

decimal places, so we have to use an asymptotic method involving exponentials or (what is the same thing) logarithms. A fairly workable table of logarithms to a great many decimals thus is instrumental in calculating a value of a numerical function which may be correct to only one or two decimal places but is nevertheless correct to something like 100 significant figures.

Here is another interesting example. Let D be a positive integer; then there is a remarkable connection between the fractional part of $\exp(\pi\sqrt{D})$ and the number of classes of binary quadratic forms of determinant $-D$. When the number of classes is quite small this fractional part may be very close to zero or one. For example for $D=163$ we have

$$e^{\pi\sqrt{163}} = 262537412640768743.99999999999250072597 \dots$$

To discover just how close to an integer this exponential is requires about 35 decimal place accuracy even if one is not interested in just that integer to which it is so close.¹ There is reason to believe that such remarkable approximations to integers by exponentials of this type do not persist. Experiments on such questions manifestly require exceedingly accurate tables of logarithms.

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¹ This integer is really very interesting since it is $744+(2^8 \cdot 3 \cdot 5 \cdot 23 \cdot 29)^2$ where 744 is the constant coefficient in the Fourier series development of Klein's celebrated absolute modular invariant.

RMT 81—Supplementary Note.

Through an oversight a review was not included in this issue of PROJECT FOR COMPUTATION OF MATHEMATICAL TABLES, *Tables of Circular and Hyperbolic Sines and Cosines for Radian Arguments*. Prepared by the Federal Works Agency, Work Projects Administration for the State of New York, conducted under the sponsorship of the National Bureau of Standards. New York, 1940, xx, 405 p. A detailed review will appear in our next issue. We simply note here that the volume includes tables of $\sin x$ and $\cos x$ for $x=[0.0000(0.0001)1.9999; 9D]$, and for $x=[0.0(0.1)10.0; 9D]$.