(22) The NYMTP found also 37 errors of a unit in the last decimal. All errors of this table are corrected in the sixth printing of Jahnke & Emde.
(23) Miller has also found 9 errors of a unit in the column b_0 and 4 in b_1. See also Note (24).
(24) Miller has also found 17 errors of a unit in the last decimal in the column $\frac{1}{2}rb_0/b_1$ up to $r = 5$. See also Note (23).
(25) This table of powers (for powers greater than 1) has been completely checked by Miller. All errors are given here. The errors in the powers of 1.68 are due to taking $x^6 = 1.66x^5$, and those in 2.85 to taking $x^{10} = 2.84x^9$. They could have been detected by differencing.
(26) Note that d is a mean difference, so that the values of the tabulated functions at 1.00 cannot be accurately obtained; for the upper line of each table this value is 1.0987, and for the lower line 0.4551.

L. J. C.

UNPUBLISHED MATHEMATICAL TABLES

Unpublished mathematical tables are listed in RMT **186** (Lehmer); QR 14.

32[A].—William Pitt Durfee (1855–1941), *Factor Table of the Sixteenth Million*. Unique ms. calculated during the years 1923–1929, and the property of the American Mathematical Society in New York City, since December 1935.

The following description of the ms. was published in *Scripta Math.*, v. 4, 1936, p. 101:

"The table comprises 500 separate sheets, $8\frac{1}{2} \times 14$ inches, each accounting for 2000 numbers, but as the multiples of 2, 3, 5, and 7 are omitted, the actual entries on each sheet number about 416. The entries are in long-hand, in black ink, except that those numbers whose lowest prime factor is 11 have been interpolated in red. They are arranged in parallel columns, three centuries to a column, the last four digits only of each number being written; and opposite each its lowest prime factor. If the number is prime a bar is drawn across the corresponding space in the column of prime factors. The arrangement is thus closely similar to that in the published tables covering the first nine millions (Burckhardt, Dase and Rosenberg, Glaisher).

"At the foot of each column the number of primes for each century is noted, and the total number of primes on preceding sheets, the number on that sheet, and the total. In the lower right-hand margin there are listed the number of entries on preceding sheets, on that sheet, and the total, with a similar notation in red ink for the entries whose least prime factor is 11."

The computations, which were made by the stencil method, have not been generally checked except by the author of the table. However, D. H. L. discovered one error where 15485303 is entered as a prime, when 109 is a factor. The total number of primes in this million is 60,465.

33[A].—U. S. Dept. of Commerce, Weather Bureau, *Table of* $(\log e)/x$, computed by, and in possession of, the Bureau.

The Table of $(\log e)/x$ is for $x = [100(1)999; 6S]$.

L. P. Harrison

Weather Bureau

MECHANICAL AIDS TO COMPUTATION

14[Z].—G. W. King, "Punched-card tables of the exponential functions." *Rev. Sci. Instruments*, v. 15, 1944, p. 349–350.

G. B. Thomas & G. W. King, "Preparation of Punched-card tables of logarithms," *ibid.*, p. 350.

With the rapid increase in recent years of the use of punched-card machines in computations of all kinds, the versatility of the machines has been adapted to many uses.

One obvious method of increasing the number of possible applications has been to prepare cards punched with the values of numerical functions which can be combined with other values by multiplication or addition. Of this type are exponential and logarithmic functions.

The first note indicates the usefulness of punched-card tables of the exponential function. From the relationship, $\exp(a+b) = (\exp a)(\exp b)$, one sees that this function is readily adapted to calculation by punched-card machines, any desired degree of accuracy being easily attained. Thus, the master set of cards might contain values of $\exp \pm n$ over the range $n = 0.00(0.01)12.00$. A second set of 100 cards for $n = 0.0000(0.0001)(0.0099)$ would then multiply the original range a hundredfold.

One of the most important problems in the use of punched cards is the verification of the numbers punched on them. For this purpose the author suggests the use of the formula

$$\sum_{k=0}^{N-\Delta k} e^{-k} \frac{1-e^{-N}}{1-e^{-\Delta k}}.$$

Partial sums of the cards are compared with the values of this function computed for suitably chosen values of Δk.

The second note suggests the use of punched-card tables of $\log x$. The authors suggest that the verification of the cards be made by comparing partial sums of the cards with computed values of $\log n!$ from the formula:

$$\sum_{k=1}^{n} \log k = \log n!.$$

Tables such as those of J. BOCCARDI: *Tables logarithmiques des factorielles jusqu'à 10,000!*, Cavaillon, 1932, or F. J. DUARTE: *Nouvelles tables logarithmiques à 36 decimales*, Paris, 1933, are available for this purpose.

H. T. D.

NOTES

29. EARLY DECIMAL DIVISION OF THE SEXAGESIMAL DEGREE.—In *MTAC*, p. 33, 100 (corrigenda), 129–130, it has been already noted that decimal division of the degree was used by RUFFI in a Latin codex of about 1450, as well as by BRIGGS, in tables prepared before 1633, the idea having been suggested to him by Viète in a work published in 1600. The decimal division of the degree was also advocated by SIMON STEVIN (1548–1620) of Bruges in 1585, in his Flemish work, 1. *De Thiende*, published at Leiden. A facsimile of this 36-page booklet is given in H. BOSMANS, *La 'Thiende' de Simon Stevin*, Antwerp and The Hague, 1924; a French edition of *De Thiende*, *La Disme*, occupied p. 132–160 of Stevin's 2. *La Pratiqve D'Arithmetiqve*, also published at Leiden in 1585; there are copies of this work in the Plimpton Library of Columbia University, the Royal Library of Belgium, University of Liége, etc. A facsimile of this edition was published by G. SARTON in "The first explanation of decimal fractions and measures (1585). Together with a history of the decimal idea and a facsimile (no. XVII) of Stevin's Disme," *Isis*, v. 23, 1935, p. 230–244. A French edition, ed. by A. Girard, appeared also in a new edition of 3. *L'Arithmetiqve*, Leiden, 1625, p. 823–849, and in 4. S. STEVIN, *Les Oeuvres Mathematiques*, Leiden, 1634, p. 206–213. There were other Flemish editions, 5–6, in 1626 (in the Brown University Library), and in 1630.

This was the first work to set forth the theory of decimal fractions. It was translated into English by 7. ROBERT NORTON, "engineer and gunner" (d. 1635, *Dict. Nat. Biog.*), son of the poet THOMAS NORTON (1532–1584),