82/62; that this has been allowed to pass indicates that this table was not compared, *after typing*, with Table 1, as it shoud have been.

In Table 4 there are three omissions, and two cases in which the same factors have been assigned to two numbers, namely $27 \times 63 = 1701$ (not 1702) and $49 \times 95 = 4655$ (not 4645).

Summarizing, we take the view that the standard of table-making shown in this volume is not high enough to meet modern requirements. In other words, the author has not been fair to his users. He, and his publishers, and the engineering public, should be grateful to Mr. Johnston for his complete duplication of the numerical work.

L. J. C.

UNPUBLISHED MATHEMATICAL TABLES

References have been made to Unpublished Mathematical Tables in no. 7 of *MTAC*, part II, Bibliography under: Airey, BAASMTC 2, 3, 4, 5, 6, 7, 8 (Bickley, Mrs. Cashen, Comrie, Gwyther, Hartley, Johnston, Jones, J. C. P. Miller, Thompson), Comrie, Corrington & Miehle, Darmstadt Technische Hochschule, H. T. Davis, J. Fischer, W. Fischer, Kohler, Morse & Haurvitz, NYMTP 5, 6, 7, 8, 9, 10, 11, 12, 13, Tveritin. See also the first article of this issue, referring to a table by Miller & Bickley).

24[A].—*Table of $n!/m!$*. Ms. prepared by, and in possession of H. E. Salzer, NYMTP, 150 Nassau St., New York City.

The quantities $n!/m!$ were computed for $n = 1(1)42$, $m = \overline{n - 2(-2)1}$ if n is odd, 2 if n is even. Exact values or 20 significant figures are given. A similar larger table in L. Potin, *Formules et Tables Numériques relatives aux Fonctions Circulaires, Hyperboliques, Elliptiques*, Paris, Gauthier-Villars, 1925, p. 842–849, contains $n!/m!$ for $n = 1, 2, \cdots 50$ and $m = (n - 1), (n - 2)$ down to either 1 or $n - 25$ (exact values). In Potin's notation, he tabulates $A_m^n \equiv m(m - 1)(m - 2) \cdots (m - n + 1)$ for m up to 50 and n up to 25.

The overlapping parts of the two tables were compared and the following errors are to be noted in Potin's values: In A_{34}^{24}, the sixth from the right group of three figures, *for* 403, *read* 463. A_{25}^{4}, *for* 303100, *read* 303600. The following obvious errors occur in the text on p. 841: "pour m et n variant de 1 à 50" is incorrectly stated, since the n goes only as far as 25. The formula $A_m^n = A_m^{n-1}(m + n - 1)$ should read $A_m^n = A_m^{n-1}(m - n + 1)$; and on the last line, A_{m+1}^n should read A_{m+i}^n.

H. E. Salzer

25[A].—*Table of the Coefficients of the Central Factorial Polynomials.* Ms. prepared by, and in possession of H. E. Salzer, NYMTP.

This table lists the quantities B_{2m}^{2n+2}, the exact values of the coefficients of x^{2m} in the polynomials $x^2(x^2 + 1^2)(x^2 + 2^2) \cdots (x^2 + n^2)$, of degree $2n + 2$, for $n = 1(1)20$, $\overline{2m = 2n + 2(-2)2}$, i.e. up to polynomials of the 42nd degree. The coefficients of x^{2m} in the central factorial polynomials $x^{[2n+2]} \equiv x^2(x^2 - 1^2)(x^2 - 2^2) \cdots (x^2 - n^2)$ are simply $(-1)^{m+n+1}B_{2m}^{2n+2}$ which are also denoted by $D^{2m}0^{[2n+2]}/(2m)!$ where $D^{2m}0^{[2n+2]}$ are usually known as the "central derivatives of zero." The values of B_{2m}^{2n+2} were obtained from the recurrence formula $B_{2m}^{2n+2} = n^2B_{2m}^{2n} + B_{2m-2}^{2n}$, starting with $B_2^2 = 1$, $B_m^2 = 0$ for $m \neq 2$, and all values on the final manuscript were checked by the relation $\sum_{m=1}^{n+1} (-1)^{m+n+1}B_{2m}^{2n+2} = 0$.

The quantities B_{2m}^{2n+2} play an important role in the calculus of finite differences whenever central factorial polynomials are to be expressed in power series. They are used to calculate derivatives from central differences, as well as in numerical integration with central differences. Other uses are in interpolation using central differences and the calculation of generalized Bernoulli numbers. The numbers $D^{2m}0^{[2n+2]}/(2m)!$ have been scarcely tabulated before, there being a few values, up to $n = 4$, given in J. F. STEFFENSEN, *Interpolation*, p. 59 and as far as $n = 5$ in H. T. DAVIS, *Tables of the Higher Mathematical Functions*, v. 2, p. 215.

H. E. SALZER

26[A].—*Table of Coefficients for expressing Powers in Terms of Factorials.* Ms. prepared by, and in possession of H. E. SALZER, NYMTP.

This table gives the quantities C_n^m, the coefficients of $x^{(m)} \equiv x(x-1)(x-2)\cdots(x-m+1)$ in the expansion $x^n = \sum_{m=1}^{n} C_n^m x(x-1)(x-2)\cdots(x-m+1)$. The table gives the exact values of all coefficients through $m = 20$. The numbers C_n^m are usually denoted by $\Delta^m 0^n/m!$, i.e. $\Delta^m x^n/m!$ when $x = 0$, where $\Delta^m 0^n$ are commonly called "differences of zero." In his *Calculus of Finite Differences*, C. JORDAN calls the quantities C_n^m, "Stirling numbers of the second kind" and denotes them by a Gothic S in place of the C. He gives a very complete discussion of the quantities, p. 168–199, and a small table, up to $n = 12$, on p. 170. H. T. DAVIS gives a similar table in his *Tables of the Higher Mathematical Functions*, v. 2, p. 212, and in Steffensen's *Interpolation*, p. 55, there is a table up to $n = 10$. All values in these tables were found to be correct. The present table was calculated from the recurrence formula $C_{n+1}^m = C_n^{m-1} + mC_n^m$, starting with $C_n^1 = 1$ and $C_n^m = 0$ for $m > n$. All values were checked by the formula in Jordan, p. 188, equation (15) which is $\sum_{m=1}^{n+1} C_{n+1}^m = \sum_{m=1}^{n} (1+m)C_n^m$.

The quantities C_n^m occur in many important problems of finite differences, probability and number theory. They are used in converting power series into factorial series, summation of series, calculation of Bernoulli numbers, inversion of sums and series, expression of differences in terms of derivatives, expression of series of reciprocal factorials as power series, in the study of the operator $x\dfrac{d}{dx}$, formulation of power moments in terms of factorial moments, and in the theory of interpolation.

H. E. SALZER

EDITORIAL NOTE: The first table of C_n^m, up to $n = 9$, was given by JAMES STIRLING (1692–1770) in his *Methodus Differentialis* . . ., London, 1730, p. 8. Unfortunately it was not until after Mr. Salzer's report was in type that we observed that the table in question was not "unpublished," but was given by CAYLEY over 60 years ago, Camb. Phil. So., *Trans.*, Vol. 13, 1883, p. 2; *Coll. Math. Papers*, v. 11, 1896, p. 145–146. See also *MTAC*, p. 318, T. XXII, up to $n = 25$.

27[L].—FRANKLIN VICTOR RENO (1911–), *Tables of the Legendrean Elliptic Integral of the First Kind and the Sine-Amplitude Function.* Ms. in possession of the author, Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland.

The two quantities here tabulated are

$$u = F(\phi, k^2) = \int_0^\phi d\phi/[1 - k^2 \sin^2 \phi]^{\frac{1}{2}},$$

and its inverse, $sn(u, k^2)$. The values of u are given to 4D for $k^2 = 0(.01)1$, and $\phi = 0(1°)90°$. The values of its inverse are tabulated for $k^2 = 0(.01)1$, and $s = 0(.01)1$, where $s = u/k$.

The tables were computed to a considerable number of decimal places beyond those tabulated in order to insure that these latter would be correct to, and including the last figure set down. Linear interpolation will yield a result correct to within one digit in the fourth decimal place.

F. V. RENO

EDITORIAL NOTE: In the tables of N. SAMOĬLOVA-ÎAKHONTOVA, RMT **50**, p. 6, is a table of u for $\phi = 0(1°)\ 90°$, and $k^2 = [0(.01)1;\ 5D]$, Δ. The tables of L. M. MILNE-THOMSON, *Die elliptischen Funktionen von Jacobi*, Berlin, Springer, 1931, include a table of $sn(u, k^2)$, for $k^2 = 0(.1)1$ and $u = 0(.01)3$, with Δ; also for $k^2 = 1$, and $u = 3(.1)6.5$.

28[L].—GEORGE WELLINGTON SPENCELEY (1886–). *Tables of the seven elliptic functions A, D, (Jacobi Theta Functions) F, E, sn, cn, dn,* prepared during the years 1938–43, with the seasonal assistance of N.Y.A. students, Miami University, Oxford, Ohio.

These tables were inspired by, and follow precisely, the pattern designed by GREENHILL, and carried out by R. L. HIPPISLEY (*Smithsonian Mathematical Formulae and Tables of Elliptic Functions*, Washington, D. C., first reprint, 1939). Hippisley's tables, to 10D, consist essentially of the four elliptic functions A, D, F, E, computed for modular angle $\theta = 5°(5°)80°(1°)89°$. His F column is the traditional value; his E column is the "periodic part of $E(\varphi)$"; his A, D columns are "normalized Jacobi Theta Functions."

The author's tables are more complete in that they are computed to 15D for modular angle $\theta = 1°(1°)89°$, and also include the sn, cn, dn functions. In these tables the E column has its traditional value. The *ms* shows 15D for all columns, with an error not greater, it is believed, than $\pm 2 \times 10^{-15}$. The original intention of the author was to cut to 12D on publication.

G. W. SPENCELEY

MECHANICAL AIDS TO COMPUTATION

11[H, Z].—D. P. ADAMS, "The quintic 'hypernom' for the equation $x^5 + Ax^3 + Bx^2 + Cx + D = 0$, a graphical method for finding the roots of polynomial equations through the fifth degree," *J. Math. Phys.*, M.I.T., v. 22, 1943, p. 78–92. 17.6 × 25.5 cm.

This paper describes a chart for the approximate determination of the roots of the reduced quintic

$$x^5 + Ax^3 + Bx^2 + Cx + D = 0.$$

By direct use of the chart, the real positive roots not exceeding 3.5 may be located in case the coefficients A, B, C, D do not exceed 10 in absolute value. By the simple expedient of changing the signs of B and D, the negative roots greater than -3.5 likewise may be determined. The author's statement that root values lie between 0 and 10 appears to be an error.

The chart consists of

(1) Three vertical linear scales for A, B, and C at the side margins,
(2) A family of curves D = constant,[1]
(3) Two families of vertical lines (L lines and M lines) bearing indices ranging from 1 to 32,
(4) Two identical horizontal non-linear "root" scales at the upper and lower margins.

The chart is unusual in that an auxiliary "S" curve must be plotted on it, a curve characteristic of the particular equation under investigation (depending, in fact, on A, B, and C only). Perhaps this feature accounts for the name "hypernom." The intersections (if any) of this S curve with the curve of the D family corresponding to the constant term of our equation determine the positive roots of the equation. The plotting of the S curve (which is preferably done on tracing paper placed over the chart), is fairly simple and consists in determining a series of points by drawing parallel lines across the grid of L and M lines.

The chart is intended as an aid to Horner's or Newton's method. The paper has six illustrative problems showing the use of the chart in various cases. Well-known methods of finding the 2 or 4 complex roots, once the real roots of the quintic have been found, are discussed. In all these examples the residual cubics and quartics happen to have coefficients not exceeding 10 in absolute value and roots less than 3.5. Some treatment of the question of quickly transforming quintic equations with large roots or large coefficients in order to be