supposed to have come into use much more recently than 1826, but they are illustrated by Legendre in volume 2 of his *Traité des Fonctions Elliptiques*, Paris, 1826, paragraphs 672, 673, 676, as the following extracts show:

672, p. 28. "Je remarquerai que lorsque les différences quatrièmes $\delta^4 A$ sont assez grandes pour que les différences suivantes $\delta^5 A$ aient quelqu'influence dans les interpolations, il conviendra de prendre $\delta^4 A - \frac{7}{10}\delta^5 A$ au lieu de $\delta^4 A$." He states that, x being the fraction of tabular interval required in the interpolate (he is here using forward differences) the relevant terms are $\frac{x(x-1)(x-2)(x-3)}{1.2.3.4}\left(\delta^4 A + \frac{x-4}{5}\delta^5 A\right)$; then—"comme $\delta^5 A$ est censé très petit par rapport à $\delta^4 A$, si l'on donne à x une valeur moyenne $\frac{1}{2}$, le terme $\frac{x-4}{5}\delta^5 A$ se reduira à $-\frac{7}{10}\delta^5 A$."

673, p. 29. Similarly a sixth difference can be "thrown back," as we should now say, to the preceding fifth, by taking $\delta^5 A - \frac{3}{4}\delta^6 A$ in place of $\delta^5 A$, $-\frac{3}{4}$ being, as before, the mean value of $\frac{x-5}{6}$ obtained for $x = \frac{1}{2}$.

676, p. 31. "Pour avoir le milieu entre deux termes consécutifs A, $A\text{I}$ d'une suite dont les différences deviennent progressivement plus petites qu'une quantité donnée, il est bon d'avoir recours aux termes qui précèdent et qui suivent les deux termes proposés."

Legendre is interpolating into the middle of an interval, and in modern notation the formula he gives is the following:

$$f_{\frac{1}{2}} = \frac{f_0 + f_1}{2} - \frac{1}{2}\frac{(\Delta_0^2 + \Delta_1^2)}{8} + \frac{1.3}{2.4}\frac{\Delta_0^4 + \Delta_1^4}{32} - \frac{1.3.5}{2.4.6}\frac{\Delta_0^6 + \Delta_1^6}{128} + \cdots$$

precisely the Everett formula for interpolation at 0.5 tabular interval.

C. R. Cosens

QUERIES—REPLIES

12. Log Log Tables (Q 4, p. 131; QR 9, p. 336).—In answer to this question it would seem to be in order to draw attention to Slide Rules with log log scales, which permit ready solution of complicated problems in raising of powers, and extracting a root besides obtaining values of natural logarithms, and hyperbolic functions. For example, to evaluate

$2.31^{5.67}$; $(.371)^{1/15.8}$; $2.87^x = 73.7$; $x^{.378} = .582$; $e^x = .748$; $\ln 31$, $e^{\pm 3}$

and various hyperbolic functions of 3.

The Keuffel & Esser Co., Hoboken, N. J., has manufactured (1) The log log duplex trig slide rule with trigonometric scales to represent degrees and minutes; (2) The log log duplex decitrig slide rule, with trigonometric scales to represent degrees and decimals of a degree; (3) The log log duplex vector rule. In Jan. 1943 these rules, 10 in. size, were respectively listed at \$12.50, \$12.50, \$13.50; and the 20 in. size at \$27.00, \$27.00, and \$31.50. Reproductions of the scales of these rules are given in *K & E, Slide Rules and Calculating Instruments*, New York, 1941, p. 313f–313i; and also in figs. 5–7 of the last of the following works:

C. N. Pickworth, *The Slide Rule: A Practical Manual*, fourteenth ed., Manchester, London, etc., 1916; "Slide rules with log.-log. scales," p. 84–92, "log-log duplex slide rule," p. 115.

J. E. THOMPSON, *A Manual of the Slide Rule, its History, Principle and Operation*, New York, Van Nostrand, 1930; fifth printing, 1942; "The log-log scale and the log-log duplex slide rule," p. 117–133; "Settings and typical problems involving the log-log scales," p. 185–203. Thompson's work is also in T. O. SLOANE, *Speed and Fun with Figures*, New York, Van Nostrand, 1939.

H. O. COOPER, *Slide Rule Calculations*, Oxford Univ. Press, 1931; "Log-log scales," p. 81–91, 118–125.

L. M. KELLS, W. F. KERN & J. R. BLAND, *The Log Log Duplex Decitrig Slide Rule no.* 4081, *A Manual*, New York, Keuffel & Esser, c. 1943, 103 p.

R. W. FRENCH, *Engineers' Slide Rule*, St. Louis, Mo., J. S. Swift, 1941; "Log log scales," p. 66–83.

J. J. CLARK, *The Slide Rule and Logarithmic Tables including a ten-place table of logarithms . . .*, third ed. *including a table of natural trigonometric functions*, Chicago, F. J. Drake, 1943 [c. 1941], "The log-log scale," p. 90–103.

E. J. HILLS, *A Course in the Slide Rule and Logarithms*, Boston, Ginn, 1943; "The log log scales," p. 84–87.

The first log log slide rules (linear and circular) were invented in 1814 by PETER MARK ROGET (1779–1869) a physician in Manchester and London, and his paper on "Description of a new instrument for performing mechanically the involution and evolution of numbers," R. So. London, *Trans.*, v. 105, 1815, p. 9–28 + plates II–IV, led to his election, in 1815, as a fellow of the Royal Society. He was also a discoverer of the path for the knight moving on a chess-board so as to cover every square once and only once (*Phil. Mag.*, v. 16, 1840, p. 306f, 498f). For a biography of Roget see R. So. London, *Proc.*, v. 18, 1870, p. xxviii–xl. He was secretary of the Society for 21 years.

After Roget there were many others connected with various log log slide rules. For example J. A. BURDON (Acad. d. Sci., Paris, *Comptes Rendus*, v. 58, 1864, p. 573–576), F. BLANC (W. v. DYCK, Deutsche Math.-Ver., *Katalog mathem. u. mathem.-phys. Modelle, Apparate u. Instrumente*, Munich, 1892, p. 145f) from which, in particular, values of sinh x and cosh x could be immediately read off, and W. SCHWETH, Ver. deutscher Ingenieure, *Z.*, v. 45, 1901, p. 567–8, 720. In his article on "Calculating machines" in v. 4, 1929, p. 553, of the *Encyclopædia Britannica*, fourteenth ed., the late D. BAXANDALL stated that "The log-log scale was reinvented and applied to the slide rule by Captain J. H. THOMSON in 1881, and by Prof. JOHN PERRY in 1902." That this statement is incorrect so far as Perry is concerned, at least, may be noted by consulting *Nature*, v. 67, 1902, p. 141, which tells us that Perry was merely making Roget's ideas more generally known. Nevertheless the "Peter and Perry" log log rule was manufactured by Albert Nestler of Lahr, Baden, Germany, and "Perry's new slide rule" by A. G. Thornton of Manchester, England (see *Engineering Record*, v. 58, Dec. 12, 1908, current news suppl., p. 37, and Pickworth's *Slide Rule*, p. 91f). Pickworth describes also log log rules introduced by John Davis & Son, Ltd., of Derby, England (H. C. DUNLOP and C. S. JACKSON, *Slide Rule Notes*, London, 1901); and A. W. Faber, of Stein (near Nürnberg), Germany. Thomson's rule was exhibited in the International Exhibition of Inventions, London, 1885, with full recognition of its earlier discovery by Roget (*Van Nostrand's Engin.*

Mag., v. 33, 1885, p. 517). More information about log log slide rules is contained in Baxandall's *Catalogue of the Collections in the Science Museum, South Kensington . . . Mathematics I. Calculating Machines and Instruments*, London, His Majesty's Stationery Office, 1926, p. 56–58.

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13. TABLES OF $N^{3/2}$ (Q **5**, p. 131; QR **8**, p. 204, **11**, p. 336).—A further contribution to the bibliography of these tables is *American Civil Engineers' Handbook*, ed. by T. MERRIMAN and T. H. WIGGIN, fifth ed., New York, Wiley, 1930, p. 1312–1314. This table is for N = [.01(.01).1(.1)10(.5)20; 4S].

In *MTAC*, p. 336 a reference was given to T. 38 in King's *Handbook of Hydraulics*, third ed. 1939, p. 103–112. In this same work, p. 117–121, is T. 40 "Discharge in cubic feet per second per foot of length over sharp-crested wiers, without velocity of approach correction by the Francis formula $Q = 3.33H^{3/2}$," for H = [.0(.001)1.5(.01)6.99; 5S], the values Q of T. 40 being 3.33 times corresponding values in T. 38. A slide rule giving the values of the Francis' wier formula, $Q = 3.33H^{3/2}\ (B - .2H)$ is illustrated in J. N. ARNOLD, *Special Slide Rules*, Purdue Univ., Lafayette, Indiana, *Engineering Bulletin*, v. 17, no. 5, 1933, p. 14–15. In this formula Q is, in cu. ft./sec., the water discharge, with head of water H, of a rectangular notch wier of breadth B ft. Furthermore, the values of $N^{3/2}$, to a certain accuracy, could be read off at once from log log slide rules discussed in QR **12**.

R. C. A.

CORRIGENDA ET ADDENDA

P. 211, l. 20, *omit* of the first kind.

P. 215, l. 17, end of line, *for* (p = 2, 4, 6), *read* (p = 4, 6, 8).

P. 217, $\mathbf{C}_2$ 6, *add* , p = 1(1)18.

P. 220, last l., *for* δ^2, *read* δ_m^2. P. 225, l. 12, *for* $w_{-n}(-x)$, *read* $w_n(-x)$.

P. 231, l. −11, *for* odd integer, *read* odd-integer.

P. 234, **D8A**, l. 5, *for* a_α, *read* a_2

P. 240, l. −3, in two places, *for* $x^{\frac{1}{2}}$, *read* x^2; l. −5, *for* 10, *read* 8; l. −7, before the integral, *add*: $2x^{n-m+1}e^{-x^2}p^{m-n+1}$.

P. 252, l. 23–24, substitute the following sentence: The terms $\mathrm{her}_n x$, $\mathrm{hei}_n x$ are given in Watson's *Bessel Functions*, p. 81 and are used in Dwight 3_1; $\mathrm{yer}_n x$, $\mathrm{yei}_n x$ were added by J. C. P. MILLER in the Liverpool *Index*.

P. 256, **E2**, l. 1, *for* $(X/V)^{\frac{1}{2}}$, *read* $\frac{1}{2}x(X/V)^{\frac{1}{2}}$.

P. 257, after entry **E14**, *add* For improved forms of θ and ϕ given in nos. 1–14 we are indebted to the Liverpool *Index*.

P. 271, for l. 12 *read* The asymptotic forms of the ber, bei, ker, kei, functions and their derivatives are given in the natural form by Dwight 1_3 and 3_1. A modified form quoted by Watson is

P. 272, l. 2, *read* $u = \frac{1}{4}\pi(2r - 1 + 4s)$.

P. 285, l. 32, *for* OSAKA and, *read* OKAYA &.

P. 287, l. 10, *for* DINNIK 15, *read* DINNIK 14.

P. 292, l. 12, *for* $\mathcal{J}$ *read* J.

P. 295, l. 19, *for* 1/3, Prescott, *read* 1/3; Prescott.

P. 300, NYMTP 8, l. 1–3, *delete* $K_0(x) = E_0(x)$ . . . x = 0(.001).03; and.

P. 330, l. 11, *for* m = 20, *read* n = 20.

P. 333, l. 16, and 17, editorial slips for which the author was not responsible: *for* 0(0°.001)3°, *read* 0°.01(0°.01)2°.99, and *for* 0°.001, *read* 0°.01.