96[A, C, D, E].—H. S. Uhler, "Recalculation of the modulus and of the logarithms of 2, 3, 5, 7 and 17," Nat. Acad. Sci., *Proc.*, v. 26, 1940, p. 205–212. 17.5×25.8 cm.

In the calculation of the table in RMT **95** the series

$$\ln \frac{p}{q} = 2\left\{\frac{p-q}{p+q} + \frac{1}{3}\left(\frac{p-q}{p+q}\right)^3 + \frac{1}{5}\left(\frac{p-q}{p+q}\right)^5 + \cdots\right\} \equiv 2S[(p-q)/(p+q)],$$

with $p-q=1$, played an important role. With $p=5041=71^2$, $5040=2^4\cdot 3^2\cdot 5\cdot 7$

$$\ln 71 = 2\ln 2 + \ln 3 + (\ln 5 + \ln 7)/2 \quad S(1/10081).$$

Similarly for $p=226$, ln 113 involves $S(1/451)$. Thus in the present paper, we have $S(1/5)$, $S(1/239)$, $S(1/2449)$, $S(1/4999)$, and $S(1/8749)$, in connection with ln 2, ln 3, ln 5, ln 7 and ln 17. J. C. P. Adams calculated the first four of these to 262D (1878 and 1887); see MTE **8**. These are here extended, with certainty on the author's part, to 328D. The values are also given of the following: arctan (1/451) to 215D; arctan (1/577) to 335D; arctan (1/2449), arctan (1/4999), and arctan (1/8749) each to 330D; and arctan (1/10081) to 216D.

Adams found M correct to 271D (1887). From his own ln 2 and ln 5 Uhler determined M, correct to 328D.

Five other values found in RMT **94** are here extended, viz: e^{10} to 289D; e^{-10} to 293D; and sin 10, cos 10, cos 20, each to 284D. These latter ranges are also supplementary to results in RMT **81**.

R. C. A.

97[A, K].—H. S. Uhler, "The coefficients of Stirling's series for log $\Gamma(x)$," Nat. Acad. Sci., *Proc.*, v. 28, 1942, p. 59–62. 17.5×25.8 cm.

When n is a positive integer, the asymptotic series of RMT **95** becomes

$$\ln \Gamma(x) = (1/2)\ln 2\pi + (x+1/2)\ln x - x + \sum_{m=1}^{\infty} (c_m/x^{2m-1}) + R,$$

where $c_m \equiv (-1)^{m-1}B_m/[(2m-1)(2m)]$. The table of the paper contains the first 71 values of c_m, many of which have recurring periods within the range of the table; c_{15} is given to 103S. Values of 100! to 158S, and of ln (100!) to 156S, are also given.

MATHEMATICAL TABLES—ERRATA

In this issue we have referred to Errata in RMT **89** (Blakesley, Forti, Hayashi, Sakamoto), RMT **92** (Lowan *et al.*, Moors, Bayly, Gauss, Heine, Hobson, Tallquist), RMT **94** (Glaisher), RMT **95** (Parkhurst, Serebrennikov), UMT **2** (Airey), N **4** (Gifford, C. G. Survey), N **5** (C. G. Survey), N **6** (Gifford), and in the first article of this issue (Callet, Brandicourt and Roussilhe, Jordan, Service Géog. 1914).

5. U. S. Coast and Geodetic Survey, Special Publication, no. 231, *Natural Sines and Cosines to Eight Decimal Places*, 1942; see RMT **77.**

End-figures are missing cos 1°44′41″ and 42″, namely: 0 and 5 respectively.

L. J. C.

Sin 36° *for* 0.587 78255, *read* 0.587 78525.

F. W. Hoffman, 689 East Ave., Pawtucket, R. I.

6. A. N. Lowan, N. Davids, A. Levenson, "Table of the zeros of the Legendre polynomials," 1942; see RMT **92.**

$n=11$, *for* $x_2=0.519096129110681$ *read* $x_2=0.519096129206812$
$n=12$, *for* $x_1=0.125333408511469$ *read* $x_1=0.125233408511469$
$n=12$, *for* $x_2=0.367831498918180$ *read* $x_2=0.367831498998180$

A. N. Lowan, and R. C. A.

7. J. T. Peters, *Zehnstellige Logarithmentafel, Erster Band, Zehnstellige Logarithmen der Zahlen von 1 bis 100 000 nebst einem Anhang mathematischer Tafeln*. Berlin, 1922. All of the errors noted below are in the "Anhang," arranged and calculated by J. T. PETERS and J. STEIN, p. i–xxviii, 1–195.

Table 3, p. 47, $1/42^5$, groups 5 and 6

for 85453 21863 *read* 85452 31863

It would seem that the check described in the Introduction (sum of the $1/42^n$, to 32D = 1/41) must have been applied in ms., not in proof. The columns involved satisfy this check as amended but not as printed. P. VII Stirling's series for log $n!$, *for* $\frac{B_3}{5 \cdot 6 \cdot n}$ *read* $\frac{B_3}{5 \cdot 6 \cdot n^5}$

C. R. Cosens, Engineering Laboratory, Cambridge, England, Nov., 1941

Table 13, contains, mainly ln$N$, $N = 2(1)146$, and all following prime numbers to 9973. This Table was computed by one of the great calculators of logarithms, J. Wolfram, Lieutenant of the Dutch artillery. After six years of intense application he computed ln N, $N = 1(1)10009$, to 48D. They were first published in J. C. Schulze, *Recueil de Tables Logarithmiques, Trigonometriques et autres nécessaires dans les Mathématiques Pratiques*, [also t. p. in German], v. 1, Berlin, 1778, p. 189–259. Space is left for the logarithms of six numbers 9769, 9781, 9787, 9871, 9883, 9907 which Wolfram had, up to 1778, been prevented from computing by a serious illness. These were supplied two years later by Schulze in *Berliner Astronomisches Jahrbuch für das Jahr 1783*, Berlin, 1780, p. 191, as given to him by Wolfram, and also, from an independent calculation by Barzellini, "Oberbuchhalter der Grafschaften Görz und Gradisca." There have been various reprints or revisions of Wolfram's table; the first of these was in G. Vega, *Thesaurus Logarithmorum Completus*, Leipzig, 1794; this is the basis of the table of P. & S., in which there are at least the following ten errors.

	N	pentad	for	read	first discovered by
1.	829	4	67458	97458	Escott
2.	1087	10	598	597	Cosens
3.	1409	4	21666	21696	Gray
4.	3967	6	91589	91389	Duarte
5.	6343	3	23897	33897	Steinhauser
6.	7247	7	24102	25102	Duarte
7.	8837	4	42054	42354	Duarte
8.	8963	7	38152	38153	Duarte
9.	9623	4	83304	83305	Duarte
10.	9883	10	194	193	Cosens

No. 10 was given correctly by Wolfram but incorrectly by Vega and P. & S.; compare *Scripta Mathematica*, v. 3, 1936, p. 99–100, and v. 4, 1937, p. 293. There are two cases where Wolfram and Vega (1794 and 1923 reprint) were wrong, while P. & S. were correct, viz: ln 1087 (pentad 6), and ln 3571 (pentad 4). E. B. Escott communicated the result in no. 1 to L. J. Comrie in 1924.

Nos. 2 and 10 were found by C. R. Cosens in 1939, after recalculating the logarithms to 55D, assuming the accuracy of Grimpen's ln of primes to 127, to 82D.

No. 3 was indicated in Peter Gray, *Tables for the Formation of Logarithms & Anti-Logarithms to twenty-four or any less number of Places*, London, 1876, p. [39].

No. 5 was given in Anton Steinhauser, *Hilfstafeln zur präcisen Berechnung zwanzigstelliger Logarithmen zu gegebenen Zahlen und der Zahlen zu 20 stelligen Logarithmen*, Vienna, 1880, p. 1.

Nos. 4, 6, 7, 8, 9, were given by F. J. Duarte, *Nouvelles Tables Logarithmiques à 36 Décimales*, Paris, 1933, p. XXII.

R. C. A.

In Table 1, p. 7, M, the modulus of common logarithms is given to 282D. There are at least 14 incorrect digits among the last 19, viz.: 47 48049 05993 55353 05.

In Table 13, p. 152, ln 2, ln 3, ln 5, ln 7 are each given to 272 places of decimals and the last 9 or 10 digits in each are erroneous as follows:

in	for	read
ln 2	81 06850 15	70 95326 37
ln 3	92 45403 15	75 60690 11
ln 5	604 17624 80	580 59722 57
ln 7	210 03537 95	183 10810 25

All five of the values as printed by P. & S. were taken from a paper by J. C. Adams, Royal So. London, *Proc.*, v. 27, 1878, p. 92–93. Corrections were given by Adams in *Proc.*, v. 42, 1887, p. 24–25. See also J. C. Adams, *Scientific Papers*, Cambridge, v. 1, 1896, p. 464, 469–477. Adams states that M is now true certainly to 272 and probably to 273D.[1]

I have recently evaluated ln 127 to about 104 places to test the illustrative value given by P. & S. p. XXVII. The last figure [82nd] should be 7 instead of 4. This finding agrees very well with the comment on p. XXVIII which reads: " . . . ; die Endziffern des so bestimmten ln 127 weichen nur um 4 Einheiten von dem vorher erhaltenen Werte ab."

P. & S. apparently failed to compare their (?) 61-place Table 14b of ordinary logarithms (p. 156–162) with the appropriately abbreviated 84-place mantissas quoted from A. Grimpen (p. XXV). For the numbers 31, 43, 47, and 59 the difference (P. & S. minus Grimpen) equals +0.8, −1.0, −0.51, and +1.35 respectively. Nevertheless P. and S. state (p. III) that they tried to attain an accuracy "of half a unit in the last decimal place." In Table 14b, p. 158, ln 227 is incorrect; in the last 6 digits *for* 49465 6, *read* 49565 6.

H. S. Uhler, Dept. of Physics, Yale University
New Haven, Conn., Oct. and Nov. 1936

"A Table of the Common or Brigg's Logarithms for all Numbers to 100; and all Primes, to 1100, true to sixty one Figures" was first given in a work by Abraham Sharp (1651–1742), *Geometry Improved* . . . , London, 1717, p. [56]–[60]. It has been reprinted many times, for example, in the first stereotyped edition of François Callet, *Tables Portatives de Logarithmes*, Paris, 1795, and on to the 1899, and possibly later editions. P. & S. copied their table from Callet's work. Sharp gave also log π correct to 61D (p. 36–37).—Editor.

I now report the following 37 other errors of P. & S., who boast of last figure accuracy (p. III, l. 11–12):

page	no.	for last figures	read	abbreviation o
XXIII and 1	log π	6	5	4 999
XXIV	ln 23	81	82	81 984
XXIV	ln 41	59	60	59 623
XXIV	ln 59	73	74	73 593
XXIV	ln 61	11	12	11 854
XXV	ln 71	59	60	59 662
XXV	ln 73	32	33	32 763
XXV	ln 97	54	53	53 422
XXV	ln 103	97	96	95 917
XXV	ln 107	64	66	65 573
XXV	log 17	5795	5796	5795 684
XXV	log 71	7501	7500	7499 931
XXV	log 101	0771	0770	0770 238
XXV	log 113	6837	6838	6837 823

[1] H. S. Uhler's recent researches have shown that even this statement concerning M is not absolutely correct; the substitution in the *Anhang* for the 264th to the 282nd digits should be as follows:

for 47 48049 05993 55353 05 *read* 53 83562 22813 95603 05.

Adams's new 271st to 277th digits were 21868 25 so that his 272nd digit should be "2," not "1"; see RMT **96**. J. W. L. Glaisher in his article on "Logarithms," in the ninth edition of the *Encyclopædia Britannica* (1882), had the incorrect value of M given by Adams in 1878, without the statement of Adams at that time that he did not claim his value to be correct beyond 262D or 263D. The corrected value of 1887 (but still slightly incorrect, as we have seen) is in the eleventh edition of the *Britannica* (1911).—Editor

151	$\ln(1-9\cdot10^{-4})$	485	486	485 507
152	$\ln(1+8\cdot10^{-4})$	567	566	566 326
151	$\ln(1-7\cdot10^{-4})$	859	860	859 672
151	$\ln(1-5\cdot10^{-4})$	785	786	785 574
152	$\ln(1+5\cdot10^{-4})$	340	339	339 355
151	$\ln(1-2\cdot10^{-4})$	811	810	810 371
151	$\ln(1-1\cdot10^{-4})$	734	735	734 571
152	$\ln(1+1\cdot10^{-4})$	402	401	401 071
151	$\ln(1-8\cdot10^{-5})$	613	614	613 570
152	$\ln(1+8\cdot10^{-5})$	796	797	796 578
151	$\ln(1-6\cdot10^{-5})$	899	898	898 045
151	$\ln(1-5\cdot10^{-5})$	846	845	845 433
152	$\ln(1+5\cdot10^{-5})$	980	981	980 850
151	$\ln(1-4\cdot10^{-5})$	446	445	445 439
151	$\ln(1-3\cdot10^{-5})$	774	773	773 336
151	$\ln(1-1\cdot10^{-5})$	682	683	682 540
151	$\ln(1-9\cdot10^{-6})$	599	597	597 457
151	$\ln(1-8\cdot10^{-6})$	358	357	357 389
151	$\ln(1-7\cdot10^{-6})$	606	605	604 608
151	$\ln(1-5\cdot10^{-6})$	448	447	447 389
152	$\ln(1+5\cdot10^{-6})$	457	458	457 806
151	$\ln(1-1\cdot10^{-6})$	858	857	857 267
152	$\ln(1+1\cdot10^{-6})$	523	524	523 684

H. S. Uhler, 8 Jan. 1943

On 2 February 1943 Mr. Uhler drew my attention to the fact that five more last-figure errors in Grimpen's 84-place table on p. XXV are suggested by comparison with H. M. Parkhurst's 102-place table (see RMT **86,** p. 20); in the cases of log 23, log 41, log 61 and log 97 there should be unit increases, but in the case of log 83 there should be a unit decrease. I found that Parkhurst and Grimpen were in complete agreement in the cases of log 31, log 43 and log 59, referred to above; hence it is Sharp's terminal digits which seem then to be slightly erroneous. On 3 May 1943 Mr. Uhler reported that he had completely checked both of Grimpen's tables, p. XXIV–XXV, and that the only errors were those in the terminal figure indicated above.—Editor.

The correct value of π to 707D was calculated by William Shanks and may be found on p. 1 of the *Anhang* by P. & S., and in G. Peano, *Formulario Mathematico*, 5 ed., v. 5, Turin, 1908, p. 250. Shanks gave the value of π to 607D in his *Contributions of Mathematics, comprising chiefly the Rectification of the Circle . . .*, London, 1853, p. 86–87. That the last 8 digits were incorrect, was shown when he extended his value of π to 707D, giving at the same time arctan (1/5) and arctan (1/239), each to 709D, R. So. London, *Proc.*, v. 21, 1873, p. 319. But there were still errors in the 460–462nd, and in the 513–515th decimal places. These were corrected in the value Shanks gave, *idem*, v. 22, 1874, p. 45. Two new errors here introduced in the 326th and 680th decimal places were easily checked from the arctangent values referred to above, and used by Shanks in computing the value of π. See RMT **95.** A reference may be given to J. P. Ballantine "The best (?) formula for computing π to a thousand places," Amer. Math. Mo., v. 46, 1939, p. 499–501.

R. C. A.

UNPUBLISHED MATHEMATICAL TABLES

We have referred to unpublished mathematical tables (a) of Comrie in RMT **82**; and (b) of Riche de Prony, of Sang, of Peters, and of Princeton University, in the first article of this issue.

2[D].—J. R. Airey, *Sines and Cosines in Radian Arguments*. Ms. in Mr. Comrie's possession.

After the death of Airey in 1937, his calculations and manuscript tables came into my possession. Most of these had, of course, been published, although in many cases, e.g., the Fresnel integral, more decimals (usually within a unit of the last decimal) thus became available.