

Coordinates for projection of maps, in various scales; Areas of quadrilaterals of the earth's surface of varying extents in latitude and longitude.

III. *Physical Tables*, 1897. Publication 8171, eighth revised edition by F. E. FOWLE (1932), first reprint, 1934, liv, 686 p. The introductory pages deal mainly with various formulae and units. The 874 tables include the following: Mathematical Tables; Tables of Mechanical Properties, Acoustics, Aerodynamics, Viscosity, Radiation, The Eye and Radiation, Electromotive Forces, Electrolysis, Atomic Structure, Radioactivity, Meteorology, Geodesy, Geophysics, Terrestrial Magnetism, Astronomy, Nebulae. The Mathematical Tables included exponential functions and their logarithms, diffusion integral, exponential integral, gamma function, zonal spherical harmonics, cylindrical harmonics, elliptic integrals.

To these three volumes was added

IV.—Publication no. 1871. *Smithsonian Mathematical Tables, Hyperbolic Functions*, 1909, of which the fifth reprint was published in 1942, lii, 321 p. Corrections of errors were made in each of the reprints. The Secretary of the Smithsonian wrote in part as follows: "Hyperbolic functions are extremely useful in every branch of pure physics, and in the applications of physics whether to observational and experimental sciences or to technology. Thus whenever an entity (such as light, velocity, electricity, or radioactivity) is subject to gradual extinction or absorption, the decay is represented by some form of Hyperbolic Functions. Mercator's projection is likewise computed by Hyperbolic Functions. Hence geological deformations invariably lead to such expression, and it is for that reason that Messrs. BECKER and VAN ORSTRAND, who are in charge of the physical work of the United States Geological Survey have been led to prepare this volume." Compare RMT 89.

The final volume in this series is

V.—Publication 2672. *Smithsonian Mathematical Formulae and Tables of Elliptic Functions, Mathematical Formulae and Tables of Elliptic Functions prepared by EDWIN P. ADAMS . . . Tables of Elliptic Functions prepared under the Direction of GEORGE GREENHILL*, by R. L. HIPPISELY, 1922; first reprint, 1939, viii, 314 p. In this reprint a few errors have been corrected. It was not originally intended to include any tables of functions in this volume, but merely to give references to such tables. But an exception was made in favor of the Greenhill-Hippisley table calculated on a new plan (in 1922) and not otherwise available. Greenhill wrote the introduction to these tables (p. 243–258). F. R. Moulton is the author of section X (p. 220–242) on "Numerical solution of differential equations." Sections VIII–IX are devoted to formulae and bibliography of "Differential equations." Other topics treated are Algebra, Geometry, Trigonometry, Vector analysis, Curvilinear coördinates, Infinite series, and Special applications of analysis.

## QUERIES

2. SCARCE MATHEMATICAL TABLES.—In what libraries of the world, public or private, may the following books be found:

- A. Achille Brocot, *Calcul des Rouages par Approximation, Nouvelle Méthode*. Paris, l'auteur, 1862. 97 p.
- B. Achille Brocot, *Table de Conversion en Décimale des fonctions ordinaire a l'Usage du Calcul des Rouages par approximation. Méthode Nouvelle*. Paris, P. Dupont, 1862, 51 p.

C. *Berechnung der Räderübersetzung. Herausgegeben von dem Verein "Hütte." Bearbeitet nach Calcul des Rouages par Approximation, Nouvelle Méthode* par Achille Brocot. Berlin, 1871, xvi, 52 p.

D. *Idem*, second ed., Berlin, 1879, 67 p.

E. [Henry Goodwyn], *A Table of Circles arising from the Division of a Unit or any other Whole Number, by all the Integers from 1 to 1024, being all the pure Decimal Quotients that can arise from this source.* London, 1823 v, 118 p. Published anonymously.

We have already noted (RMT 87, p. 21) that *A* and *B* were formerly in the Bibliothèque Nationale, Paris.

R. C. A.

QUERIES—REPLIES

2. TABLES TO MANY PLACES OF DECIMALS (Q1; QR1).—The functions which occur in the solution of the problems of applied mathematics are of numerous types but many of them are associated with the differential equation of the hypergeometric function and its generalizations. Now it happens that the logarithmic case of this equation is of frequent occurrence and the desired solution consists of two parts one of which is multiplied by a logarithm. The Legendre function of the second kind

$$Q_n(z) = \frac{1}{2} \int_{-1}^{+1} dt P_n(t)/(z-t)$$

is a type of such a function and in computations it is generally convenient to use the recurrence relation

$$(n+1)Q_{n+1}(z) + nQ_{n-1}(z) = (2n+1)zQ_n(z)$$

which is satisfied by each of the two terms of which  $Q_n(z)$  is composed. Now when  $z$  exceeds unity these two terms are very nearly equal, but of opposite sign: consequently the desired value of  $Q_n(z)$  is the difference of two quantities which must be known very accurately. The first two functions are

$$Q_0(z) = \frac{1}{2} \ln[(z+1)/(z-1)], \quad Q_1(z) = zQ_0(z) - 1$$

and so the logarithm giving the value of  $Q_0(z)$  must be found to a large number of places of decimals. In some calculations that were made in connection with a hydrodynamical problem use was made of the values given by J. C. Adams in his note on the value of Euler's constant (R. So. London, *Proc.*, v. 27, 1878, p. 88-94, *Scientific Papers*, v. 1, Cambridge, 1896). The short tables of  $Q_n(z)$ , published in *Messenger of Math.*, v. 52, 1923, p. 71-78, were actually calculated with the aid of the recurrence relation and it was found that the difference between  $nQ_{n-1}$  and  $(2n+1)zQ_n$  was generally quite small. Further calculations have been made by this method for many integral values of  $z$  so as to have about 30D in the value of  $Q_n(z)$  many of the figures being zeros. Such accuracy may not ordinarily be needed but expansions in series of  $Q_n(z)$  are useful in the solution of problems in hydrodynamics and sometimes the coefficients are large. It should be mentioned that a resolution of a function into two terms one of which has a logarithmic factor occurs naturally in the evaluation of certain integrals which occur in potential theory. When, for instance, the integrand is of form  $P_n(t)/(z-t)$ , the subtraction of an integral whose integrand is  $P_n(z)/(z-t)$  gives an integral whose integrand is a polynomial. Sometimes the integral-logarithm takes the place of the logarithm in the first