be a complex variable. If

$$f(x) = \sum_{n=0}^{k} a_n x^n = 0$$

is the equation under discussion, the machine is set to draw the map M in the w-plane of the circle

$$|z| = r$$

under the transformation

$$w = \sum_{n=1}^{k} a_n r^n (\cos n\theta + i \sin n\theta).$$

The number of times this map encircles the point $w = -a_0$ is the number of roots within the circle $|z| = r$. By choosing different values of r and inspecting the different maps, a value of r is soon found whose map passes through the point $w = -a_0$. This value of r is the absolute value of one root of $f(x) = 0$. The argument of this root is the angle θ read on the machine just as the tracing point reaches the point $w = -a_0$.

To draw these maps the sine elements of the machine are used to drive the tracing point up and down while the cosine elements are used to drive the drawing board back and forth. The amplitude (for a fixed r) set on the nth crosshead is $|a_n|r^n$. If a_n is not real, its argument is set by giving the crosshead an appropriate initial angular displacement. A dozen or more of these maps are illustrated in the paper.

A second method applicable to real roots only is given. Suppose that one wishes to find all the real roots of $f(x) = 0$ that lie between $-r$ and $+r$. Substitute $x = r\cos\theta$ in $f(x) = \sum_{n=0}^{k} a_n x^n$. Since $\cos^n \theta$ is a linear combination of $\cos n\theta$, $\cos (n-2)\theta$, $\cdots$, the polynomial $f(x)$ is reduced to a series of cosines which the machine can draw in one application. The real roots of $f(x) = 0$ (and indeed any other features of the graph of $f(x)$ for $-r \leq x \leq r$) can then be read from this graph. The paper offers convincing evidence that this harmonic synthesizer is a powerful tool for studying polynomials and their roots.

D. H. L.

[1] *Bell Laboratories Record*, v. 16, 1937, R. L. DIETZOLD, "The isograph—a mechanical root-finder," p. 130–134; R. O. MERCNER, "The mechanism of the isograph," p. 135–140.

NOTES

10. BRIGGS AND VIETA.—That Briggs in early years of the seventeenth century should have computed a "canon of sines," and of other functions, for every hundredth of a degree in the quadrant, is a notable fact, and that he was probably mainly led to this choice of argument by a passage in Vieta's *Relatio Calendarii vere Gregoriani*, Paris, 1600 is of special interest. The passage on p. 1 of his *Trigonometria Britannica* (1633) where Briggs tells us this is, in translation from the Latin, as follows: "Therefore we divide any circumference into 360 parts which we call degrees; and each degree we divide sexagesimally into minutes and seconds, etc. But I, induced by the authority of Vieta '*pag.* 29. *Calendarij Gregoriani*,' and on the advice of others, divide degrees by the decimal system into 100 primary parts, and each of these into 10 parts, of which each one is divided in the same way. And these parts give a much easier, and not less certain method of calculation." With too hasty a glance at this passage I made an incorrect statement (which might have been corrected with the change of a single word) on page 33, line 12; see Corrigenda, p. 100. Since Vieta's *Relatio* of 1600 is not to be found in any of the larger libraries of America, I turned to the reprint on p. 447–503 of Vieta's *Opera Mathematica*, Leyden, 1646. But I failed to find anywhere a passage such as seemed to be referred to by Briggs. That this same difficulty was experienced by such a distinguished table-maker and authority as J. W. L. Glaisher, who had the original work of Vieta before him, is shown by the following note, in his handwriting, which I happen to possess (in several places the writing is difficult to read):

"80 pages 400 [should be 40] double pp. as the rectos are numbered 1 to 40."

"I have looked through *Francisci Vietae Fontenaeensis . . . Relatio Kalendarii vere Gregoriani* . . . 1600 (Colophon-excudebat Parisiis . . .) without finding either on p. 29 or anywhere else anything about the decimal division of the degree. Without wanting to say that the book does not contain anything of the kind, it is not unlikely that the reference is a wrong one. Vieta always in the [this?] book seems to use when he can dec[imal] p[arts] viz. fra[ctions] with 100 . . . as the den[ominato]r."

On laying this note before Alexander Pogo, an outstanding authority not only on the Gregorian but also on many ancient calendars, he cleared up the difficulty completely in the following communication.

R. C. A.

Vieta does not always use fractions with 1000,000,000 as denominator. He uses them on p. 487 of the Leyden 1646 edition for the first time, with an explanation indicating that not all his readers are ready to accept them; he uses the same denominator, 1000,000,000, again three times on p. 489, once on p. 497; he uses 1000 as denominator three times on p. 494; 10000 on p. 497; 100,000,000,000 on p. 497. That seems to be all.

And here is mathematical proof that leaf (folio, not page) 29 of the Paris 1600 edition is actually meant. There are 80 pages in the Paris edition, hence leaf 29 would correspond to p. 57 or p. 58 of the Paris edition. The Leyden edition contains the *Relatio* on pages 447 to 503, or on 57 pages, including a title page (p. 447); dividing 57 pages of the Leyden edition in the ratio of 57 : 80 or of 58 : 80, we find page 41 of the reprint, i.e., the page $446 + 41 = 487$ of the Leyden edition. This is the page on which Vieta introduces his $29\frac{530,592,348}{1000,000,000}$ days, after having given the clumsy execontades and $29\frac{22,313}{42,053}$. As far as I am concerned, this is Briggs's reference—fol. 29 (recto or verso?) of the Paris 1600 edition; page 487 of the Leyden 1646 edition can be quoted as the practically certain equivalent of fol. 29 of the Paris 1600 edition.

The idea of a decimal fraction of a degree is obviously contained in Vieta who gives decimal fractions of a day, instead of hours, minutes, seconds, etc. Briggs did not say that Vieta subdivides a degree decimally. Briggs must have seen the Paris equivalent of p. 487 of the Leyden edition, where Vieta says:

"At qualium dierum $29\frac{22,313}{42,053}$ seu aliter $29\frac{530,592,348}{1000,000,000}$ est mensis unus, talium menses duodecim sunt dierum $354\frac{367,108,176}{1000,000,000}$."

A better case for the simplicity of decimal subdivisions of a day or of a degree, avoiding the sexagesimal fractions, would be hard to find. You multiply 29.530,592,348 by 12, and you get the result.

A. Pogo

arvard University Library