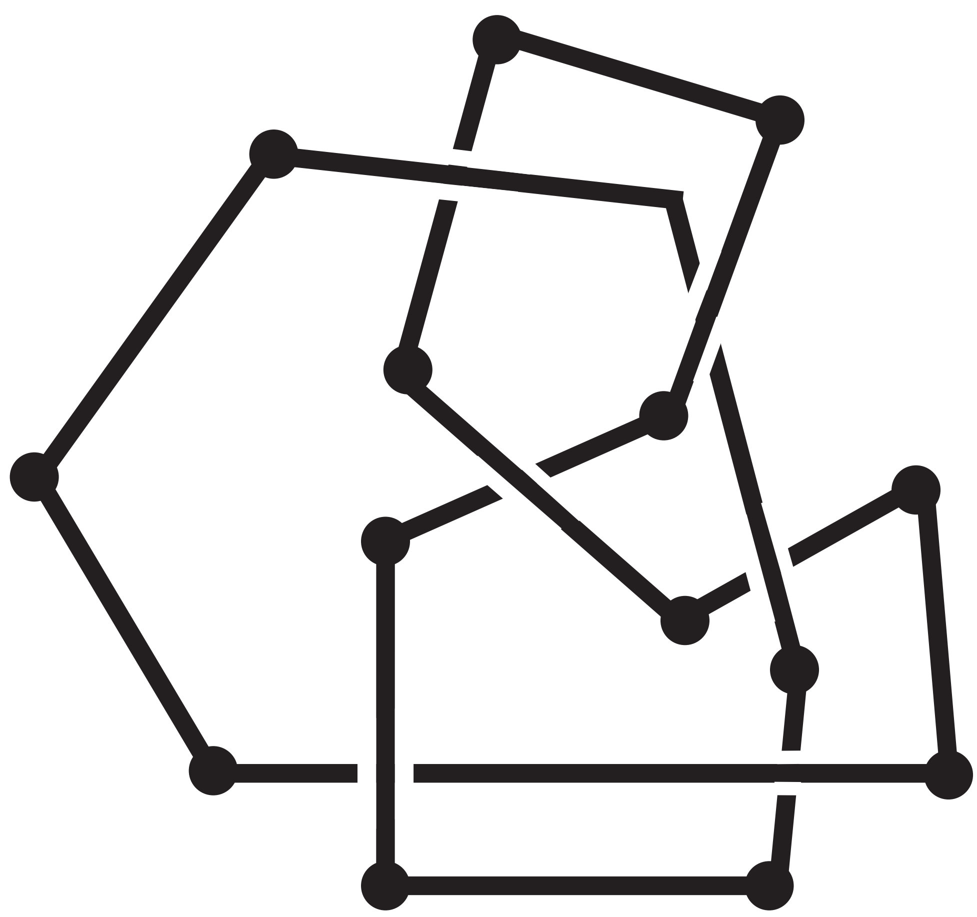
THE FIRST QUESTION ONE MIGHT ASK IS, “WHAT IS THE SMALLEST NUMBER OF ATOMS NEEDED TO CONSTRUCT A NONTRIVIALLY KNOTTED MOLECULE?”

Let's model each atom by a point in space, called a vertex, and each bond between atoms by a stick between the corresponding vertices. Then a cyclic molecule, knotted or not, is modelled by a sequence of sticks glued end-to-end so that the last one is also glued to the first.



A stick version of a knotted molecule.

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WE THEN ASK THE RELATED QUESTION, "HOW MANY STICKS DOES IT TAKE TO CONSTRUCT A NONTRIVIAL KNOT?"

Of course, we may not be able to make a knotted molecule with this number of atoms. There are additional constraints such as bond angles, and lengths of bonds. But the answer to this question should at least give us a minimum number of atoms we will need to construct a knotted molecule.

CHALLENGE QUESTION:
WHAT IS THE FEWEST NUMBER OF STICKS NEEDED TO MAKE A TREFOIL KNOT? TEN, NINE, FIVE...?
TRY IT OUT!