MATHEMATICAL TABLES—ERRATA

References have been made to errata in RMT **143** (Dwight), **145** (Ko and Wang), **150** (Legendre), **154** (Adams, Dwight, Webster), N**18** (Silberstein), N**19** (Carrington, Jahnke & Emde), and N**20** (Boys, Byrne, Knappen).

29. Bessel and Hansen Tables of Bessel Functions.

It is well known that Bessel published in 1826 a table of $J_n(x)$, $n = 0, 1$; $x = [0.00(0.01)3.20; 10D]$. Because of misinformation published in sources usually highly authoritative it is not so well known that one of P. A. Hansen's tables of 1843 was of $J_n(x)$, $n = 0, 1$; $x = [0.0(0.1)20; 6D]$. Among others this was reprinted by Lommel in 1868.

In the absurd paragraph of 16 lines on "Tables de fonctions de Bessel" in *Encyclopédie des Sciences Mathématiques*, II, 5, 2, p. 228, one reads (omitting footnote references) "F. W. Bessel a calculé des tables des fonctions J_0 et J_1 pour des arguments variant de $x = 0$ à $x = 3, 2$. P. A. Hansen les a étendues jusqu'à $x = 10$, E. Lommel jusqu'à $x = 20$." [Hansen did not extend Bessel's 10-place table for hundredths of a unit to $x = 10$, and Lommel did not extend Hansen's table.] In Watson's *Bessel Functions*, 1922, p. 655, there are three misleading statements, "Hansen constructed a Table of $J_0(x)$ and $J_1(x)$ to six places of decimals with a range from $x = 0$ to $x = 10.0$ with interval 0.1 . . . Hansen's table was reprinted . . . by Lommel who extended it to $x = 20$." E. M. Horsburgh repeats one misleading phrase and makes one new error, on p. 57 of his *Modern Instruments and Methods of Calculation*, 1914, "P. A. Hansen's extension of Bessel's table is reproduced by . . . E. Lommel. . . . It gives $J_0(x)$ and $J_1(x)$ from $x = 0$ to $x = 20$ at intervals of .01 throughout the lower part of the range." It would seem as if all three authors did not recognize that $J_n(x) = I^n_{2x}$ (Hansen) $= J^n(x)$ (Lommel) [$= I^n_x$ (Bessel)].

R. C. A.

30. E. W. Brown with the assistance of H. B. Hedrick, *Tables of the Motion of the Moon*, New Haven and London, 1919. Compare *MTAC*, p. 29.

Apart from the erratum on p. xiv of these *Tables*, two lists of errata were published by Brown, namely: (1) in *Astr. J.*, v. 34, 1922, p. 54; and (2) in Yale Univ., Astr. Observatory, *Trans.*, v. 3, 1926, p. 157; in l. 4 from the bottom, *for* 92-1′, *read* 82-1′. Eight other errors sent to us are now presented. The one in Section II was supplied by D. H. Sadler, Superintendent of H. M. Nautical Almanac Office, three of those in Sections I and VI came from W. J. Eckert, Director of the Nautical Almanac Office, Washington, and the rest, in Sections I, III, VI, were taken by Dirk Brouwer, Director of the Yale University Observatory, from two copies of the *Tables* used by the late E. W. Brown.

Section I

P. 44, l. 3, *for* 70, *read* 71.
80, l.-2, *for* $n^{-\frac{2}{3}}$, *read* $n^{+\frac{2}{3}}$.
109, l. 10, *for* log cos ω, *read* log sin ω.

Section II

P. 31, arg. 84 for 1966, *for* 2305, *read* 2405.

Section III

P. 160, col. 17, arg. 16.5, *for* 19608, *read* 19598.
P. 187, col. 153, arg. 9.0, *for* 23878, *read* 23868.

Section VI

P. 63, col. 47, arg. 180, 190, 200, *for* 110, 117, 112, *read* 210, 217, 212.
P. 92, 0°59′0″, *for* 007 4464, *read* 007 4454.

31. EDWIN CHAPPELL (1883–1938), *A Table of Coefficients to Facilitate Interpolation by Means of the Formulæ of Gauss, Bessel and Everett.* London, The Author, 1929, viii, 27 p. 22.3 × 28 cm.

As the author of these tables is no longer living, it may be of interest to mention that the balance of the 110 copies printed (by Chappell himself) are now available from Scientific Computing Service Ltd., 23 Bedford Square, W.C. 1.

Besides the two corrections given by the author on page 2, the formula on the second page of the Introduction for $G^{vi} = B^{vi}$ should be preceded by a minus sign. On pages 26 and 27 the values of $G^{vi} = B^{vi}$ for 0.38 and 0.62 (which are the same) are both in error: *for* 0.004 56167, *read* 0.004 56157. The occurrence of this error (which was found about ten years ago by Mr. D. H. SADLER) in two places shows that it is an error in recording the original calculations, rather than one of composition. The fact that the values in Table I for 0.380 and 0.620 are correct shows that a comparison between the two tables was not made.

This table was compiled at the suggestion of the present writer, whose copy is inscribed "3/110. With the author's compliments and in recognition of the suggestion which has resulted in this book." The calculations were done on the first Brunsviga Dupla calculating machine imported into England—a machine which, by curious fate, also came into my hands some years after Chappell's death.

L. J. C.

EDITORIAL NOTE: This well printed and bound little volume of Chappell is worthy of a place in any large mathematical library, along with his *Five-figure Mathematical Tables: consisting of logs and cologs of numbers from* 1 *to* 40,000, *illogs (antilogs) of numbers from* .0000 *to* .9999, *lologs (logs of logs) of numbers from* 0.00100 *to* 1,000, *illologs (antilologs) of numbers from* $\bar{6}$.0 *to* 0.5000, *Together with an Explanatory Introduction and Numerous Examples; also, trigonometrical functions and their logs of angles from* 0°–90° *at intervals of* 1 *minute, With Subsidiary Tables.* Edinburgh, London, Chambers, and New York, Van Nostrand, 1915, xvi, 320 p. Compare Q **4**. Chappell was much interested in Pepysiana, and published (1933–36) at least four items concerning Samuel Pepys and his bibliography.

32. H. B. DWIGHT, *Tables of Integrals and Other Mathematical Data*, New York, 1934. See RMT **154.** Most of the following errata were contributed by the author.

P. vii, *for* page numbers 113, 114, 114, 117, 118, 121, *read* respectively: 111, 112, 112, 115, 116, 119.

93.1. Change $3aX$ to $3a^2X$.

After the sub-headings on pages 101, 102 and 144, insert ($a > 0$).

Omit No. 591.

603.2 For the part in square brackets, *read* [$\sin x \neq 0$]; and *for* logsin x, *read* $\log|\sin x|$.

603.4. For the part in square brackets, *read* [$\cos x \neq 0$]; and *for* logcos x, *read* $\log|\cos x|$. (See P. Franklin, *Differential Equations*, New York, 1933, p. 275).

P. 127, last line, *for* $\pi/4$, *read* π.

630.2, last line, *read* [$x^2 < \pi^2/4$].

673.19, *for* 657.6 *read* 657.8.

679.19, *for* 657.5, *read* 657.7.

Since a statement is made in 810.8 about the size of the error from using the asymptotic series, it would be better to add $(1/\pi)K_0(x)$ to 810.6; and $(K_n(x)/\pi)e^{i(n+\frac{1}{2})\pi}$ to 810.7.

815.2 *for* $s = 1$, *read* $s = 0$.

836.4 *for* ker x, *read* ker$'x$.

850.1 *for* It, *read* The integral.

851.4. Change to the asymptotic series:

$$\Gamma(n+1) = n^{n+\frac{1}{2}}e^{-n}(2\pi)^{\frac{1}{2}}\left[1 + \frac{1}{12n} + \frac{1}{288n^2} - \frac{139}{51840n^3} - \cdots\right].$$

Omit No. 861.9.

Table 1030: *read* $\log_{10}$ cosh .32 = .02187; sinh 1.20 = 1.5095; tanh 3.15 = .99633; tanh 3.55 = .99835.

Table 1050: *read* bei′ 3.7 = + 0.131 486 760. From x = 4.6 to 6.4, delete the last 3 digits of the values of bei′x.

P. 214, last line, *for* 109, *read* 122.

33. C. C. FARR, "On some expressions for the radial and axial components of the magnetic force in the interior of solenoids of circular cross-section," R. So. London, *Proc.*, v. 64, 1899, table p. 199–202. Reprinted without change in all five editions of JAHNKE and EMDE, 1909–1943.

On comparing with our recently calculated table for $dP_n(\cos\theta)/d\theta$, $n = 1(1)7$, we found 53 last figure errors, of which the 12 of more than a unit in the last figure are as follows: n = 3, 31°, *for* − 2.0654, *read* − 2.0656; n = 4, 5°, *for* − 0.8570, *read* − 0.8567; n = 4, 64°, *for* 1.6306, *read* 1.6300; n = 5, 54°, *for* 2.0173, *read* 2.0178; n = 5, 64°, *for* 1.5420, *read* 1.5418; n = 5, 67°, *for* 1.1187, *read* 1.1183; n = 5, 83°, *for* − 1.4831, *read* − 1.4827; n = 6, 23°, *for* − 3.0917, *read* − 3.0902; n = 6, 48°, *for* 2.3715, *read* 2.3713; n = 6, 57°, *for* 1.1936, *read* 1.1934; n = 6, 83°, *for* − 1.4487, *read* − 1.4484; n = 7, 70°, *for* − 1.984, *read* − 1.934. None of these 53 errors occur in the corresponding table of H. Tallqvist, *Grunderna af Teorin för Sferiska Funktioner jämte Användningar inom Fysiken*, Helsingsfors, 1905, p. 422–431.

We checked also the table of $P_n(\cos\theta)$, $n = 1(1)7$, $\theta = 0°(1°)90°$, on p. 121–123 of the fifth edition of JAHNKE and EMDE, and found no error.

MATHEMATICAL TABLES PROJECT

New York City

EDITORIAL NOTE.—The table of $P_n(\cos\theta)$ here referred to, is the one by HOLLAND, JONES and LAMB, originally with 92 errors (*MTAC*, p. 136–137). These were faithfully reproduced in JAHNKE & EMDE 1; but EMDE eliminated the errors, pointed out by TALLQVIST, in editions 2–5.

34. GREENHILL's first zero of $J_{-1/3}(x)$.

In Cambridge Phil. So., *Proc.*, v. 4, 1881, p. 68, G. GREENHILL gives the first positive root of $J_{-\frac{1}{3}}(x) = 0$ as 1.88; this was reproduced in the first edition (1909) of JAHNKE and EMDE, *Tables of Functions*, p. 106, and in A. DINNIK, (a) Russkoe fiziko-khimicheskoe Obshchestvo, *Zhurnal, Chast' fizicheskaia͡* v. 42, 1911, p. 4 of "Tablit͡sy funktsiĭ Besseli͡a $J_{\pm\frac{1}{3}}$ i $J_{\pm\frac{2}{3}}$"; (b) *Archiv Math. Phys.*, s. 3, v. 18, 1911, p. 338.

J. R. AIREY gave the more accurate value 1.8663, in *Phil. Mag.*, s. 6, v. 41, 1921, p. 203. Y. IKEDA in *Z. angew. Math. u. Mech.*, v. 5, 1925, p. 81 derived the value 1.86635 0858. In a table of the first five zeros of $J_{\pm\frac{1}{3}}$ and $J_{\pm\frac{2}{3}}$, Dinnik gave the value 1.860 (Akad. Nauk, U.R.S.R., Kiev, *Prirodnicho-tekhnichniĭ viddil*, 1932, p. 12). D. H. L. writes, "more accurately this zero is 1.86635 08588 73895 . . . ."

R. C. A.

35. K. HAYASHI, *A. Sieben- und mehrstellige Tafeln der Kreis- und Hyperbelfunktionen und deren Produkte sowie der Gammafunktion* . . . , Berlin, 1926, table of $\sin^{-1} x$ for radian argument x = [0.00000(0.00001)0.001; 20D], [0.0010(0.0001)0.0999; 10D] and [0.100(0.001)1; 7D], p. 5–86; *B. Fünfstellige Funktionentafeln* . . . , Berlin, 1930, table of $\sin^{-1} x$, for x = [0.00(0.01)1; 5D], p. 2–4.

With the exception of the 100 values given to 20D all of Hayashi's inverse sines were proof-read against the Project's manuscript of the same function, see UMT **7**, p. 94–95, and the following errors were discovered:

A

x	*for*	*read*	*x*	*for*	*read*
.0132	.01320 03883	.01320 03834	.479	.4995154	.4995152
.0168	.01690 07904	.01680 07904	.481	.5017937	.5017950
.0257	.02570 28099	.02570 28299	.528	.5562452	.5562438
.0532	.05322 51278	.05322 51268	.543	.5744904	.5740056
.0558	.05585 89975	.05582 89975	.548	.5798714	.5799714
.0582	.05823 29154	.05823 29064	.589	.6298207	.6298209
.0626	.06264 09575	.06264 09580	.640	.6944985	.6944983
.0627	.06276 11548	.06274 11548	.657	.7171234	.7168325
.0660	.06604 80057	.06604 80102	.676	.7422209	.7423209
.0691	.06915 51094	.06915 51084	.683	.7519620	.7518620
.0693	.06935 56180	.06935 55890	.690	.7614939	.7614891
.0732	.07326 53661	.07326 55287	.696	.7698129	.7698116
.0759	.07597 29638	.07597 30638	.705	.7824131	.7824231
.0886	.08871 63294	.08871 63291	.707	.7852467	.7852472
.0891	.08921 83127	.08921 83145	.732	.8212534	.8212529
.132	.1323913	.1323864	.744	.8390418	.8390370
.152	.1525922	.1525915	.776	.8872990	.8882990
.198	.1993227	.1993171	.837	.9922139	.9917776
.267	.2702835	.2702787	.856	1.0274823	1.0274821
.387	.3973754	.3973758	.863	1.0411481	1.0411781
.390	.3906316	.4006316	.914	1.1530364	1.1530362
.408	.4202672	.4202624	.927	1.1863332	1.1863334
.457	.4746204	.4746195	.933	1.2026605	1.2026609
.467	.4858971	.4858950	.948	1.2473617	1.2468920
.470	.4891804	.4892908	.951	1.2564570	1.2564542
.472	.4915880	.4915580			

In addition to the above, errors of a unit in the last place (a) a unit too large, (b) a unit too small (including doubtful rounding off), occur at the following values of x:

(a) $x=.0015$, .0109, .0134, .0138, .0177, .0238, .0381, .0389, .0402, .0539, .0647, .0666, .0746–.0748, .0817, .0819, .0924, .0963–.0969, .304, .329, .359, .422, .468, .506, .531, .539, .567, .595, .628, .691, .694, .703 and .742.

(b) $x=.0209$, .0272–.0279, .0305, .0512, .0614, .0616, .0619, .0707, .0715, .0836, .0876, .0878, .0889, .0903, .0914, .0916, .0923, .0927–.0929, .0947, .0949, .0957, .0973, .0976, .0980, .0981, .0985, .0989, .167, .180, .181, .195, .346, .368, .369, .395, .453, .458, .516, .520, .529, .579, .582, .583, .602, .617, .633, .635, .637, .667, .740, .773, .986 and .994.

B

$x=.21$, *for* .21156, *read* .21157; $x=.39$, *for* .39063, *read* .40063; $x=.47$, *for* .48918, *read* .48929.

Although in 1932 Hayashi issued a pamphlet of corrections to his *Sieben- und mehrstellige Tafeln* he failed to note any errors in his $\sin^{-1} x$ values.

H. E. Salzer and E. Isaacson

Mathematical Tables Project
New York City

Editorial Notes.—It seems desirable to elaborate the incidental reference to the 12-page pamphlet in which Hayashi published an extensive list of corrections. It is entitled *Berichtigung in Hayashis sieben* [*sic*] *u. mehrstellige Tafeln* (1926), Fukuoka, 1932. 19.1 x 26.4 cm. The photoprint of this pamphlet in the Library of Brown University was made from an original in the Library of Columbia University. There are 8 pages (with 3 columns on a page) of corrections; in e^x and e^{-x}, $1\frac{1}{2}$ cols. (about 75 corrections); in $\sin x$ and $\cos x$, about 11 cols. (550 corrections), etc. Eleven corrections of the *Berichtigung*, and six additional corrections have been made by hand (presumably Hayashi's). L.J.C. writes as follows, however: "This pamphlet has the same percentage of error as the original volume and, what is more, there are some cases where things that were right in the original are put wrong by these so-called corrections."

We may also add notes regarding errors in radian tables of $\tan^{-1} x$ in the above mentioned works of Hayashi, *A* and *B*. In *A* the tables are for $x=[0.00000(0.00001)0.001; 20D]$, $[0.0000(0.0001)0.0999; 10D]$, $[0.000(0.001)2.999; 8D]$, $[3.00(0.01)10.0(0.1)20(1)50; 7D]$. In his *Tables of* $\tan^{-1} x$ *and* $\log(1+x^2)$, (*Tracts for Computers*, no. XXIII, 1938), L. J. Comrie states that Hayashi's tables were first checked through the first quadrant, i.e. up to $x=1.570$, and 100 errors exclusive of errors in the last decimal were found. Comrie remarked further that over 100 errors were found for later values of x. In the above mentioned Hayashi pamphlet 71 corrections are given for these tables. In B, 13 errors not mentioned in his errata occur in the table for $\tan^{-1} x$, $x = [0(0.01)10.00; 5D]$, and are listed in *The Table of Arc Tan x* by the MATHEMATICAL TABLES PROJECT (New York, 1942); see RMT **90**.

36. P. R. E. JAHNKE and F. EMDE, *Table of Functions*, first–fifth eds., 1909–1943. Compare RMT **113**; MTE **21**, **23**, **33**, **34**, **37**. Page references are to the first and fifth editions.

P. 54 (62), $F(15°, 32°)$, *for* 0.5604, *read* 0.5603. P. 56, $F(60°, 5°)$, *for* 0.08745, *read* 0.08735, error in first ed. only. P. 61 (69), $E(15°, 16°)$, *for* 0.2788, *read* 0.2790; $E(20°, 16°)$, *for* 0.2786, *read* 0.2788; $E(25°, 16°)$, *for* 0.2790, *read* 0.2786. P. 64 (72), $E(50°, 64°)$, *for* 1.0072, *read* 1.0007; $E(80°, 64°)$, *for* 0.9027, *read* 0.9072; $E(90°, 86°)$, *for* 0.9926, *read* 0.9976. P. 68 (85), $E(8°)$, *for* 1.5630, *read* 1.5632, error in first and second eds. only; $K(86°48')$, *for* 4.2744, *read* 4.2746; $K(87°36')$, *for* 4.5619, *read* 4.5609; $K(89°36')$ *for* 6.3504, *read* 6.3509. Except in the two cases noted these errors occur in all five editions.

S. P. GLAZENAP, *Matematicheskie i Astronomicheskie Tablitsy*, Leningrad, 1932, p. 214–215.

37. P. R. E. JAHNKE and F. EMDE, *Table of Functions*, third, fourth and fifth eds., 1938–1943. Compare RMT **113**, and MTE **36**.

On p. 79 the following recurrence formula is given for the complete Legendre elliptic integral:

$$4(n+1)^2 \int \mathbf{E} x^n dx - (2n+3)(2n+5) \int \mathbf{E} x^{n+1} dx = 2x^{n+1}[\mathbf{E} - (2n+3)(1-x)\mathbf{K}].$$

It is readily verifiable that the right hand member should be

$$2x^{n+1}\{[2n+1-(2n+3)x]\mathbf{E} + (1-x)\mathbf{K}\}.$$

I. OPATOWSKI

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EDITORIAL NOTE.—This formula was given correctly in the source indicated by EMDE, namely: K. F. MÜLLER, *Archiv f. Elektrotechnik*, v. 17, 1926, p. 337.

38. MATHEMATICAL TABLES PROJECT, *Tables of Arc Tan x*, New York, 1942. Compare RMT **90**.

P. 27, $x = 2.554$, 2.557–2.559, insert missing decimal points in function.
P. 27, $x = 2.570$, insert missing decimal point in argument.
P. 41, $x = 3.999$, insert missing decimal point in function.
P. 68, $x = 6.645$, insert missing 1 in function.
P. 160 (some volumes, not all) $x = 5450$, insert missing 1 in function.

Mrs. RUTH CAPUANO

50 Church St., New York City

39. MATHEMATICAL TABLES PROJECT, *Tables of the Exponential Function e^x*, New York, 1939.

The integral part of the entry corresponding to the argument 2.3979 on p. 240 should be 11 in lieu of 10.

A. N. LOWAN

40. MATHEMATICAL TABLES PROJECT, *Table of Natural Logarithms*, 1941.

V. 1, p. 82, argument, *for* 9023, *read* 8023; v. 2, p. 355, argument, *for* 85201, *read* 85301.

J. W. WRENCH, Jr.

41. A. J. THOMPSON, *Tables of the Coefficients of Everett's Central-Difference Interpolation Formula*, London, second ed., 1943. See RMT **148**.

An error in the first edition (1921) was pointed out by CHAPPELL (see MTE **31**).

Page 10, ϵ_2 for $\phi = 0.468$, *for* .06091 51280, *read* .06091 61280. A recent comparison by Mr. E. S. DAVIS has shown that there are no other discrepancies between the two editions. The columns ϵ_2 and ϵ_4 have been checked by Miss D. P. KILNER by comparison with the appropriate columns of my Burroughs-script tables of 4-point and 6-point Lagrangean coefficients. From these checks, and Thompson's known high standard (no errors are known in his monumental *Logarithmetica Britannica*), there is every reason to believe that the rest of his figures are correct.

The following trivial corrections in the text have been pointed out to me in letters from Thompson and J. C. P. MILLER.

Page	line	*for*	*read*	authority
viii	12	$\delta^8{}_{1.0}$	$\delta^8 u_{1.0}$	J.C.P.M.
32	−2	δ^2 to δ^{10}	ϵ_2 to ϵ_{10}	A.J.T.

L. J. C.

42. H. WEBER, *Theorie der Abelschen Functionen vom Geschlecht* 3, Berlin, 1876. The following corrections in Table II, p. 183 (The complete system of odd characteristics), were given in a letter, dated 10 February 1933, from the late H. S. WHITE to the late W. F. OSGOOD.

"Left-hand column, 7th characteristic from the top, instead of $p\begin{pmatrix}000\\011\end{pmatrix}$, $\cdots$, it should read (at least this is one of the 8 correct systems)

p	β_1	β_2	β_3	β_4	β_5	β_6	β_7
$\begin{pmatrix}000\\111\end{pmatrix}$	111 111	001 011	110 100	101 011	010 111	100 100	011 110

Otherwise, replace this characteristic, no. 7, left, by the no. 11, right, inverted, with its 7β's also inverted.

"In system 9, right-hand column, replace β_4 by $\begin{matrix}100\\110\end{matrix}$.

In system 8, right-hand column, replace β_3 by $\begin{matrix}111\\100\end{matrix}$.

In system 12, right-hand column, replace β_4 by $\begin{matrix}011\\101\end{matrix}$.

In system 13, right-hand column, replace β_4 by $\begin{matrix}100\\101\end{matrix}$.

In system 11, left-hand column, replace β_4 by $\begin{matrix}010\\111\end{matrix}$.

In system 17, right-hand column, replace β_4 by $\begin{matrix}100\\101\end{matrix}$.

It is singular that Weber should have let so many errors slip by! And notice that five of them are in column β_4. That must indicate something about his field of vision."

43. H. E. H. WRINCH and D. M. WRINCH, *Phil. Mag.*, s. 6, v. 47, 1924, p. 63, $I_0(30) = 7.81674 \times 10^{11}$; R. C. COLWELL and H. C. HARDY, *Phil. Mag.*, s. 7, v. 24, 1937, p. 1046, $I_0(30) = 5.70 \times 10^{11}$.

The EDITORS brought these contradictory results in tables, already referred to in *MTAC* (p. 139–140), to my attention, and suggested that it would be a matter of interest to have the correct result determined.

In the asymptotic expansion

$$I_0(x) = \frac{e^x}{(2\pi x)^{\frac{1}{2}}}\left\{1 + \frac{1^2}{1!8x} + \frac{1^2\cdot 3^2}{2!(8x)^2} + \frac{1^2\cdot 3^2\cdot 5^2}{3!(8x)^3} + \cdots\right\},$$

upon setting $x = 30$, and evaluating 25 terms of the series, each to 22 decimal places, I found the sum of the terms in the braces to be

1.00424 76530 20713 59155(8).

This calculation was performed twice,—first by means of a calculating machine, the second time long-hand. The results agreed perfectly.

The value of e^{30} was found in the W.P.A. *Tables of the Exponential Function* e^x (1939), p. 533, to 19S. Subsequently I checked and extended this approximation to 38S. I have unpublished values of $\pi^{\frac{1}{2}}$ and $1/\pi^{\frac{1}{2}}$ to 317 and 310 decimal places respectively. These were calculated from π and $1/\pi$, respectively, and the product of the square roots was formed with the assistance of a machine, and was found to consist of a sequence of 309 consecutive 9's. Consequently I have great confidence in the accuracy of these roots. The value of $15^{\frac{1}{2}}$ was determined to 40D. By multiplication I found $e^{30}15^{\frac{1}{2}}/30\pi^{\frac{1}{2}}$ $= 7.78366\,06884\,04464\,04193\,55906\,75 \times 10^{11}$; and finally, $I_0(30) = 7.81672\,29782\,39774\,8972$ $\times 10^{11}$, correct to 20S.

Hence the value of Wrinch and Wrinch is nearly correct, the error being in the last significant figure which they give. The value of Colwell and Hardy is entirely incorrect.

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UNPUBLISHED MATHEMATICAL TABLES

References have been made to Unpublished Mathematical Tables in RMT **138** (Davis, Mills), and RMT **157** (Davis, Math. Tables Project).

21[B].—*Tables of Fractional Powers*, Mss. prepared by, and in possession of, the Mathematical Tables Project, 50 Church St., New York City.

Of these 12 tables 6 are of A^x, and 6 of x^a, as follows:

I: $A = 2(1)9$, and $x = [0.001(0.001)0.01(0.01)0.99;\ 15D]$.
II–IV: $A = 10$, π, and Euler constant, $x = [0.001(.001)1.000;\ 15D]$.
V: $A = 0.01(0.01)0.99$, and $x = [0.001(0.001)0.01(0.01)0.99;\ 15D]$.
VI: $A = p10^{-3}$, $p =$ the primes between 101 and 997, and $x = [0.001(0.001)0.01$-$(0.01)0.99;\ 15D]$.
VII–XI: $x = [0.01(0.01)9.99;\ 15D]$ and $a = \pm\frac{1}{4}, \pm\frac{1}{2}, \pm\frac{3}{4}, \pm\frac{1}{3}, \pm\frac{2}{3}$.
XII: $a = 0.01(0.01)0.99$. For any a the values of x^a were computed at the interval 0.001 in x approximately up to that value of x for which the derivative of the function (which is ∞ at $x = 0$) is in the neighbourhood of unity. All entries are to 7D.

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22[D, H].—*Roots of the equation* $\tan x + ax = 0$, $a = 3/(8\pi)$, $3/(12\pi)$, $3/(16\pi)$. Ms. in possession of the Department of Aeronautical Engineering, University of Michigan, Ann Arbor, Mich. Compare Q. **8**.

The first three roots of each of these equations were calculated to 5S, and the next five roots to 3S. First, we obtained an approximate value x_n^0, graphically, and then applied