computers had been reduced to 250. Since the WPA is to be liquidated on 1 February 1943 the New York Mathematical Tables Project is also to be terminated by that date. Had all the computers in the Project been dispersed, the loss to scientific research would have been irreparable. We learn, however, that 47 members of the group have been taken over by the U. S. Bureau of Standards and by the Hydrographic Office.

Unless a much more adequate nucleus of the present Project is preserved, with calculating machines, a large number of mathematical tables now in process may never come to publication.

R. C. A.

QUERIES

1. Tables to Many Places of Decimals.—Various tables, computed to from 15 to 60 or more places of decimals, have been published in the distant past, and continue to appear in recent times; several tables of this kind are referred to in this issue of *MTAC*. In what specific problems of research do their solutions make such tables highly desirable? There are doubtless many problems of this kind which might be formulated by the specialists in different fields. An appeal for contributions of such formulations is herewith made. Professor Lehmer, a specialist in the theory of numbers, makes an interesting start in this direction in QR **1**, extracted from a personal letter dated 7 November 1942.

R. C. A.

QUERIES—REPLIES

1. Tables to Many Places of Decimals.—As to the sort of problems in which great accuracy in logarithms is helpful, you cannot draw the line between 20 figure problems and over 100 digit problems. I could not pretend to describe all such problems but those with which I have had something to do are of the following type: There is defined some numerical function whose value is an integer for each integer value of the variable. Quite often the function is defined by words (such as the number of ways that such and such a thing can be done). It often happens that when the argument of the function is only fairly large the value of the function is extravagantly large. Even so, this value is an integer and it is either even or odd (or perhaps we should like to know if it is divisible by 23 to check up on a conjecture). Perhaps there is no workable formula with which to calculate an isolated value of the function. Perhaps even a recursion formula for calculating a complete table of the function is not feasible. Then we may turn to an approximate formula, a series consisting of infinitely many terms which are values of an analytic function [often a Bessel function]. If enough terms of this series are taken so that the sum of the remainder terms is less than 1/2 in absolute value then the integer we are looking for is the nearest integer to the value we get from the series. Since these terms must add up to an extravagantly large amount some terms must be calculated with extreme accuracy. We do not have tables of Bessel functions to say 100

decimal places, so we have to use an asymptotic method involving exponentials or (what is the same thing) logarithms. A fairly workable table of logarithms to a great many decimals thus is instrumental in calculating a value of a numerical function which may be correct to only one or two decimal places but is nevertheless correct to something like 100 significant figures.

Here is another interesting example. Let D be a positive integer; then there is a remarkable connection between the fractional part of exp $(\pi\sqrt{D})$ and the number of classes of binary quadratic forms of determinant $-D$. When the number of classes is quite small this fractional part may be very close to zero or one. For example for $D=163$ we have

$$e^{\pi\sqrt{163}} = 262537412640768743.999999999999250072597 \cdots .$$

To discover just how close to an integer this exponential is requires about 35 decimal place accuracy even if one is not interested in just that integer to which it is so close.[1] There is reason to believe that such remarkable approximations to integers by exponentials of this type do not persist. Experiments on such questions manifestly require exceedingly accurate tables of logarithms.

D. H. L.

[1] This integer is really very interesting since it is $744+(2^6\cdot 3\cdot 5\cdot 23\cdot 29)^3$ where 744 is the constant coefficient in the Fourier series development of Klein's celebrated absolute modular invariant.

RMT **81**—Supplementary Note.

Through an oversight a review was not included in this issue of PROJECT FOR COMPUTATION OF MATHEMATICAL TABLES, *Tables of Circular and Hyperbolic Sines and Cosines for Radian Arguments*. Prepared by the Federal Works Agency, Work Projects Administration for the State of New York, conducted under the sponsorship of the National Bureau of Standards. New York, 1940, xx, 405 p. A detailed review will appear in our next issue. We simply note here that the volume includes tables of sin x and cos x for $x=[0.0000(0.0001)1.9999; 9D]$, and for $x=[0.0(0.1)10.0; 9D]$.