184[A, B, C, D, E, M].—EDWARD SCHWITZER ALLEN (1887–), *Six-Place Tables. A Selection of Tables of Squares, Cubes, Square Roots, Cube Roots, Fifth Roots and Powers, Circumferences and Areas of Circles, Common Logarithms of Numbers and of the Trigonometric Functions, the Natural Trigonometric Functions, Natural Logarithms, Exponential and Hyperbolic Functions, and Integrals.* Sixth ed., New York, McGraw-Hill, 1941. xxiii, 181 p. 10.7 × 17.7 cm. Leatherette $1.50. Compare RMT **49**.

Through revisions, including enlargements, and the addition of marginal index and flexible covers, in new editions (1925, 1929, 1931, 1935, 1941) since the original 1922 pocket volume, this work has attained to its present very useful form. At many places of the compilation, the thorough competence of the author is in evidence. In this latest edition three new tables are added, namely: (a) T. VIII, a four-place table of sine, cosine, tangent, cotangent, and log sin, log cos, log tan, for radian argument [0(.01)3.20; 4D], included at the suggestion of electrical engineers; (b) T. IX, log $n!$, n = [1(1)100; 6D]; and (c) a four-place table of complete elliptic integrals K, E, for $\sin^{-1} k = 0(1°)65°(0°.5)80°(0°.2)89°(0°.1)89°.9$ added to T. XIII, containing 167 integrals.

T. VI is a six-place table of all six of the natural trigonometric functions for each minute of the quadrant; T. XI, for e^x, e^{-x}, sinh x, cosh x, tanh x, x = [0(.01)1(.05)2.4(.1)5(1)10; mostly 4D], and $\Gamma(x)$, x = [0(.01).99; 4D], and $\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$, x = [0(.01)2.99; 4D]. Thus not all tables are six-place; T. III of circumferences and areas of circles with diameters from 1/64 to 100 is mostly 6S. The last Table, XIV, is devoted to mathematical constants. In the xxiii pages of the Introduction the theory of logarithms and explanation of tables are given, with exercises for solution.

R. C. A.

185[A, B, D, E, G, H, I].—FRITZ EMDE, *Tables of Elementary Functions . . . With* 83 *figures. Tafeln elementarer Funktionen . . . mit* 83 *Textfiguren.* Leipzig and Berlin, Teubner, 1940. Two title-pages. On the back of the second title-page is, in part, *Copyright vested in the Alien Property Custodian, 1944, pursuant to law. Published and distributed in the Public Interest by authority of the Alien Property Custodian under License No. A-709. Lithoprinted by Edwards Brothers, Inc., Ann Arbor, Michigan, U. S. A.*, 1945. xii, 181 p. 17.7 × 26.1 cm. $3.20.

There was incidental reference to this work in *MTAC*, p. 108, Oct. 1943 (when no copy of the work was available in this country), in the course of reviews of the six editions or reprints then available of JAHNKE & EMDE, *Tables of Functions*. The first 78 p., devoted to Elementary Functions in the third edition of 1938, did not appear in the fourth or fifth editions but were reprinted in the sixth. The work under review was intended to be a revision, and in various directions greatly to elaborate the 1938 material. Larger type is used and the type-page is about 25% larger than that of the *Tables of Functions* of JAHNKE & EMDE. Instead of X sections in the original 78 pages there are now XVII sections in the 172 pages. We shall now briefly indicate most of what is new. Section I, **Powers,** p. 1–19 (old, p. 1–11); the new tables are of squares and reciprocals, tables of cubes and reciprocals of the squares, powers of 2, 3, 5, 7, 11, $x^{\frac{1}{2}}$, $(10x)^{\frac{1}{2}}$, $(x/10)^{1/3}$, $x^{1/3}$, $(10x)^{1/3}$. Section II, **Factor Table, Factorials,** etc., p. 20–25 (new): Factor table of 49–9997; n!, 1/n!, n!!, 1/n!!, n = 1(1)20; multiples of $\log e = M$, x/M, $2x/\pi$, $\pi x/2$. Section III, **Auxiliary tables for computation with complex numbers,** p. 26–33 (old, p. 12–19), slight rearrangements. Section IV, **Quadratic equations,** p. 34–37 (new), graphs and tables for the solution of

$x^2 + px \mp q^2 = 0$. Section V, **Cubic equations,** p. 38–47 (old, p. 20–30), much the same. Section VI, **Equations of the fourth degree,** p. 47–72 (new), elaborate graphs, and tables for finding real and complex roots of biquadratic equations. Section VII, **Circular functions, angles in degrees,** p. 73–79 (new), tables of $\sin x$, $\cos x$, $\tan x$, $\sqrt{2} \sin x$, $\sqrt{2} \cos x$, $1 - \cos x$; also of integrals; graphs; etc. Section VIII, **Angles in quadrants,** p. 80–89 (new), tables of amp. $\frac{1}{2}\pi x$, $\sin x$, $\sinh \frac{1}{2}\pi x$, $e^{-\frac{1}{2}\pi x}$, $\cos x$, $\cosh \frac{1}{2}\pi x$, $e^{\frac{1}{2}\pi x}$, $\tan x$, $\tanh \frac{1}{2}\pi x$, $\coth \frac{1}{2}\pi x$, $\sqrt{2} \sin x$, $\sqrt{2} \cos x$, $1 - \cos x$, $\cosh \frac{1}{2}\pi x - 1$; also graphs. Section IX, **Hyperbolic functions, angles in radians,** p. 90–115 (old, p. 52–60), various formulae, functions of a multiple sector, powers, derivatives, integrals, approximate values for small sectors, gudermannian; graphs and altitude chart; numerous tables include those of $\sin x$, $\cos x$, $\tan x$, $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$, $\sinh x$, $\cosh x$, $\tanh x$, $\sinh^{-1} x$, $\cosh^{-1} x$, e^x, e^{-x}, $1 - \cos x$, $\cosh x - 1$; tables of $\ln x$ and $\log x$. Section X, **Special functions,** p. 116–130 (old, p. 36–39, 44–51), tables of e^{-x^2}, Planck's radiation function, Langevin's function $\coth x - 1/x$, $\rho \tan \rho$, $-\cot \rho/\rho$, $-\tan \rho/\rho$, ρ in right angles; numerous graphs including Hayashi's $\sinh x \sin x$, $\cosh x \cos x$, $\sinh x \cos x$, $\cosh x \sin x$. Section XI, **Transcendental equations,** p. 130–131 (old, p. 30–31). Section XII, **Circular and hyperbolic functions of a complex variable,** p. 132–145 (old, p. 60–76), some revisions. Section XIII, **The function** $[\tan(i^{\frac{1}{2}}r)]/i^{\frac{1}{2}}r$, p. 146–155 (new), graphs and table of the function. Section XIV, **Chebyshev's polynomials,** p. 156–157 (new), graphs. Section XV, **The function** z^z, p. 158 (new), graph. Section XVI, **Approximate calculations with polynomials,** p. 159–162 (new), formulae, tables. Section XVII, **Some remarks on numerical calculations,** p. 163–172 (new). The tables on old pages 32–35 and 40–44 do not appear in the newer work.

On pages 173–176 is a list of nearly 100 "Useful books for the computer."

In various tables the interval in the argument is either 1 or 2 or 5 units in the last figure. The differences printed are calculated throughout for the interval 1. "The interval is chosen so great that the maximal error which might arise by linear interpolation does not exceed .5 (10^{-4}) of the function value. Further details on this point are to be found in the last chapter. This increase of the interval reduces the tables considerably, so that the values for many functions can be given in small space. The tables are not overstretched, but adapted to the demand of each function."

The work is in English and German throughout, the English translations having been prepared by G. W. O. Howe of the University of Glasgow.

Emde felt that his "book will be useful to many who never refer to the 1938 Jahnke-Emde. The latter also include most of the students of Technical Colleges. Our book may prove a help in their mathematical instruction. The numerous figures to scale, of which many show the same function in various scales, will make the functions clearer. An attentive study of the variation of the interval in the several functions will lead to greater insight."

R. C. A.

186[A, D].—H. M. Terrill & Ethel M. Terrill, "Tables of numbers related to the tangent coefficients," Franklin Institute, *J.*, v. 239, Jan. 1945, p. 66–67. 16.5 × 24.1 cm.

If $\tan x = T_1 x + T_2 \dfrac{x^3}{3!} + T_3 \dfrac{x^5}{5!} + T_4 \dfrac{x^7}{7!} + \cdots$,

$$T_n = 2^{2n}(2^{2n} - 1)B_n/2n.$$

Correcting the erroneous relation in the text, the number H_n should have been defined by $nH_n = 2^n(2^{2n} - 1)B_n$. Hence

$$H_n = T_n/2^{n-1}.$$

Defining the number $K_n = 2(2^{2n} - 1)B_n$, we have $K_n = nT_n/2^{2n-2}$, and

$$nH_n = 2^{n-1}K_n.$$

The chief tables before us give the values of H_n and K_n (each an integer) for $n = 1(1)19$. Since there is a table of T_n for $n = 1(1)30$, in Peters and Stein's *Anhang*, p. 88, of Peters' *Zehnstellige Logarithmentafel*, v. 1, Berlin, 1922, the computation of the present tables for $n = 1(1)19$ was a rather trivial matter. There are also two small tables of H_n^p and K_n^p, for $p = 1(1)4$, $n = 2(1)5$; these numbers are defined by the recurrence formulae

$$2p^2H_n^p - (p+1)(2p+1)H_n^{p+1} = H_{n+1}^p,$$
$$p^2K_n^p - (p+1)^2K_n^{p+1} = K_{n+1}^p,$$

where $H_2^p = K_2^p = 1$.

In his *Théorie des Nombres*, Paris, 1891, v. 1, p. 251, LUCAS refers to $G_{2n} = (-1)^nK_n$ as Genocchi numbers (A. GENOCCHI, *Annali d. Sci. Matem.*, v. 3, 1852, p. 395 f.), though considered by EULER (*Institutiones Calculi Differentialis*, v. 2, 1755, p. 486), and Lucas gives the first six numbers of the Terrill table. A ms. table of K_n for $n \leq 110$, by D. H. L., is in the Library of the American Mathematical Society.

R. C. A.

187[B, D, E, G, H, L].—P. R. E. JAHNKE & F. EMDE, *Tables of Functions with Formulae and Curves [Funktionentafeln mit Formeln und Kurven . . .] Published and distributed in the public interest by authority of the Alien Property Custodian under license No. A-57*, Fourth revised edition, New York, Dover Publications, 1780 Broadway, 1945. [xv], 306, 76 p. 15.4 × 29.8 cm. $3.75.

In *MTAC*, p. 106–109, 171 and 204 (corrigenda), 293–294, some information has been given regarding the five previous editions or reprints in 1909, 1933, 1938, 1941, and 1943. Now we have the sixth, with the correction of 396 errors in the fifth edition (or reprint), and the addition of a Supplementary Bibliography containing 43 titles. Material for correction of the errors was mainly furnished, either directly or indirectly, by R. C. A., L. J. C., NYMTP, and J. C. P. MILLER. In many cases these errors, on 56 pages, are only unit errors in the last decimal place. A list of the pages where the only known errors in the 1943 volume occur, followed by their number on each page, may be found to be of service. They are as follows:

9 (1), 10 (2), 34 (8), 35 (3), 39 (1), 59 (2), 62 (1), 65 (1), 66 (1), 69 (3), 70 (1), 72 (3), 74 (1), 79 (1), 85 (3), 94 (1), 102 (2), 103 (1), 105 (1), 117 (1), 124 (13), 125 (40), 128 (2), 164 (32), 166 (1), 168 (27), 177 (1), 183 (1), 204 (2), 205 (1), 206 (4), 212 (1), 216 (2), 218 (1), 220 (1), 228 (15), 229 (6), 234 (8), 235 (63), 236 (35), 237 (5), 238 (42), 239 (10), 246 (1), 247 (2), 263 (3), 266 (1), 300 (3), 301 (1), [2] (2), [4] (4), [5] (8), [6] (5), [7] (12), [9] (1), [56] (2). The notation [56], for example, refers to p. 56 in the second section, of 76 p., in the volume. There are also changes 298(6), 299(2).

In earlier pages of *MTAC* a number of these errors have been already listed. References to these, a discussion of other errors of more than a unit in the final figure, as well as a list of errors in the 1909 and 1933 editions, are given elsewhere in this issue, MTE **61**, p. 391 f.

Thus information here placed at the disposal of owners of all editions will enable them to increase the value of their editions as tools, while purchasers of the volume under review will have a greatly improved edition of an extraordinarily useful work.

R. C. A.

188[C, D].—*Five-Figure Logarithm Tables containing Logarithms of Numbers and Logarithms of Trigonometrical Functions with Arguments in Degrees and Decimals.* Published for the MINISTRY OF SUPPLY by His Majesty's Stationery Office, London, 1944. iv, 74, XXII, 29–120 p. 15 × 24 cm. 7s 6d.

"This collection of 5-figure tables of logarithms and logarithmic trigonometrical functions has been compiled as a war-time measure, primarily to meet the urgent requirements of the optical industry. Expediency can be the only justification for the juxtaposition of differing page sizes, of differing styles of types and of differing forms of printing." So begins the Preface by H. J. GOUGH, director-general of Scientific Research and Development, Ministry of Supply, writing in September 1944.

The first section of the volume is a reproduction of p. 2–73 of E. CHAPPELL, *Five-Figure Mathematical Tables* . . . , Edinburgh and London, W. & R. Chambers, 1915. Through an arrangement with the publishers the original stereotype plates were used. These reproduced pages give logarithms of numbers 10 000 to 40 000, and 4 000 to 10 000, with differences and proportional parts. See *MTAC*, p. 195. The next two parts are photolithographic reproductions of German works, under license from the Custodian of Enemy Property. The first of these has the title-page: *Die Logarithmen der Sinus und Tangenten für 0° bis 5° und der Cosinus und Cotangenten für 85° bis 90° von Tausendstel zu Tausendstel Grad als Ergänzung zu C Bremiker's 5 stelligen Logarithmentafeln, herausgegeben von* M. von ROHR . . . Berlin, Weidmannsche Buchhandlung, 1900, and the table occupies pages I–XX. The final part of the volume has the title page: *Dr. C. Bremikers Logarithmisch-Trigonometrische Tafeln mit fünf Decimalstellen, besorgt von* . . . A. KALLIUS. *Sechzehnte Stereotyp-Auflage,* Berlin, Weidmannsche Buchhandlung, 1925. The pages 30–119 contain five-figure logarithms of sines, tangents, cotangents and cosines, for the angles 0(0°.01)45°. On the final page are values of constants.

The original edition of these tables of CARL BREMIKER (1804–1877) appeared in 1872. Beginning with the third edition in 1880, A. Kallius was the editor. There was a seventeenth edition in 1937. The third unchanged edition, of the supplement by Louis Otto Moritz von Rohr (1868–), appeared in 1933.

Explanations of any kind in connection with the tables were naturally regarded as unnecessary. Some American libraries will doubtless be glad of the opportunity offered for purchasing this volume so as to get a copy of Rohr's table.

Workers in the optical industry in this country, where computing machines are so freely used, are not likely to find any use for the volume under review. Rather would they turn to such a volume as the seven-place table of natural trigonometric functions for every thousandth of a degree, prepared by Peters in 1918 for the Goerz optical establishment, now readily available in an American edition. See *MTAC*, p. 12–13.

R. C. A.

189[D].—HERMANN BRANDENBURG, *Sechsstellige trigonometrische Tafel alter Kreisteilung für Berechnungen mit der Rechenmaschine, enthaltend die unmittelbaren oder natürlichen Werte der vier Winkellinien-Verhältnisse: Sinus, Tangens, Cotangens und Cosinus des in* 90° *und* 60′ *geteilten Einheits-Viertelkreises in Unterschieden von* 10 *zu* 10 *Sekunden nebst zwei Vortafeln mit Einzelsekundenwerten der Cotangente und erweiterten Zehnsekundenwerte des Sinus und der Tangente kleiner Winkel. . . . Das Vorwort in deutscher, englischer, französischer, spanischer, holländischer, russischer, und japanischer Sprache,* Leipzig, Lorenz, 1932. Back of the title page is, in part, *Copyright vested in the Alien Property Custodian, 1944, pursuant to law. Published and distributed in the Public Interest by authority of the Alien Property Custodian under License No. A.407. Lithoprinted by Edwards Brothers, Inc., Ann Arbor, Michigan, U. S. A.*, 1945. xxii, 304 p. 20.2 × 28 cm. $5.00.

In 1923 Brandenburg published seven-place trigonometric tables containing natural values of the Sine, Tangent, Cotangent, and Cosine at interval 10″, with differences and proportional parts. There were also an introductory table of Cotangent up to 6°, and of Tangent from 84° to 90°, both at interval 1″. The construction of this work was based on the 10-place trigonometric table of RHÆTICUS, 1596, 1607, and the 15-place trigonometric table of PITISCUS, 1613 (see *MTAC*, p. 8–9, and A. DEMORGAN, *English Cyclopaedia*, ed. C. Knight, *Arts and Sciences*, v. 7, London, 1861, article "Table"). But this edition of Brandenburg's work contained over 400 errors. All of these were eliminated in the second edition of 1931, which is "practically free from error." This work was prepared chiefly for computers working with calculating machines.

From the material of these editions Brandenburg prepared six-place tables by rounding off the last decimal, and thus appeared the original of the work now under review. Vortafel 1 is of Cotangent to 3° (or of Tangent 87° to 90°) at interval 1″, to 7S; and Vortafel 2 is of Sine and Tangent to 1°, at interval 10″, mostly to 7 or 8D. The last five pages contain miscellaneous numerical data and tables. Nine errata discovered in this volume by L. J. C. were published in *MTAC*, p. 162. Six of these, and two other corrections on p. 64, are listed on the back of the dedication page (p. iv). The two corrections not previously listed in *MTAC* are as follows:

Diff. following cot 5° 42′ 30″, *for* 4903, *read* 4898
P.P. for 9/10 of 490, *for* 441.0, *read* 441,0.

The volume is well reproduced, and bound in light-blue buckram, with title in black ink along the back.

Brandenburg well remarks that a "6-figure table has one advantage that must not be underestimated, namely that it suffices for the greater part of the calculations encountered in the solution of triangles, and because of its convenience in manipulation is particularly advantageous when five or fewer figures are sufficient, in which case the unused figures are rejected. Further the owner of a 7-figure table generally equips himself with a 6-figure table also, whereas the converse is rarely the case. The 6-figure table is undoubtedly especially beloved in the computing profession, and can be pronounced the favorite among trigonometrical tables."

R. C. A.

190[F].—L. ALAOGLU & P. ERDÖS, "On highly composite and similar numbers," Amer. Math. So., *Trans.*, v. 56, Nov. 1944, p. 448–469. 19.5 × 27.5 cm.

This paper contains two tables (p. 467–469) having to do with so-called abundant numbers. If the sum $\sigma(n)$ of the divisors of n exceeds $2n$, then n is called ***abundant***. In case $\sigma(n)>\sigma(m)$ for all $m<n$, then n is called ***highly abundant***. In case $\sigma(n)/n>\sigma(m)/m$ for all $m<n$, then n is called ***superabundant***.

The first table lists all highly abundant numbers $\leqslant$ 10080. This table errs by including the non-highly abundant number 2280 and by excluding the highly abundant numbers 1920 and 2160. The number of highly abundant numbers $\leqslant$ 10080 is thus 84, not 83. The table also contains the minor errata:

$n = 1080$	*for* $2^3.3.5$, *read* $2^3.3^3.5$
$n = 4620$	*for* $\sigma(n) = 16120$, *read* 16128.

This table was based on Glaisher's tables[1] of $\sigma(n)$ for $n \leqslant 10^4$.

The second table lists all superabundant numbers $< 10^{18}$ in their factored form together with values to 3D of $\sigma(n)/n$ in each case. These numbers are similar to Ramanujan's[2] highly composite numbers which have more divisors than any smaller number.

D. H. L.

[1] J. W. L. GLAISHER, Table I in BAASMTC, *Mathematical Tables, volume VIII, Number-Divisor Tables*, Cambridge, 1940.

[2] S. RAMANUJAN: "Highly composite numbers," London Math. So., *Proc.*, s. 2, v. 14, 1915, p. 347–409; *Collected Papers*, Cambridge, 1927, p. 78–128.

191[L].—R. P. BELL, "Eigen-values and eigen-functions for the operator $d^2/dx^2 - |x|$," *Phil. Mag.*, s. 7, v. 35, Sept. 1944, p. 584–585. 16.9 × 25.3 cm.

T. I gives values of $J_{2/3}(z)$ and $J_{-2/3}(z)$, for $z = [0(.02)1; 4D]$. These were calculated from the following series:

$$J_{2/3}(z) = .69783z^{2/3}(1 - .1500z^2 + .00703z^4 - .00016z^6 + \cdots), \text{ and}$$
$$J_{-2/3}(z) = .59255z^{-2/3}(1 - .7500z^2 + .07031z^4 - .00251z^6 + .000047z^8 - \cdots).$$

Interpolation in T. I gave the first zero of $J_{2/3}(z) - J_{-2/3}(z)$ as $z = .6856$, not quite agreeing with the value given by H. M. MACDONALD, R. So. London, *Proc.*, v. 90A, 1914, p. 54, where we find .6854. The eigen-values λ_n of $\left(\frac{d^2}{dx^2} - |x| + \lambda\right)\psi = 0$ are given by

$$J_{2/3}(\tfrac{2}{3}\lambda_n^{3/2}) - J_{-2/3}(\tfrac{2}{3}\lambda_n^{3/2}) = 0, \ n \text{ even},$$
$$J_{1/3}(\tfrac{2}{3}\lambda_n^{3/2}) - J_{-1/3}(\tfrac{2}{3}\lambda_n^{3/2}) = 0, \ n \text{ odd}.$$

The first six values of λ_n are given, $n = [0(1)5; 3D]$, in T. II.

In T. III are the eigen-values λ_n of $\left(\frac{d^2}{dx^2} - |x^q| + \lambda\right)\psi = 0$ for $q = 1, 2, 4, \infty$, $n = [0(1)5; 2 \text{ or } 3D]$.

The error in Watson's *Bessel Functions*, p. 751, noted by J. C. P. MILLER, *MTAC*, p. 367, was already published by Bell, on p. 584.

R. C. A.

192[L].—R. CLARK JONES, "On the theory of the directional patterns of continuous source distributions on a plane surface," Acoustical So. Amer., *J.*, v. 16, Jan. 1945, p. 158–171. 19.8 × 26.6 cm.

Computations are made for distributions of finite extent with a distribution function of type

$$A(u) = (1 - u^{2m})^n$$

with 10 different choices of (m, n) for both the circular and the linear cases. These choices are:

$$(1, -1),\ (1, 0),\ (1, 1),\ (1, 2),\ (1, 3),\ (1, 4),\ (2, 1),\ (2, 2),\ (3, 1),\ (4, 1).$$

There are the following 21 tables: T. I.—$J_0(\alpha)$ and $J_0^2(\alpha)$ to 5S, $10\log J_0^{-2}(\alpha)$ to 4S, for $\alpha = 0(.4)20$. Also first 12 values of α for maxima of $J_0^2(\alpha)$ (zeros of $J_1(\alpha)$) to 3D. T. II.—$Q_N = 2J_1(\alpha)/\alpha$ and Q_N^2 to 6S, $10\log Q_N^{-2}$ mostly 4S, $\alpha = 0(.1)20$. Also first 12 values of α for maxima of Q_N^2 (zeros of $J_2(\alpha)$) to 3D. T. III.—$Q_N = 8J_2(\alpha)/\alpha^2$ and Q_N^2 to 5S, and $10\log Q_N^{-2}$ to 4S, $\alpha = 0(.4)20$. Also first 11 values of α for maxima of Q_N^2 (zeros of $J_2'(\alpha)$) to 3D. T. IV.—$Q_N = 48J_3(\alpha)/\alpha^3$ and Q_N^2 to 5S, and $10\log Q_N^{-2}$ to 2D, $\alpha = 0(.4)20$. Also first 11 values of α for maxima of Q_N^2 (zeros of $J_4(\alpha)$), to 3D. T. V.—$Q_N = 384J_4(\alpha)/\alpha^4$ and Q_N^2 to 5S, and $10\log Q_N^{-2}$ to 2D, $\alpha = 0(.4)20$. Also first 10 values of α for maxima of Q_N^2 (zeros of $J_5(\alpha)$), to 3D. T. VI.—$Q_N = 3840J_5(\alpha)/\alpha^5$ and Q_N^2 to 5S, and $10\log Q_N^{-2}$ to 4S, $\alpha = 0(.4)20$. Also first 10 values of α for maxima of Q_N^2 (zeros of $J_6(\alpha)$). T. VII.—$Q_N = 12J_2(\alpha)/\alpha^2 - 24J_3(\alpha)/\alpha^3$ and Q_N^2 to 5S, and $10\log Q_N^{-2}$ to 4S, $\alpha = 0(.4)20$. T. VIII.—$Q_N = 120J_3(\alpha)/\alpha^3 - 720J_4(\alpha)/\alpha^4 + 1440J_5(\alpha)/\alpha^5$ and Q_N^2 to 5S, and $10\log Q_N^{-2}$ to 4S, $\alpha = 0(.4)20$. T. IX.—$Q_N = 16J_2(\alpha)/\alpha^2 - 64J_3(\alpha)/\alpha^3 + 128J_4(\alpha)/\alpha^4$, and Q_N^2 to 4S, and $10\log Q_N^{-2}$ to 4S, $\alpha = 0(.4)20$. T. X.—$Q_N = 20J_2(\alpha)/\alpha^2 - 120J_3(\alpha)/\alpha^3 + 480J_4(\alpha)/\alpha^4 - 960J_5(\alpha)/\alpha^5$ and Q_N^2 to 4S, and $10\log Q_N^{-2}$ to 4S. T. XI.—$Q_N = \sin\beta/\beta$ to 6S, and Q_N^2 to 5S, and $10\log Q_N^{-2}$ to 5S, $\beta = 0(.4)20$. Also first 5 values of β, to 2D, for maxima of Q_N^2. T. XII.—$Q_N = 3(\pi/2\beta)^{\frac{1}{2}}J_{3/2}(\beta)/\beta$ to 6S, Q_N^2 to 5S, and $10\log Q_N^{-2}$ to 5S, $\beta = 0(.4)20$. Also first 5 values of β, 2–3D, for maxima of Q_N^2. T. XIII.—$Q_N = 15(\pi/2\beta)^{\frac{1}{2}}J_{5/2}(\beta)/\beta^2$ to 6S, and Q_N^2 to 5S, and 10log Q_N^{-2} to 4S, $\beta = 0(.4)20$. Also first 4 values of β, to 2D, for maxima of Q_N^2. T. XIV.—$Q_N = 105(\pi/2\beta)^{\frac{1}{2}}J_{7/2}(\beta)/\beta^3$ to 6S, Q_N^2 to 5S, 10log Q_N^{-2} to 4S, $\beta = 0(.4)20$. Also first 4 values of β, to 2D, for maxima of Q_N^2. T. XV.—$Q_N = 945(\pi/2\beta)^{\frac{1}{2}}J_{9/2}(\beta)/\beta^4$ to 6S, Q_N^2 to 5S, 10log Q_N^{-2} to 4S, $\beta = 0(.4)20$. T. XVI.—$Q_N = (\pi/2\beta)^{\frac{1}{2}}[5J_{3/2}(\beta)/\beta - 10J_{5/2}(\beta)/\beta^2]$ and Q_N^2 to 5S, 10log Q_N^{-2} to 4S, $\beta = 0(.4)20$. T. XVII.—$Q_N = (\pi/2\beta)^{\frac{1}{2}}[45J_{5/2}(\beta)/\beta^2 - 270J_{7/2}(\beta)/\beta^3 + 540J_{9/2}(\beta)/\beta^4]$, and Q_N^2 to 5S, and 10log Q_N^{-2} to 4S, $\beta = 0(.4)20$. T. XVIII.—$Q_N = (\pi/2\beta)^{\frac{1}{2}}[7J_{3/2}(\beta)/\beta - 28J_{5/2}(\beta)/\beta^2 + 56J_{7/2}(\beta)/\beta^3]$, and Q_N^2 to 5S, 10log Q_N^{-2} to 4S, $\beta = 0(.4)20$. T. XIX.—$Q_N = (\pi/2\beta)^{\frac{1}{2}}[9J_{3/2}(\beta)/\beta - 54J_{5/2}(\beta)/\beta^2 + 216J_{7/2}(\beta)/\beta^3 - 432J_{9/2}(\beta)/\beta^4]$, and Q_N^2 to 5S, and 10log Q_N^{-2} to 4S, $\beta = 0(.4)20$. T. XXI.—$Q_N = \cos\beta/[1 - (2\beta/\pi)^2]$ and Q_N^2 to 5S, 10log Q_N^{-2} to 4S, $\beta = 0(.4)20$. T. XXII.—$Q_N = \sin\beta/[\beta(1 - (\beta/\pi)^2)]$ and Q_N^2 to 5S, 10log Q_N^{-2} to 4S, $\beta = 0(.4)20$.

R. C. A. & H. B.

193[L].—NATIONAL DEFENSE RESEARCH COMMITTEE, Division 6, *Scattering and Radiation from Circular Cylinders and Spheres*, prepared by A. N. LOWAN of the N. Y. M. T. P., & by P. M. MORSE, H. FESHBACH, M. LAX, of the M. I. T. Underwater Sound Laboratory. Report February 1945. iii + 124 leaves, 21.5 × 28 cm. Printed on only one side of each leaf, by the photo-offset process from manuscript. These tables are available only to certain Government agencies and activities.

In problems of wave motion external to spheres and cylinders the Bessel functions of the first and second kinds are of importance. The tables 1–5, 8–12 in this report are all for $x = 0(.1)10$ with 2D, but the number of significant figures generally varies from 3 to 5. In tables 6, 7, 13 and 14, 6S are given and the ranges are $x = 0(.1)10$, $n = 0(1)20$. The same range for n is used for all the other tables except 10 and 11 where $n = 0(1)15$. The quantities tabulated are:

Cylindrical Functions	*Spherical Functions*
1. $\alpha_n(x) = \tan^{-1}[-xJ_n'(x)/J_n(x)]$	8. $\alpha_n(x) = \tan^{-1}[-xj_n'(x)/j_n(x)]$
2. $\beta_n(x) = \tan^{-1}[-xN_n'(x)/N_n(x)]$	9. $\beta_n(x) = \tan^{-1}[-xn_n'(x)/n_n(x)]$
3. $\delta_n(x) = \tan^{-1}[-J_n(x)/N_n(x)]$	10. $\delta_n(x) = \tan^{-1}[-j_n(x)/n_n(x)]$
4. $\delta_n'(x) = \tan^{-1}[-J_n'(x)/N_n'(x)]$	11. $\delta_n'(x) = \tan^{-1}[-j_n'(x)/n_n'(x)]$
5. $\gamma_n(x) = \tan^{-1}[\tan\delta_n \sec\alpha_n \cos\beta_n]$	12. $\gamma_n(x) = \tan^{-1}[\tan\delta_n \sec\alpha_n \cos\beta_n]$
6. $C_n(x) = \lvert J_n(x) + iN_n(x)\rvert$	13. $D_n(x) = \lvert j_n(x) + i\,n_n(x)\rvert$
7. $C_n'(x) = \lvert J_n'(x) + iN_n'(x)\rvert$	14. $D_n'(x) = \lvert j_n'(x) + i\,n_n'(x)\rvert$

The function $N_n(x)$ is the Bessel function of the second kind denoted by $Y_n(x)$ in *MTAC*. The functions $j(z)$, $n(z)$ are defined by the equations

$$j_n(z) = (\pi/2z)^{\frac{1}{2}}J_{n+\frac{1}{2}}(z), \qquad n_n(z) = (\pi/2z)^{\frac{1}{2}}N_{n+\frac{1}{2}}(z).$$

A useful list of properties of the functions is given. In the table of contents the expressions for the spherical functions $\delta_n(x)$ and $\delta_n'(x)$ in 10–11 are given incorrectly.

H. B.

194[L].—ÎA. M. SEREBRIĬSKIĬ, "Obtekanie tel vrashcheniîa" [Flow past bodies of revolution], Akad. Nauk, SSSR, Otdelenie tekhnichnikh Nauk, Institut Mekhaniki, *Prikladnaîa Matematika i Mekhanika, Applied Mathematics and Mechanics*, v. 8, June 1944, p. 103, 104. 16.7 × 25.5 cm.

There is a table of Legendre functions $P_n(\mu)$, $dP_n(\mu)/d\mu$, for $n = 1(1)5$, $\mu = [-1(.05) -.9(.1)+.9(.05)1$; 4D], where $\mu = \cos\theta$; θ is given to the nearest 1′ for each value of μ.

There are also graphs of $Q_n(\lambda) = Q_n(\cosh\psi)$, and of $\delta_n(\psi)$, for $\psi = .1(.05).35$, $n = 1(1)5$, where $dQ_n(\lambda)/d\lambda = \delta_n(\psi) - 1/\psi^2$.

H. B.