UNPUBLISHED MATHEMATICAL TABLES

References are made to unpublished tables of PETERS in RMT **116**, of DAVIS in Q **5**, and of DAVIS and of TALLQVIST in RMT **118**.

10[A].—HERBERT ELLIS SALZER and ABRAHAM HILLMAN (1918–   ), *The First* 120 *Factorials*. Ms. in possession of the authors; a copy is in the Library of Brown University.

The exact values of the first 120 factorials were obtained as follows: Direct multiplication (checked by duplicate calculation at every step) was used to obtain the first 100 factorials. An additional check was performed by the independent calculation of every fourth or fifth factorial, and a manuscript was prepared which was proofread against the original. The value of 100! was found to agree with that of H. S. Uhler in Nat. Acad. Sci., *Proc.*, v. 28, 1942, p. 61. Then the table was extended to 120!. The largest table of factorials before this was given in J. T. Peters, *Zehnstellige Logarithmentafeln*, v. 1, Berlin, 1922, Anhang, p. 58, containing the first 60 factorials, all of which were found to be correct when proofread against this table. Finally, to close the gap of possible uncertainty, due to proofreading, the final manuscript was checked again by the independent computation of every value from 60! to 120!. Thus the authors are confident that no error exists in their manuscript.

In L. Potin, *Formules et Tables Numériques relatives aux Fonctions Circulaires, Hyperboliques, Elliptiques*, Paris, 1925, p. 836, are the first 50 factorials, which contain the following errors: 18! has an extra 0 at the end; 38! has the first group of ten numbers written as 52302 26147 which should read 52302 26174; 45! has the first group of ten numbers written as 11962 22086 which should read 1 19622 22086; and 50! has an error in the fifth group of ten numbers written as 75415 68960 which should read 76415 68960.

H. E. SALZER and A. HILLMAN

Mathematical Tables Project,
New York City

11[D, G].—*Table of Chebyshev Polynomials*. Preliminary Ms. prepared by, and in possession of, the PROJECT FOR COMPUTATION OF MATHEMATICAL TABLES, 50 Church St., New York City.

The Chebyshev Polynomials $C_n(x)$ and $S_n(x)$ are defined by the expansions of $2\cos n\theta$ and $\sin (n+1)\theta/\sin\theta$ in terms of the argument $x = 2\cos\theta$. These polynomials were computed to twelve decimal places for $n = 1, 2, \cdots, 12$ and for x ranging from 0 to 2 at intervals of 0.001. The only similar table in existence is contained in an unpublished manuscript of the Tables Committee of the British Association for the Advancement of Science; this manuscript lists twelve-place values of $C_n(x)$ for $n = 1, 2, \cdots, 12$ and for x ranging from 0 to 1 at intervals of 0.01.

Chebyshev Polynomials arise in many phases of mathematical analysis and are of particular practical importance as a tool for approximating empirical or analytic functions and for evaluating power series. An exhaustive bibliography of Chebyshev Polynomials is contained in *A Bibliography on Orthogonal Polynomials* (Nat. Res. Council, *Bull.* no. 103, 1940). See also G. SZEGÖ, *Orthogonal Polynomials* (Amer. Math. So., *Colloquium Publications*, v. 23, 1939).

A. N. LOWAN

12[L].—*Tables of Jacobi Elliptic Functions*. Preliminary Ms. prepared by, and in possession of, the PROJECT FOR COMPUTATION OF MATHEMATICAL TABLES, 50 Church Street, New York City.

These are Tables of $sn(u, k)$, $cn(u, k)$ and $dn(u, k)$, to 15D, for $k^2 = 0(0.01)1$, and for $u = pK$, where $p = 0.01$ and $p = 0.1(0.1)1$, and K is the complete elliptic integral corresponding to the modulus k.

From the computed values one can generate the values of the functions under consideration for imaginary arguments. Specifically, $sn(iu, k) = i\, sn(u, k')/cn(u, k')$; $cn(iu, k) = 1/cn(u, k')$; $dn(iu, k) = dn(u, k')/cn(u, k')$ where $k' = \sqrt{1 - k^2}$. It is also possible to generate the values of the function $am(u, k) = \sin^{-1} sn(u, k)$.

With the aid of the known addition formulae, it is further possible to generate the values of the function in question for all p's ranging from 0 to 1 at intervals of 0.01.

The three Jacobi elliptic functions were computed by two independent methods. (a) From their known expressions in terms of the theta functions, as defined in E. T. Whittaker and G. N. Watson, *Modern Analysis.* (b) By the AGM (arithmetico-geometric means) method, involving the sequence of number-triplets, as described, for instance, in L. V. King, *On the Direct Numerical Calculation of Elliptic Functions and Integrals*, Cambridge, Univ. Press, 1924.

The following auxiliary functions were also obtained in the course of the computations.

(1). The complete elliptic integral K for $k^2 = [0(0.01)1; 17S]$.

(2). $q^n = [\mathrm{Exp}\,(-\pi K'/K)]^n$ for $n = 1/4, 1/2, 1, 2, 3, 4, 6, 8, 9, 12, 16, 20$, and 25, to 17D and $-\log q$ to 15D; K' is the complete elliptic integral corresponding to the complementary modulus k'.

(3). The three theta functions: $\vartheta_2(\pi u/2K, q)$, $\vartheta_3(\pi u/2K, q)$, and $\vartheta_4(\pi u/2K, q)$ for $u/K = 0$, 0.01, and 0.1, to 17S.

(4). The first six sets of number triplets of the AGM methods to 17D.

The only known tables of the Jacobi Elliptic functions *sn*, *cn* and *dn* are contained in L. M. Milne-Thomson, *Die elliptischen Funktionen von Jacobi*, Berlin, Springer, 1931. This is a 5-place table in which *sn*, *cn* and *dn* are tabulated as a function of k^2 ranging from 0 to 1 at intervals of 0.1 and of the argument u whose range and interval are as follows: for the *sn* function, $u = 0(0.01)3(0.1)6.5$; for the *cn* function, $u = 0(0.01)3.50$; for the *dn* function, $u = 0(0.01)4$.

Some of the auxiliary functions listed under (1), (2), (3), and (4) are also tabulated in K. Hayashi, *Tafeln der Besselschen, Theta-, Kugel- und anderer Funktionen*, Berlin, Springer, 1930.

A. N. LOWAN

13[I, K].—*Table of coefficients for inverse interpolation.* Ms. prepared by, and in possession of the PROJECT FOR COMPUTATION OF MATHEMATICAL TABLES. 50 Church St., New York City.

Although many tables have been published to facilitate direct interpolation, there has been to date no table of coefficients to handle the more cumbersome task of inverse interpolation. To meet this need, the Project has computed the polynomial coefficients of the ratios of differences of various order in the formula obtained by the inversion of the Everett formula for direct interpolation (H. T. DAVIS, *Tables of the Higher Mathematical Functions*, v. 1, Bloomington, Ind., 1933, p. 82–83). Specifically: The coefficients of the five fourth order terms were calculated to ten decimal places at intervals of 0.001 of the argument $m = (u - u_0)/(u_1 - u_0)$. Also a short table giving the values of the coefficients of the ten sixth order terms was computed at intervals of 0.1. It was not necessary to compute the coefficients of the second order terms, since these coefficients are tabulated at intervals of 0.0001 in the table of Lagrangian interpolation coefficients to be published by the Columbia Press. Compare UMT **6**, p. 94. The table here described will be of particular value whenever inverse interpolation is to be carried out in a table with a fairly large tabular interval.

A. N. LOWAN and H. E. SALZER

14[L].—JAMES NICHOLAS SARMOUSAKIS (1912–), *Tables of associated Legendre functions with complex argument.* Manuscript and film in possession of the author. Film copies at the Mathematical Tables Project, New York City, and in the Library of Brown University.

The Tables of $P_n^m(ix)$ and of $Q_n^m(ix)$ are as follows:

$P_n^m(ix)$, $x = 0(.1)5$ for $n = 1(1)5$, $x = 0(.1)2.5$ for $n = 6$, $x = 0(.1)1.4$ for $n = 7(1)9$, $x = 0.1(.1)1$ for $n = 10(1)12$, $x = 0.1(.1)0.8$ for $n = 13, 14$, $x = 0.7$ for $n = 15(1)18$, and $m = 0(1)n$.

$Q_n^m(ix)$, $x = 0(.1)1.4$ for $n = 0(1)9$, $x = 0.1(.1)1$ for $n = 10(1)12$, $x = 0.1(.1)0.8$ for $n = 13, 14$, $x = 0.7$ for $n = 15(1)18$, and $m = 0(1)(n + 1)$. The tables range from 4S to 20S, mostly between 10S and 15S.

The computation of these tables was done only once and the sole auxiliary tables used in their preparation were the *Tables of the First Ten Powers of the Integers from* 1 *to* 1,000, New York, Project for the Computation of Mathematical Tables, 1938. A few miscellaneous checks have been made. As yet there has been no appropriate rounding off to a suitable number of significant figures.

I have used the tables to evaluate relative dissociation constants of substituted benzoic acids which were computed on the basis of an electrostatic theory of substituent effects.

J. N. SARMOUSAKIS

Aberdeen Proving Grounds

Such tables are also of use for computing potentials, flow and diffusion about disks and rods.—EDITOR.

MECHANICAL AIDS TO COMPUTATION

6[Z].—SIMPSON LEROY BROWN (1881–), "A mechanical harmonic synthesizer-analyzer," Franklin Institute, *Jn.*, v. 228, 1939, p. 675–694.

The 30-term synthesizer-analyzer described in this paper is perhaps the largest machine of its kind having so few harmonic components. It is 15 feet long and 7 feet high and weighs nearly a ton. This may be compared with the old Michelson and Stratton[1] 80-term machine which is about 3 feet long and 4 feet high. The reasons for building the machine on a large scale were to gain accuracy and to save labor. The driving mechanism, for example, is a train of 22 spur gears which, though of only commercial grade, are large enough so that more accuracy is obtained than with small expensive precision made gears.

Two simple harmonic motions, out of phase by 90°, are generated at both ends of 15 rotating shafts by Scotch crossheads. This gives 15 sine terms and 15 cosine terms in the corresponding Fourier series. The two fundamental motions are capable of amplitudes up to 16 centimeters while even the components of highest frequency may be set with amplitudes as great as 4 centimeters. Thus the machine draws curves well over a foot in overall width. The 30 harmonic motions are added, in the usual manner, by an endless fine chain passing over pulleys.

Harmonic analysis to, say, 60 terms could be performed by pencil and paper methods, better still by commercial computing machines; it may also be done entirely mechanically with a 60-term analyzer. The author points out that a combination of the two methods is not only possible but indeed desirable, and that by using an accurate synthesizer with comparatively few harmonic elements one can make analyses that include many harmonic elements by applying a little pencil and paper before and after using the analyzer.

This may be illustrated as follows. Let $y = f(x)$ be the subject of analysis, a periodic function defined empirically by 64 of its equally spaced arguments $(y_1, y_2, \cdots y_{64})$. We wish