interchanging the roles of the input and output sides; the machine can integrate an empirical curve but cannot differentiate one.

This limitation is not a serious one, for the author shows that any differential equation of the form $f(x, y, y', \cdots, y^{(n)})=0$ can be solved provided only that the function f is not hypertranscendental in character, that is, provided f considered as a function of any of its arguments satisfies a differential equation of the form $P(x, y, y', \cdots, y^{(m)})=0$ where P is a polynomial. The solution can be accomplished with a finite, but sufficiently large, number of units, and without any information supplied to the machine by the operator beyond the connections between the units demanded by the particular problem. Furthermore, even though f is hypertranscendental, it will still be possible to approximate the solution in the sense that a function y can be obtained which makes the left member of the given equation uniformly less than a given positive number ϵ. Or, the operator can supply the hypertranscendental function to the machine.

That differential analyzers are not in general use is due to their prohibitive cost, being of the order of many thousands of dollars for the larger machines. In paper (ii) there is described in detail the construction of a small scale analyzer which, while relatively inexpensive, is claimed to have an average accuracy of one half percent. The authors have reduced the cost of the machine by limiting the number of integrators to four, by eliminating certain refinements such as backlash compensators, automatic speed controls, etc., and by using standard parts for the construction. The resulting cost of materials is given as about £50, but the time required to assemble the parts must have been considerable. One wonders if the construction of machines following the authors design might not be brought within the range of the abilities and interests of groups of amateur hobbyists if fostered by mathematicians after the fashion set by astronomers in encouraging the construction and use of reflecting telescopes. That the completed machines are in the nature of glorified toys must have been in Hartree's mind when he in 1935 succeeded in making a demonstration model out of toy meccano parts.[1]

P. W. KETCHUM

NOTES

4. GIFFORD AND C. G. S. TABLES.—In RMT **77** the improvements in the Table of *Natural Sines and Cosines* published by the Coast and Geodetic Survey, as compared with Emma Gifford's volume, on which it was mainly based, were not made sufficiently clear. The Gifford volume is defective in that
(a) The arrangement of sines throughout the quadrant so that sines and cosines of a given angle have to be sought in different places.
(b) Consecutive values are in rows rather than in columns.
(c) The number of errors of more than a unit in the last decimal place is very large. The C. G. S. table is a notable improvement by virtue of
(a′–b′) Its semi-quadrantal arrangement, in columns, of the sines on one page and the cosines on the opposite page. (That an arrangement with sines and cosines on the same page would have been still better can hardly be gainsaid.)
(c′) The number of errors of more than a unit in the last decimal place being almost negligible. One may add (d) cost of Gifford 40 shillings, (d′) cost of C. G. S. 1.75 dollars.

R. C. A.

5. COAST AND GEODETIC SURVEY VERSUS PETERS.—In MTE **1**, p. 25, lines 6–12 from the bottom, six entries in C. G. S., *Natural Sines and Cosines* (RMT **77**) were called into question by quoting the corresponding results in Peters' *Eight-figure Table of the Trigonometrical Functions* (RMT **78**). On

[1] D. R. Hartree and A. Porter, "The construction and operation of a model differential analyser," Manchester Lit. and Phil. So., *Mem. and Proc.*, v. 79, 1935, p. 51–71, +2 plates.—EDITOR.

5 March, 1943, G. M. Clemence, of the U. S. Naval Observatory, wrote as follows: "As a matter of academic interest I have taken these six sines out of Andoyer's 15-place tables, obtaining the following results for the decimals from the fifth to the twelfth inclusive:

17°15′19″	2954	4997	68°49′06″	3946	5041
33 20 15	6973	5006	71 09 45	3813	4969
61 01 41	5699	5037	74 02 28	5922	4959

It appears that Peters is correct in every instance, the errors in the Survey tables being in units of the twelfth decimal, 5003, 5006, 5037, 5041, 5031, 5041. These errors are hardly important in computation, the largest being less than 1 percent greater than the unavoidable error in using an 8-place table."

6. The Eight-figure Table of J. T. Peters and L. J. Comrie.—In RMT **78** we referred to *Eight-figure Table of the Trigonometrical Functions for Every Sexagesimal Second of the Quadrant*, produced by the British War Office in 1939 and 1940 from the German original, and pointed out that this volume was unfortunately not available even to British scientists. L. J. Comrie has set forth the merits of this great table as follows:

"(1) All four functions of any angle are on the same line in four adjacent columns.

(2) Consecutive values are in columns, not in lines.

(3) The printing is perfect.

(4) The table is free from error. Not one of the discrepancies between Peters and Gifford's *Sines* and *Tangents* (which have upwards of 700 errors) was traced to Peters. The mechanical printing of the printer's copy, and Peters' known standard of proof reading (including the reading against an independently prepared copy) are such as to inspire the utmost confidence."

Since another table of Peters (see RMT **79**) has been "published and distributed in the public interest by authority of the Alien Property Custodian," we urge most strongly that at an early date this larger work be also made available to scientific workers.

7. Smithsonian Tables.—It seems desirable to place on record some notes concerning these five volumes of Washington tables, of uniform format, about 16×24 cm. Among the early publications of the Smithsonian Institution was an important volume of Arnold Henry Guyot (1807–1884), *Tables, Meteorological and Physical*, 1852; second edition revised and enlarged, 1858; fourth edition, 1884, xxv, 747 p. Thereafter the work was divided into the following three different publications:

I. *Meteorological Tables*, 1893. This is now Publication 3116, fifth revised edition (corrected to 1931), reprinted with corrections 1939, xiii, 282 p. The 116 tables include those which are: Thermometrical; Hygrometrical; Geodetical; Involving Conversions of Linear Measures; Measures of Time and Angle, and Measures of Weight; For Determining Heights and Conversions involving Geopotential.

II. *Geographical Tables*, 1894. Publication 854, third edition, second reprint, 1929, cv, 182 p. This last edition was prepared by R. S. Woodward. The introductory pages set forth useful formulae, etc., in Mathematics, Geodesy, Astronomy, and Theory of errors. Then follow 42 tables which include, for example, Natural sines, cosines, tangents, cotangents, to 4D;

Coordinates for projection of maps, in various scales; Areas of quadrilaterals of the earth's surface of varying extents in latitude and longitude.

III. *Physical Tables*, 1897. Publication 8171, eighth revised edition by F. E. Fowle (1932), first reprint, 1934, liv, 686 p. The introductory pages deal mainly with various formulae and units. The 874 tables include the following: Mathematical Tables; Tables of Mechanical Properties, Acoustics, Aerodynamics, Viscosity, Radiation, The Eye and Radiation, Electromotive Forces, Electrolysis, Atomic Structure, Radioactivity, Meteorology, Geodesy, Geophysics, Terrestrial Magnetism, Astronomy, Nebulae. The Mathematical Tables included exponential functions and their logarithms, diffusion integral, exponential integral, gamma function, zonal spherical harmonics, cylindrical harmonics, elliptic integrals.

To these three volumes was added

IV.—Publication no. 1871. *Smithsonian Mathematical Tables, Hyperbolic Functions*, 1909, of which the fifth reprint was published in 1942, lii, 321 p. Corrections of errors were made in each of the reprints. The Secretary of the Smithsonian wrote in part as follows: "Hyperbolic functions are extremely useful in every branch of pure physics, and in the applications of physics whether to observational and experimental sciences or to technology. Thus whenever an entity (such as light, velocity, electricity, or radioactivity) is subject to gradual extinction or absorption, the decay is represented by some form of Hyperbolic Functions. Mercator's projection is likewise computed by Hyperbolic Functions. Hence geological deformations invariably lead to such expression, and it is for that reason that Messrs. Becker and Van Orstrand, who are in charge of the physical work of the United States Geological Survey have been led to prepare this volume." Compare RMT **89.**

The final volume in this series is

V.—Publication 2672. *Smithsonian Mathematical Formulae and Tables of Elliptic Functions, Mathematical Formulae prepared by* Edwin P. Adams . . . *Tables of Elliptic Functions prepared under the Direction of* George Greenhill, by R. L. Hippisley, 1922; first reprint, 1939, viii, 314 p. In this reprint a few errors have been corrected. It was not originally intended to include any tables of functions in this volume, but merely to give references to such tables. But an exception was made in favor of the Greenhill-Hippisley table calculated on a new plan (in 1922) and not otherwise available. Greenhill wrote the introduction to these tables (p. 243–258). F. R. Moulton is the author of section X (p. 220–242) on "Numerical solution of differential equations." Sections VIII–IX are devoted to formulae and bibliography of "Differential equations." Other topics treated are Algebra, Geometry, Trigonometry, Vector analysis, Curvilinear coördinates, Infinite series, and Special applications of analysis.

QUERIES

2. Scarce Mathematical Tables.—In what libraries of the world, public or private, may the following books be found:

A. Achille Brocot, *Calcul des Rouages par Approximation, Nouvelle Méthode*. Paris, l'auteur, 1862. 97 p.

B. Achille Brocot, *Table de Conversion en Décimale des fonctions ordinaire a l'Usage du Calcul des Rouages par approximation. Méthode Nouvelle*. Paris, P. Dupont, 1862, 51 p.