

C. *Berechnung der Räderübersetzung. Herausgegeben von dem Verein "Hütte." Bearbeitet nach Calcul des Rouages par Approximation, Nouvelle Méthode* par Achille Brocot. Berlin, 1871, xvi, 52 p.

D. *Idem*, second ed., Berlin, 1879, 67 p.

E. [Henry Goodwyn], *A Table of Circles arising from the Division of a Unit or any other Whole Number, by all the Integers from 1 to 1024, being all the pure Decimal Quotients that can arise from this source.* London, 1823 v, 118 p. Published anonymously.

We have already noted (RMT 87, p. 21) that *A* and *B* were formerly in the Bibliothèque Nationale, Paris.

R. C. A.

QUERIES—REPLIES

2. TABLES TO MANY PLACES OF DECIMALS (Q1; QR1).—The functions which occur in the solution of the problems of applied mathematics are of numerous types but many of them are associated with the differential equation of the hypergeometric function and its generalizations. Now it happens that the logarithmic case of this equation is of frequent occurrence and the desired solution consists of two parts one of which is multiplied by a logarithm. The Legendre function of the second kind

$$Q_n(z) = \frac{1}{2} \int_{-1}^{+1} dt P_n(t) / (z-t)$$

is a type of such a function and in computations it is generally convenient to use the recurrence relation

$$(n+1)Q_{n+1}(z) + nQ_{n-1}(z) = (2n+1)zQ_n(z)$$

which is satisfied by each of the two terms of which  $Q_n(z)$  is composed. Now when  $z$  exceeds unity these two terms are very nearly equal, but of opposite sign: consequently the desired value of  $Q_n(z)$  is the difference of two quantities which must be known very accurately. The first two functions are

$$Q_0(z) = \frac{1}{2} \ln[(z+1)/(z-1)], \quad Q_1(z) = zQ_0(z) - 1$$

and so the logarithm giving the value of  $Q_0(z)$  must be found to a large number of places of decimals. In some calculations that were made in connection with a hydrodynamical problem use was made of the values given by J. C. Adams in his note on the value of Euler's constant (R. So. London, *Proc.*, v. 27, 1878, p. 88-94, *Scientific Papers*, v. 1, Cambridge, 1896). The short tables of  $Q_n(z)$ , published in *Messenger of Math.*, v. 52, 1923, p. 71-78, were actually calculated with the aid of the recurrence relation and it was found that the difference between  $nQ_{n-1}$  and  $(2n+1)zQ_n$  was generally quite small. Further calculations have been made by this method for many integral values of  $z$  so as to have about 30D in the value of  $Q_n(z)$  many of the figures being zeros. Such accuracy may not ordinarily be needed but expansions in series of  $Q_n(z)$  are useful in the solution of problems in hydrodynamics and sometimes the coefficients are large. It should be mentioned that a resolution of a function into two terms one of which has a logarithmic factor occurs naturally in the evaluation of certain integrals which occur in potential theory. When, for instance, the integrand is of form  $P_n(t)/(z-t)$ , the subtraction of an integral whose integrand is  $P_n(z)/(z-t)$  gives an integral whose integrand is a polynomial. Sometimes the integral-logarithm takes the place of the logarithm in the first

of two terms representing an integral. This is the case, for instance, when an integral used by T. H. Havelock is resolved into two parts each of which satisfies a certain recurrence relation. The integral in question represents a logarithmic case of the confluent hypergeometric function and occurs in the paper, "The method of images in some problems of surface waves," R. So. London, *Proc.*, A iv. 115, 1927, p. 268-280. Many figures may be needed, then, in tables of the integral-logarithm and in the value of Euler's constant which occurs in many expressions for this function. Tables to many places of decimals are needed occasionally for the solution of transcendental equations. In his paper "Comparaison de la méthode d'approximation de Newton à celle dite des parties proportionnelles," *Nouv. Annales d. Math.*, s. 2, v. 18, 1879, p. 218-231, L. Maleyx calculates the root of

$$e^x - x^e = \frac{1}{2}\pi$$

which lies between 1 and  $e$  by two different methods and finds that with 6 substitutions the method of proportional parts is more accurate than the Newton-Raphson method. He says that his calculations were made with the aid of the excellent tables of Fédor Thoman, *Tables de Logarithmes à 27 Décimales pour les Calculs de Précision*, Paris, 1867

H. B.

### CORRIGENDA

Omit last four lines page 25.

See Note 2 of the article "Tables and trigonometric functions in non-sexagesimal arguments" in this issue of *MTAC*, p. 44.

P. 20, l. 4 from bottom, for "II, III, IX," read "II, III, IX, XIII."