MATHEMATICAL TABLES—ERRATA

Reference has been made to Errata under RMT**187** (Jahnke & Emde), **189** (Brandenburg), **190** (Alaoglu & Erdös), **191** (Watson), **193** (N. D. R. C.); N**30** (Girard), **36** (Jacobi).

61. E. JAHNKE & F. EMDE, *Tables of Functions*. Editions as in RMT **113**, p. 106f. The errors here listed are almost wholly supplementary to those discussed p. 108–109, 136–137, 161, 196, 198, 204 (corrigenda, p. 109, 161), 286, 287, 294, 308 (corrigenda, p. 137), 325.

The popularity on both sides of the Atlantic of the Dover Publications reprint (1943) of these well-known tables makes a list of their errors timely. The following lists do not pretend to be complete, but as they put on record the results of systematic examination of some of the tables, they may well relieve anxieties about the correctness of certain tables. I wish to place on record the great share that J. C. P. MILLER has had in these examinations, and in perfecting this list. While references have been previously made in *MTAC* to contributions of NYMTP we present here a complete list of other discovered errors exceeding a unit in the last decimal place.

The original tables (1909) were followed by a flood of corrections, mostly in *Arch. Math. u. Physik*. That perfection was not attained in the later editions is evident. The surviving compiler (Emde) is not entirely to blame, as many of the tables have been taken from diverse sources (always freely acknowledged); it was obviously impossible to recompute every such table.

Table I isolates errors that occur in the 1909 edition only. Some more known, but obvious or unimportant, errors have been omitted. In both lists, with few exceptions (the principal being in the table of powers in the 1933 edition), errors of a unit in the last decimal are not given, although often statistics of their frequency will be found in the notes. The practical view-point is that the labour of making such end-figure alterations would not be justified; nor is it fair to classify them as errors if the author has not claimed absolute accuracy in the last decimal.

One of my most treasured possessions is the late J. R. Airey's personal copy of the 1909 edition of these tables. It is covered with beautifully written notes, references and corrections, and has 120 pages of related formulae, etc. pasted in—all neatly transcribed, with their sources if they are taken from published works. Whenever Airey visited me, he *always* came hugging this book mysteriously! It is now shared with J. C. P. Miller, on whom Airey's mantle has fallen.

In the date column, . . . denotes that the quantity to be corrected is not given; *C* denotes that it is correct. Page numbers in square brackets in the 1938 column are a reminder that these pages are at the end of the reprint (1943) by Dover Publications.

In the column "authority," A = Airey (marked in his personal copy, with no source ascribed), C = Comrie, M = Miller, *A.M.*, followed by a number, denotes *Archiv für Mathematik und Physik*, dritte Reihe, and the volume number. The complete references here are:

v.	year	pages	authors
15	1909	372–4	Bruns, Egloff, Emde, Greenhill, Kritaniger, Witt.
16	1910	285–6	Baruch, Debye, Emde, Goldscheider, Naetsch, Russell.
17	1910	286	Dinnik, Goldscheider, Jahnke.
18	1911	204	Dinnik.
18	1911	374	Emde, Jahnke, Schaefer.
20	1912	95	Escott.
21	1913	191	Emde, Escott, Jahnke, Nagaoka.
22	1913	95	Wagner.
22	1914	285	Airey.
22	1914	376	?
24	1915	94	Kalähne.

TABLE I—Errors in 1909 edition only

1933–8	1909			*for*	*read*	authority
C	1	$x \tan x$	$x = 0.2$	.04045	.04054	M
			$x = 1.4$	8.2627	8.1170	
			$x = 1.5$	21.1523	21.1521	
		$\frac{\tan x}{x}$	$x = 0.2$	1.0112	1.0136	M
			$x = 0.5$	1.0927	1.0926	
			$x = 1.2$	2.1434	2.1435	
			$x = 1.4$	4.2157	4.1413	
C	3	$\cos x \cosh x = 1$. $x = 0$ is also a root				*A.M.*, 18
...	6	e^x for $x = 6.0$		403.00	403.43	*A.M.*, 20
...	10	Penultimate line		$-\mathrm{Ar\,Sin}\, ix$	$-i\,\mathrm{Ar\,Sin}\, ix$	*A.M.*, 16
...	11	Line−10		$\sin 2x$	$\mathrm{Sin}\, 2x$	*A.M.*, 16
C	12	Last line, middle integral		$\mathrm{Tg}\frac{x}{2}$	$\mathrm{Tg}\, x$	*A.M.*, 22
C	12	$d\,\mathrm{Ar}\frac{1}{\mathrm{Tg}\, x}$		$\frac{1}{\mathrm{Tg}\, x}$	Ctg	*A.M.*, 22
...	19	Last line of Section 1		$Ei(ix)Ci(x)$	$Ei(ix)=Ci(x)$	*A.M.*, 16
...	20	Line 4		$-\frac{1}{2}\pi \cos 2x$	$\frac{1}{2}\pi(1-\cos 2x)$	*A.M.*, 16
C	22, 23	$Ci(x)$ for $x = 40(5)75$, $120(10)170$, insert an additional cipher after the decimal point				M
C	26	$S(1.6)$		.6383	.6389	(12)
		$C(1.8)$		.3363	.3337	
		$S(2.3)$		.5525	.5531	
...	27	Line 2		$\frac{1}{2\sqrt{2}}$	$\frac{\pi}{2\sqrt{2}}$	*A.M.*, 16
		Line 7		2^{nx}	2^{2x}	*A.M.*, 16
		Line 3 of Section 5		See note		A(1)
...	28	Line 8, denominator		$x - \frac{1}{2}$	$x + \frac{1}{2}$	*A.M.*, 16
		Line 9, denominator		$\pi^2(\frac{1}{4})$	$\pi(\frac{1}{4})$	*A.M.*, 16
...	29	Line 4, replace last term by			$+\frac{1}{12}x$	*A.M.*, 16
C	29	Legend of Fig. 8		Gamma-funktion	Fakultät	A
...	36	Diagram		$\frac{1}{8}\Phi^{VI}$	$\frac{1}{8}\Phi^{IV}$	*A.M.*, 15
		Diagram		$\frac{1}{32}\Phi^{IV}$	$\frac{1}{32}\Phi^{VI}$	
...	44	Characteristic of logs in $\phi = 0°, 5°, 10°$		1	$\bar{1}$	*A.M.*, 15
	45	$r = 48$ $\phi = 10°$		0	$\bar{1}$	M
...	47	Line 2, last term in numerator		d v	dn v	*A.M.*, 16
...	51	Line−3		Amp x	Amp u	*A.M.*, 16
...	67	$\log q$ for $\alpha = 64° 5'$		.4087	.0087	*A.M.*, 21
		$\log q$ for $\alpha = 69° 55'$		.1195	.1159	
C	71	Upright side. Denominators		dv	πdv	*A.M.*, 16
				dv_1	πdv_1	
...	72	Legend of Fig. 22		1.53995	1.52995	A
...	78	Beginning of line 7		$\frac{dF}{dk}$	$\frac{dK}{dk}$	*A.M.*, 22
...	78	Line 1 of Section 3; Niven's initials		M	W.D.	*A.M.*, 16

TABLE I—(*Continued*)

1933–8	1909		*for*	*read*	authority
C	83	$P_1(.03)$	.3000	.0300	*A.M.*, 15
C	85	$P_7(1°)$	.9955	.9957	C(2)
		$P_7(3°)$	.9617	.9620	
		$P_6(7°)$	.8476	.8492	
		$P_7(7°)$	.7986	.8016	
		$P_7(13°)$	.3940	.3980	
		$P_7(18°)$	.0289	.0248	
		$P_6(19°)$	.1347	.1353	
		$P_7(19°)$	.0443	.0433	
C	86	$P_7(21°)$	.1662	.1664	C(2)
		$P_6(25°)$	.2053	.2040	
		$P_7(25°)$	.3463	.3441	
		$P_6(40°)$	.3234	.3236	
		$P_7(40°)$	.1003	.1006	
		$P_4(47°)$	.4252	.4227	
		$P_6(47°)$	.0645	.0665	
		$P_7(47°)$	.2054	.2028	
		$P_7(57°)$	.2949	.2947	
C	87	$P_6(71°)$	.1786	.1791	C(2)
		$P_7(71°)$	.1811	.1808	
		$P_6(73°)$	.1144	.1136	
		$P_7(73°)$	.2347	.2352	
		$P_6(74°)$	.0795	.0788	
		$P_7(74°)$	.2559	.2563	
		$P_2(76°)$	.4112	.4122	
		$P_6(76°)$	.0076	.0070	
		$P_7(76°)$	.2848	.2850	
		$P_6(77°)$	.0284	.0290	
		$P_7(77°)$	.2919	.2921	
		$P_7(88°)$	.0735	.0755	
...	94	Second formula for $Y_1(x)$	$J_1(x)$	$J_1(x)(\ln x-1)$	*A.M.*, 16
...	96	$\lim_{x=0} N_1(\pm ix)$	See note (3)		*A.M.*, 16
C	99	Expression for large roots of J_0	.50661	.050661	*A.M.*, 16
		Expression for large roots of J_1	.15399	.015399	
...	100	Line − 3	185	191	A
...	102	Coefficient of $\cot g^2\tau$ in $A_2(\tau)$	$\frac{7}{576}$	$\frac{77}{576}$	*A.M.*, 17
...	103	Coefficient of $\cot g^2\tau$ in $B_2(\tau)$			
		Series for $J_p(x)$	$\frac{1}{\pi}$	$\frac{1}{2\pi}$	*A.M.*, 17
...	106	Smallest root of $J_{-1/3}(x)=0$	1.88	1.8663:	A(4)
		$D(10)$	.078	.068	A
C	111	$J_0(0.62)$	.9052	.9062	*A.M.*, 22
C	115	$J_0(5.90)$	.1120	.1220	*A.M.*, 22
C	118	$-J_1(10.30)$	−	+	*A.M.*, 16
...	125	$Y_0'(5.2)$	.0816	.0846	A(5)
		$Y_0'(8.1)$	.1806	.1804	
		$Y_0'(8.5)$	.0091	.0094	
		$Y_0'(8.6)$	.0336	.0333	

TABLE I—(*Continued*)

1933–8	1909		*for*	*read*	authority
		$Y_0'(8.7)$	.0751	.0753	
		$Y_0'(8.8)$	.1157	.1160	
		$Y_0'(8.9)$	.1548	.1551	
		$Y_0'(9.0)$	.1921	.1923	
		$Y_0\ (9.5)$	.2463	.2465	
		$Y_0'(9.5)$	.3381	.3378	
		$Y_0'(9.6)$	.3574	.3571	
		$Y_0\ (9.7)$	.1755	.1752	
		$Y_0'(9.7)$	.3729	.3727	
		$Y_0'(9.8)$	.3846	.3844	
		$Y_0\ (9.9)$	.0982	.0984	
		$Y_0'(10.2)$	.3931	.3922	
...	127	$K_1(0.20)$	5.2209	5.2211	A(6)
		$K_1(0.65)$	1.8483	1.8485	
...	128	$K_1(2.4)$	.1580	.1578	A(6)
		$K_1(8.1)$	.2093	.2091	
		$K_1(8.5)$	.0408	.0411	
		$K_1(8.6)$	.0020	.0017	
		$K_1(8.7)$	.0438	.0440	
		$K_1(8.8)$	.0851	.0854	
		$K_1(8.9)$	.1251	.1255	
		$K_1(9.0)$	.1637	.1639	
		$K_1(9.5)$	.3194	.3192	
		$K_1(9.6)$	.3412	.3410	
		$K_0(9.7)$	.2012	.2009	
		$K_1(9.7)$	.3594	.3592	
		$K_0(9.9)$	.1261	.1263	
		$K_1(10.2)$	.3923	.3930	
C	129	$-N_1(0.1)$	7.0317	6.4590	M(7)
		$-N_1(0.2)$	3.3235	3.3238	M
		$N_0(3.9)$	.0237	.0234	M
		$-N_1(5.2)$	.0773	.0792	A
C	130	$-N_1(8.8)$	.0541	.0544	M(7)
		$N_0(9.9)$	.0800	.0804	M
		$-N_1(10.2)$	.2507	.2502	A
C	137	$x = 5.1$ to 6.0. Imaginary part	$-$	$+$	*A.M.*, 17
...	138	$x = 3.1$	1.8918	0.8919	
		$x = 3.3$	0.3556	1.3556	*A.M.*, 18
C	139	$x = 6.0$. Real part of $H_0^{(1)}$	$+$	$-$	*A.M.*, 16
...	140	$x = 2.4$. Real part	$-$	$+$	*A.M.*, 18
...	146	Table of ber x, etc.	See note (8)		*A.M.*
C	153	$J_7(13)$	$+$	$-$	A
...	164	Line-4	XVI	XXVI	A
...	166	Right-hand side of fifth integral	See note (9)		*A.M.*, 16
...	167	Last differential equation	$\frac{p^2}{x^2}$	$\frac{p}{x^2}$	*A.M.*, 16
...	168	c) Second equation	$-\frac{b^2}{16p^2}$	$-\frac{b^2}{16p^1}$	*A.M.*, 17
...	175	Reference to W.D. Niven	77	78	*A.M.*, 20
...	175	Reference to P. A. Hansen	170	169	*A.M.*, 15

TABLE II

1938	1933	1909		*for*	*read*	authority
9	86	22	$Si(65)$	1.5775	1.5792	M(10)
10	87	...	C_3	0,067 353 011	0,067 352 301	A. Fletcher (11)
			$H(x)$	$-\frac{571}{248\,832\,x^4}$	$-\frac{571}{2\,488\,320\,x^4}$	A. Fletcher
34	106	...	$\psi_n(x)$	$\Phi_1\left(\frac{x}{2}\right)$	$\Phi_1\left(\frac{x}{\sqrt{2}}\right)$	M
34	109	26	$C(0.1)$	.0999	.1000	(12)
			$C(0.8)$	.7230	.7228	
			$C(1.1)$	.7648	.7638	
			$C(4.5)$	.5258	.5260	
			$C(5.1)$	.4987	.4998	
			$S(7.3)$	.5199	.5189	
35	108	25	$S(1.5)$	.4153	.4155	M
39	113	...	Heading x	.01	.1	M
70	142	62	$E(15°, 54°)$	.9345	.9346	M(13)
74	146	...	Coefficient of ϵ^{17}	1701	1707	A(14)
94	160	...	In first bordered table	$\frac{-d}{ks}$	$\frac{-d}{kc}$	Todd (letter)
103	169	74	σu for $r = 123$	1.0441	1.0438	M
105	171	50	Last integral formula for Weierstrassian functions. After this integral add: where v is defined by $\wp(v) = \delta/\gamma$	$\frac{\alpha u}{\gamma}-$	$\frac{\alpha u}{\gamma}+$	Todd (letter)
108	174	...	Diagram is in error			(15)
117	183	82	Expression for $dP_7/d\theta$	$\sin 3\theta$	$\sin^3\theta$	*A.M.*, 16 (16)
124	190	88	$dP_n/d\theta$, $n = 3$, $\theta = 31°$	2.0654	2.0656	NYMTP (17)
			$n = 4, \theta = 5°$	0.8570	0.8567	
			$n = 4, \theta = 64°$	1.6306	1.6300	
			$n = 5, \theta = 54°$	2.0173	2.0178	
			$n = 5, \theta = 64°$	1.5420	1.5418	
			$n = 5, \theta = 67°$	1.1187	1.1183	
			$n = 5, \theta = 83°$	1.4831	1.4827	
			$n = 6, \theta = 23°$	3.0917	3.0902	
			$n = 6, \theta = 48°$	2.3715	2.3713	
			$n = 6, \theta = 57°$	1.1936	1.1934	
			$n = 6, \theta = 83°$	1.4487	1.4484	
			$n = 7, \theta = 70°$	1.984	1.934	A
C	223	...	$J_{1/4}(4.8)$	.3250	.3255	M(18)
			$J_{3/4}(4.8)$	.3556	.3518	
			$J_{-1/4}(4.8)$	.1149	.1158	
			$J_{-3/4}(4.8)$	.1803	.1791	
			$J_{-1/4}(5.4)$	.0942	.0940	
			$J_{-3/4}(5.4)$	.3085	.3082	
164	223	...	$J_{1/4}(7.4)$	.2925	.2923	NYMTP(19)
			$J_{1/4}(7.6)$	.2873	.2869	
			$J_{1/4}(7.8)$	.2711	.2704	
			$J_{1/4}(8.0)$	.2449	.2436	
			$J_{3/4}(6.8)$	.0447	.0449	
			$J_{3/4}(7.8)$	.2602	.2604	
			$J_{3/4}(8.0)$	.2749	.2752	

TABLE II—(*Continued*)

1938	1933	1909		*for*	*read*	authority
			$J_{-1/4}(.2)$	1.4310	1.4319	
			$J_{-3/4}(3.2)$	.3943	.3945	
168	239	147	$p = 2, n = 9$	30.571	30.569	A, M(20)
			$p = 3, n = 3$	13.017	13.015	
			$p = 3, n = 9$	32.050	32.065	
			$p = 4, n = 1$	7.586	7.588	
			$p = 4, n = 9$	33.512	33.537	
			$p = 5, n = 1$	8.780	8.771	
			$p = 5, n = 4$	18.982	18.980	
			$p = 5, n = 5$	22.220	22.218	
			$p = 5, n = 9$	34.983	34.989	
183	253	...	$x = 6.4$	$d = 193$	$d = 103$	M
206	276	163	$p = 0$ and 1. Delete all bracketed digits			M
212	...	...	$\Omega_0(0.20)$	.1266	.1268	M(21)
216	...	...	$\Omega_0(10.35)$	394	294	M
			$\Omega_0(13.25)$	135	235	
218	...	...	$\mathcal{S}_0(4.41)$	273	278	M(21)
220	...	...	$\mathcal{S}_0(8.17)$	136	236	M
228	280	...	$I_0(6.4) = J_0(6.4i)$	96.98	96.96	NYMTP
235	285	...	$I_{1/3}(1.4) = i^{-1/3}J_{1/3}(1.4i)$	1.4002	1.4000	NYMTP(22)
			$I_{1/3}(3.6)$	7.8727	7.8729	
			$I_{1/3}(3.8)$	9.3473	9.3470	
			$I_{1/3}(4.0)$	11.1136	11.1138	
			$I_{-1/3}(2.2)$	2.5226	2.5627	
			$I_{-1/3}(3.0)$	4.9710	4.7754	
			$I_{-1/3}(3.6)$	7.8831	7.8827	
			$I_{-1/3}(3.8)$	9.3546	9.3548	
			$I_{-1/3}(4.0)$	11.1204	11.1201	
			$I_{-1/3}(6.0)$	65.55	66.55	
			$I_{2/3}(2.4)$	2.6650	2.6648	
			$I_{2/3}(2.8)$	3.7343	3.7345	
			$I_{2/3}(3.6)$	7.4423	7.4429	
			$I_{2/3}(3.8)$	8.8698	8.8692	
			$I_{2/3}(4.0)$	10.5796	10.5799	
			$I_{2/3}(4.4)$	15.107	15.098	
			$I_{2/3}(5.8)$	53.77	53.71	
			$I_{-2/3}(2.4)$	2.7065	2.7067	
			$I_{-2/3}(2.8)$	3.7002	3.7603	
			$I_{-2/3}(3.4)$	6.2655	6.2657	
			$I_{-2/3}(3.6)$	7.4535	7.4531	
			$I_{-2/3}(3.8)$	8.8841	8.8773	
			$I_{-2/3}(4.0)$	10.5867	10.5864	
			$I_{-2/3}(4.4)$	15.112	15.103	
			$I_{-2/3}(4.8)$	21.621	21.623	
			$I_{-2/3}(8.0)$	418.01	415.01	*A.M.*, 22
263	313	...	$b_0(5.3)$	7.340	7.475	M(23)
			$b_0(9.5)$	108.02	108.00	
			$b_0(9.6)$	115.31	115.30	
266	316	...	$\frac{1}{2}rb_0/b_1$ for $r = 0.5$	006	008	M(24)
300	77	...	(*i*) Comrie's initials	L.T.	L.J.	C
300	...	...	(i), line 3	.0350	.0400	C

TABLE II—(*Continued*)

1938	1933	1909			*for*	*read*	authority
			(1), line 2		Mittlin	Mifflin	C
[2]	2	...	x^4 for $x = 0.83$		4747	4746	M(25)
			x^6 for $x = 0.88$		4640	4644	
[4]	4	...	x^5 for $x = 1.18$		2289	2288	
			Powers of $x = 1.68$	x^6	2222	2248	
				x^7	3732	3777	
				x^8	6270	6346	
[5]	5	...		x^9	10534	10661	
				x^{10}	17697	17910	
				x^{11}	2973	3009	
				x^{12}	4995	5055	
				x^{13}	8391	8492	
				x^{14}	14097	14267	
				x^{15}	2368	2397	
			x^{10} for $x = 1.40$		2892	2893	
[6]	6	...	x^5 for $x = 2.05$		3620	3621	M(25)
			x^7 for $x = 2.35$		3960	3958	
			Powers of $x = 2.95$	x^6	6590	6591	
				x^7	19441	19443	
				x^8	5735	5736	
[7]	7	...		x^9	16919	16920	
				x^{11}	14723	14725	
				x^{12}	4343	4344	
				x^{13}	12813	12814	
			Powers of $x = 2.85$	x^{10}	3523	3535	
				x^{11}	10041	10076	
				x^{12}	2862	2872	
				x^{13}	8156	8184	
				x^{14}	2324	2333	
				x^{15}	6624	6648	
			x^{11} for $x = 3.10$		2540	2541	
			x^{14} for $x = 3.30$		18164	18163	
[9]	9	...	$x^{0.60}$ for $x = 0.1$		0.251	0.2512	M
[15]	15	...	See note (26)				
[56]	56	13	$\int \frac{dx}{x^2-1}$		$\frac{1}{\text{Tg}\, x}$	Ctg x	*A.M.*, 22

NOTES

(1) Airey has corrected his copy from

$$\sum_{r=2}^{\infty} (1 - s_{2r+1})x^{2r} \text{ to } \sum_{r=1}^{\infty} (1 - s_{2r+1})x^{2r} + (1 - \gamma)$$

(2) In this table many end-figure changes have been made in the 1933 and 1938 editions, expecially in P_6 and P_7; only those greater than one unit are listed here. It will be noticed that, in every case where P_6 is in error, P_7 is in error by $\frac{13}{7} P_1$ times the error in P_6, thus showing that P_7 was found by recurrence, and was not checked by differencing. For further comments on the original table, see *MTAC*, p. 137.

(3) This should read, according to Goldscheider,

$$\lim_{x=0} N_1(\pm ix) = \pm i \frac{2}{\pi x} \pm \frac{ix}{\pi} \ln x - \frac{x}{2} \mp \left(\frac{1}{2} + \ln \frac{2}{\gamma}\right) \frac{ix}{\pi} \cdot$$

(4) See *MTAC*, p. 196 where the value of the root to 15D is also given.

(5) There are also 44 alterations of a unit in the last decimal, taken from Airey's table in B.A.A.S., *Report*, 1914. See also Note (6).
(6) Marked in Airey's personal copy, together with over 120 alterations of a unit in the last figure. See also *A.M.*, v. 18, p. 374 where Jahnke notes these end-figure corrections, based on the B.A.A.S., *Report*, 1911 (computed by Airey). Note the obvious connection between the erroneous arguments here and in the values covered by Note (5), which is explained by

$$Y_n\ (\text{J.\&E.}) = \frac{\pi}{2}\, Y_n\ (\text{Watson}) + \ln\frac{2}{\gamma}\, J_n$$

$$K_n\ (\text{J.\&E.}) = -\frac{\pi}{2}\, Y_n\ (\text{Watson}) = G_n\ (\text{Airey})$$

Watson's Y_n is called N_n in Jahnke & Emde.
(7) See the argument 10.2 covered by Notes (5) and (6). Miller has compared this table with Watson's Y_0 and Y_1. Besides the corrections given, he finds 67 errors of a unit, and 6 of two units. Kalähne also points out the error in $-N_1(0.1)$ in *A.M.*, v. 24, p. 94.
(8) See *A.M.*, v. 16 and 21. There are many errors, but it is not worth correcting this table, since correct values may be obtained from the later editions (up to $x = 10$) or from Dwight's *Mathematical Tables* (up to $x = 20$). For a thorough discussion of the errors see *MTAC*, p. 297f.
(9) A further term is needed, namely

$$-\frac{Z_p(\alpha x)\bar{Z}_q(\alpha x)}{p+q}$$

(10) Miller has compared these integrals with the NYMTP tables up to $x = 100$. There are also 5 end-figure errors in the 1933 and 1938 versions of this table.
(11) This error occurs also in Schlömilch, *Comp. d. höh. Analysis*, v. 2, 1879, p. 258, and in Láska, *Sammlung*, 1888–94, p. 272. A. Fletcher.
(12) There have been "errors within errors" here. A first list was given by Lash Miller and Gordon in *J. Phys. Chem.*, v. 35, 1931, p. 2872. A reference to this paper, and extracts, were given by J. H. Awbery in Phys. So. London, *Proc.*, v. 51, 1939, p. 199. Lash Miller and Gordon failed to note the error at $S(2.3)$ and gave C instead of S for the corrections at 1.6 and 7.3. The first three errors were found independently by Rankin (letter from L. Rosenhead, Feb. 1943). In a 4-figure table prepared by Comrie, $C(1.8)$ appears as .3336, and $S(2.3)$ as .5532. This table shows that there are no other errors exceeding one unit up to $u = 3.3$. This note has reference to T. I, p. 26 and T. II, p. 34, 109, 26.
(13) Traced by means of errors in Legendre's table as given by Heuman (1941).
(14) Also reported by John Todd by letter. Airey has inserted coefficients up to ϵ^{33} by hand in his copy; this one was first written (p. 47, 1909) as 1701 and later changed to 1707.
(15) G. I. Taylor has pointed out that this diagram is completely wrong near $x = -1$. Also it should be symmetrical about $\nu = -\frac{1}{2}$, not $\nu = 0$, as seems suggested.
(16) It is surprising that this published correction (which I have verified) was not incorporated in later editions.
(17) The NYMTP found also 41 errors of a unit in the last decimal. All errors of this table are corrected in the sixth printing of Jahnke & Emde.
(18) Miller has pointed out that the table in 1933 is taken from Dinnik, *A.M.*, v. 21, 1913, p. 324–6, whereas that in 1938 has been corrected to agree with Karas, *Z. angew. Math. u. Mech.*, v. 16, 1936, p. 248, who removed some of the major errors in Dinnik and subtabulated from interval 0.2 to interval 0.1. There are also 5 cases of a change of a unit in the last decimal. See *MTAC*, p. 365–366.
(19) The NYMTP found also 22 errors of a unit in the last decimal. All errors of this table are corrected in the sixth printing of Jahnke & Emde.
(20) Also 18 errors of a unit in the last decimal. Compare *MTAC*, p. 160, 282.
(21) Miller reports that there are also 40 errors of one unit in the last decimal in the table, p. 212, 214, 216. This table is theoretically identical with that on p. 218, 220 and 222, in which only 8 such end-figure errors have been found, none of which corresponds to any of the 40 just mentioned.

(22) The NYMTP found also 37 errors of a unit in the last decimal. All errors of this table are corrected in the sixth printing of Jahnke & Emde.
(23) Miller has also found 9 errors of a unit in the column b_0 and 4 in b_1. See also Note (24).
(24) Miller has also found 17 errors of a unit in the last decimal in the column $\frac{1}{2}rb_0/b_1$ up to $r = 5$. See also Note (23).
(25) This table of powers (for powers greater than 1) has been completely checked by Miller. All errors are given here. The errors in the powers of 1.68 are due to taking $x^6 = 1.66x^5$, and those in 2.85 to taking $x^{10} = 2.84x^9$. They could have been detected by differencing.
(26) Note that d is a mean difference, so that the values of the tabulated functions at 1.00 cannot be accurately obtained; for the upper line of each table this value is 1.0987, and for the lower line 0.4551.

L. J. C.

UNPUBLISHED MATHEMATICAL TABLES

Unpublished mathematical tables are listed in RMT **186** (Lehmer); QR 14.

32[A].—William Pitt Durfee (1855–1941), *Factor Table of the Sixteenth Million*. Unique ms. calculated during the years 1923–1929, and the property of the American Mathematical Society in New York City, since December 1935.

The following description of the ms. was published in *Scripta Math.*, v. 4, 1936, p. 101:

"The table comprises 500 separate sheets, $8\frac{1}{2} \times 14$ inches, each accounting for 2000 numbers, but as the multiples of 2, 3, 5, and 7 are omitted, the actual entries on each sheet number about 416. The entries are in long-hand, in black ink, except that those numbers whose lowest prime factor is 11 have been interpolated in red. They are arranged in parallel columns, three centuries to a column, the last four digits only of each number being written; and opposite each its lowest prime factor. If the number is prime a bar is drawn across the corresponding space in the column of prime factors. The arrangement is thus closely similar to that in the published tables covering the first nine millions (Burckhardt, Dase and Rosenberg, Glaisher).

"At the foot of each column the number of primes for each century is noted, and the total number of primes on preceding sheets, the number on that sheet, and the total. In the lower right-hand margin there are listed the number of entries on preceding sheets, on that sheet, and the total, with a similar notation in red ink for the entries whose least prime factor is 11."

The computations, which were made by the stencil method, have not been generally checked except by the author of the table. However, D. H. L. discovered one error where 15485303 is entered as a prime, when 109 is a factor. The total number of primes in this million is 60,465.

33[A].—U. S. Dept. of Commerce, Weather Bureau, *Table of* $(\log e)/x$, computed by, and in possession of, the Bureau.

The Table of $(\log e)/x$ is for $x = [100(1)999; 6S]$.

L. P. Harrison

Weather Bureau

MECHANICAL AIDS TO COMPUTATION

14[Z].—G. W. King, "Punched-card tables of the exponential functions." *Rev. Sci. Instruments*, v. 15, 1944, p. 349–350.

G. B. Thomas & G. W. King, "Preparation of Punched-card tables of logarithms," *ibid.*, p. 350.

With the rapid increase in recent years of the use of punched-card machines in computations of all kinds, the versatility of the machines has been adapted to many uses.