Comparison of no. **1** with no. **11** yields the following:

Ince no. **1**			*Lubkin & Stoker* no. **11**		
$\theta = 40$	$a_2 =$	$-$ 20.20794 08	$\beta = 20$	$\alpha(C_2) =$	$-$ 5.05198 (= 20.20792 ÷ 4)
$\theta = 2$	$b_3 =$	9.14062 77	$\beta = 1$	$\alpha(C_3) =$	2.28515 (= 9.14060 ÷ 4)
$\theta = 4$	$a_3 =$	10.67102 71	$\beta = 2$	$\alpha(S_3) =$	2.66777 (= 10.67108 ÷ 4)
$\theta = 24$	$b_4 =$	13.55279 65	$\beta = 12$	$\alpha(S_4) =$	3.38817 (= 13.55268 ÷ 4)
$\theta = 12$	$a_4 =$	22.97212 75	$\beta = 6$	$\alpha(C_4) =$	5.74803 (error)
$\theta = 32$	$b_5 =$	26.10835 26	$\beta = 16$	$\alpha(C_5) =$	6.52721 (= 26.10884 ÷ 4)
$\theta = 40$	$b_5 =$	22.33214 85	$\beta = 20$	$\alpha(C_5) =$	5.58302 (= 22.33208 ÷ 4)
$\theta = 40$	$a_5 =$	41.34975 44	$\beta = 20$	$\alpha(S_5) =$	10.33749 (= 41.34996 ÷ 4)
$\theta = 40$	$b_6 =$	41.43300 52	$\beta = 20$	$\alpha(S_6) =$	10.35813 (= 41.43252 ÷ 4)

In the above, differences indicate that Lubkin & Stoker's value of $\alpha(C_4)$ for $\beta = 6$ is in error, and should be 5.74303. In the other cases the discrepancies are too small for the available differences to discriminate with certainty, but in view of the fact that all Ince's values were worked out to 12 decimals by a process of successive approximation, and bear evidence of careful checking, they are prima facie the more reliable.

Comparison of Hidaka no. **9** with Ince no. **1** shows two end-figure discrepancies

	Ince no. **1**	*Hidaka* no. **9**
$\theta = 2$,	$B_3^{(5)} =$ 0.12413 61	0.12413 60
$\theta = 1$,	a_5 = 25.02085 43	25.02085 44.

In the case of the second of these, the present writer agrees with Ince, having obtained 25.02085 43454 5. . .

Comparison of Hidaka no. **9** with Goldstein no. 6 shows one discrepancy.

Goldstein no. **6**		*Hidaka* no. **9**	
$q = 0.2$	$-B_3^{(1)} = 0.16171$	$\theta = 1.6$	$-B_3^{(1)} =$ 0.16171 80

Comparison of Hidaka no. **9** with Lubkin & Stoker no. **11** confirms that $\alpha(S_1)$ for $\beta = 0.8$ should be 0.55406. It yields also the following.

Hidaka no. **9**		*Lubkin & Stoker* no. **11**	
$\theta = 1.2$	$b_3 =$ 9.06485 47	$\beta = 0.6$	$\alpha(C_3) =$ 2.26622 (= 9.06488 ÷ 4)
$\theta = 2.0$	$b_3 =$ 9.14062 77	$\beta = 1.0$	$\alpha(C_3) =$ 2.28515 (= 9.14060 ÷ 4)

Additional References

For collected accounts of ranges of the theory of solutions of the Mathieu equation, reference may be made (in addition to nos. **6**, **10**, and **16** above) to

M. J. O. STRUTT, *Lamésche, Mathieusche, und verwandte Funktionen in Physik undTechnik*, (*Ergebnisse der Mathematik*, v. 1, part 3), Berlin, 1932.

E. T. WHITTAKER and G. N. WATSON, *Modern Analysis*, Cambridge, University Press, fourth ed. 1927; Amer. reprint, 1943.

E. G. C. POOLE, *Introduction to the Theory of Linear Differential Equations*, Oxford, University Press, 1936.

W. G. BICKLEY

Imperial College of Science and Technology,
LONDON, S. W. 7, England.

RECENT MATHEMATICAL TABLES

195[A, D].—J. G. BECKERLEY, "The calculation of arg Γ $(ia + 1)$," *Indian J. Physics*, v. 15, and Indian Assoc. for the Cultivation of Science, *Proc.*, v. 24, 1941, p. 229–232. 16.5 × 23 cm.

$\Gamma(z)$ for complex values of z has been tabulated to a rather limited extent. The tables of WALTER MEISSNER, 1939, were reviewed in *MTAC*, p. 177; in his *Tables of the Higher*

Mathematical Functions, v. 1, Bloomington, Indiana, 1933, p. 269f., H. T. DAVIS tabulated $1/\Gamma(re^{i\theta})$, to 12D, for $r = -1(.1) + 1$, $\theta = 0°, 30°, 45°, 60°, 90°, 120°, 135°, 150°$; in C. P. WELLS & R. D. SPENCE, *J. Math. Phys.*, M.I.T., v. 24, Feb. 1945, p. 61, there is a table of $\Gamma[(3 + ia)/4]$ giving the argument and modulus to 4D, $a = 1(1)5$; JAHNKE & EMDE's reference, p. 21 of the 1943 edition, to a certain GINZEL table of $(x + iy)!$, is without foundation in fact.

Beckerley gives a 4-place table of arg $\Gamma(ia + 1)$ for $a = [0(.1)2; 4D]$, with indications of the methods of calculation. This function occurs frequently in problems involving positive energy hydrogen functions.[1] The author expresses the hope that "this table will be of use to physicists who are engaged in numerical calculations involving the 'Coulomb phase factors' which occur in the continuous spectrum wave functions of hydrogenic atoms."

R. C. A.

[1] A. SOMMERFELD, *Wave-Mechanics*, London, 1930, p. 290f.; G. GAMOW, *Structure of Atomic Nuclei and Nuclear Transformations*, Oxford, 1937, chap. IX, equation 15, p. 163; E. C. KEMBLE, *Fundamental Principles of Quantum Mechanics*, New York and London, 1937, p. 177.

196[A, F].—I_1. J. KAVÁN, *Rozklad Všetkých Čísel Celých od* 2 *do* 256 000 *v Prvočinitele*, and on the second title-page: *Tabula Omnibus a* 2 *usque ad* 256 000 *Numeris Integris Omnes Divisores Primos Praebens* (Observatorium Publicum, Stará Ďala, Czechoslovakia), Prague, 1934, xi, 514 p. 28.3 × 29.4 cm. See RMT **71**, *Scripta Mathematica*, v. 4, 1936, p. 338. I_2. *Factor Tables giving the Complete Decomposition into Prime Factors of All Numbers up to* 256 000. . ., with prefaces, English by B. ŠTERNBERK, and Latin by K. PETR, and an introduction by ARTHUR BEER, London, Macmillan, 1937. xii, 514 p. 28.3 × 29.4 cm. 42 shillings. II. BAASMTC, *Mathematical Tables, volume V. Factor Table giving the Complete Decomposition of all Numbers less than* 100,000 *prepared independently by* J. PETERS, A. LODGE and E. TERNOUTH, E. GIFFORD . . ., London, 1935, xv, 291 p. 21.6 × 27.9 cm. Published at 20 shillings; out of print. For brevity these tables will be referred to in what follows as Table I and Table II.

Many mathematical problems arising out of the war effort are concerned with the development of new expansions for very special functions. These expansions, in most cases, have rational coefficients. To maintain accuracy in the combination and checking of these expansions it is often best to keep denominators in factored form and, to reduce the rational numbers to their lowest terms, it is convenient to know the factors of the numerators also. In short, a good computer need not be a number theorist to profit by using a fair-sized factor table at appropriate times.

A number of small factor tables will indeed be found in machinist's handbooks and small volumes of collected tables. These usually extend to about 10000 and often omit multiples of 2, 3, and 5, the very numbers most frequently met with in work of the sort described above. These embarrassing omissions are a practical necessity in the great factor tables of the first ten millions, which are intended after all for number theorists. The two tables under review are unique in giving all the prime factors of every number in their respective ranges and at the same time being sufficiently extensive to take care of the 5- or 6-figure work most frequently required. A comparison of the two tables may be made as follows. Table I is of course more than two and a half times as extensive as Table II. However, this does not mean that the former is twice as useful as the latter. Anyone who inspects a much used copy of a factor table cannot fail to notice the frayed and soiled edges of the early pages and the comparatively new appearance of its later pages.

The two tables are well arranged, though quite differently. Table I is arranged like an ordinary table of logarithms, consecutive numbers being in adjacent columns which are headed 0, 1, ⋯, 9, with 50 lines to the page. The lines are numbered at both ends to facilitate entering the rather wide (23 cm.) table. Table II is arranged so that consecutive numbers occupy the same column and adjacent lines, an opening being devoted (in most cases) to 700 numbers, as compared with 1000 in Table I. The left page of the opening gives the factors of numbers whose last two digits are 00, 01, ⋯, 49. The average computer will no doubt find the arrangement of Table I more familiar and natural. When the factors of a long series of consecutive numbers are needed (as in some number theory problems) Table II is more convenient to use. In case the number to be factored is actually a prime both tables print the number in bold-face type, thus assuring the user that he has entered the table correctly. This is a decided improvement over earlier tables in which primes are represented by a dash.

The printing of Table I is decidedly inferior to that of Table II. There is much irregularity in type and inking. The exponents of the primes are unnecessarily small and in some cases (e.g. $N = 123725$) are of two sizes. The printing job in Table II is beautifully done, the primes being in old type and the exponents in new type.

Both tables are extremely reliable, especially Table II, which is based on three independent manuscripts and an intense program of proof reading and comparison with previous tables. The history of factor tables shows a preponderance of printer's errors over author's errors. Table I, which took 17 years to prepare, was verified in manuscript by actually multiplying together the decompositions given. Had this been done on the proof sheets, the more serious of the two following errors might have been avoided:

p. 32 $N = 15280$, *for* $2^4 \cdot 3 \cdot 191$, *read* $2^4 \cdot 5 \cdot 191$;
p. 39 argument left column, *for* 1800, *read* 1870.

These errors were discovered by J. C. P. MILLER.

For the sort of work encountered in practical problems the numbers to be factored seldom contain large prime factors. Such numbers constitute a quite small minority, so that a small factor table devoted exclusively to such numbers may nevertheless have a considerable upper limit. Such a table has been published by Cunningham.[1] It gives the factorization of all numbers up to 100000 having no prime factor in excess of 11 on nine small pages, 1196 entries in all.

D. H. L.

NOTE BY S. A. JOFFE: In the Czech introduction by KAVÁN there are two errors in quotations from CAYLEY, *Collected Math. Papers*, v. 9, p. 462–463. On p. V, box, factors of 391 should be 17·23, *not* 17·33; Cayley was correct and Kaván copied the factor 23 incorrectly. On p. VI, first box, the number corresponding to 297 should be 180, *not* 198; here Kaván copied Cayley's error, p. 463.

[1] A. J. C. CUNNINGHAM, *Quadratic and Linear Tables*, London, Hodgson, 1927, p. 162–170.

197[C, D].—*Natural and Logarithmic Haversines*, "arranged from Bowditch, American Practical Navigator, 1938 Edition, Table 34. By permission of the HYDROGRAPHIC OFFICE, United States Navy Department." New York, Macmillan, 1943, 38 p. 14.2 × 21.2 cm. Paper cover, 30 cents.

This publication is misleading in more than one respect. In the source indicated, in the above quoted footnote, the five-place table is of logarithmic and natural haversines, 0° to 120°, at interval 15″; 120° to 135° at interval 30″; 135° to 180° at interval 1′. The natural haversines are in black-face type throughout. What is "arranged" is simply to abridge this table to every minute of arc throughout, five degrees to the page, with constant uniformity in face of type. Hence this does not justify the title-page legend, *Natural and Logarithmic Haversines* by PAUL R. RIDER . . . and CHARLES A. HUTCHINSON, even though this table also occupies p. 187–222 of *Navigational Trigonometry*, published by these authors in 1943.

The other criticism is of an error made by the Hydrographic Office, rather than by these authors. Nathaniel Bowditch (1773–1838) never had anything to do with such tables as those under review. In 1844, Nathaniel's son, J. I. Bowditch (1806–1889) edited a volume of tables taken from the *New American Practical Navigator* of his father. These were called *Bowditch's Useful Tables* (247 small p.); of this volume at least a score of editions had been published up to 1932. The editions after 1868 were published by a department of the Government, now called the Hydrographic Office, and after 1880 the word *New* was dropped from the earlier title *New American Practical Navigator*. Up to 1903 neither the *Useful Tables* ("Bowditch's" had been dropped from the title), nor the *American Practical Navigator*, contained haversine tables. But both of these works had such tables in their 1911 edition. Furthermore, the Hydrographic Office in that year published the volume, with the partial title-page, "*Useful Tables from the American Practical Navigator* by Nathaniel Bowditch" (427 large p.), even though no Bowditch had anything whatever to do with most of the contents of the volume, including the haversine tables.

The most elaborate published table of haversines is that of J. C. HANNYNGTON, *Haversines, Natural and Logarithmic, used in Computing Lunar Distances for the Nautical Almanac*, London 1876, 327 folio p. This is a 7-place table (except for the first 36° of the logarithms), 0 to 180°, log haversines at interval 15″, and natural haversines at interval 10″. For the range 0 to 125° or 135° a similar excellent table was published earlier, RICHARD FARLEY, *Natural Versed Sines from* 0 *to* 125°, and *Logarithmic Versed Sines from* 0 *to* 135° ···, London, 1856, 90 p. And yet earlier another 7-place table of haversines, at interval 10″, from 0 to 120°, JAMES ANDREW, *Astronomical and Nautical Tables with Precepts for finding the Latitude and Longitude of Places* ···, London, 1805, T. XIII, p. 29–148. There is a copy of Andrew's very rare work in the New York Public Library.

R. C. A.

198[C, E].—J. R. HULL & R. A. HULL, "Tables of thermodynamic functions of paramagnetic substances and harmonic oscillators," *J. Chem. Phys.*, v. 9, 1941, p. 465–469. 19.5 × 26 cm.

This paper, which was not mentioned in *MTAC*, p. 119, contains tables of the Einstein functions z/Z, $\ln Z$, z^2e^z/Z^2 where $Z = e^z - 1$. The tables give 5S for values of z between 0 and 14.4 at intervals ranging from .02 to .8. There is a misprint in the heading of the fifth column of p. 468. Tables are also given for Q and $\ln Q$, where

$$Q = \sum_{m=-j}^{j} e^{mx};$$

$j = \frac{1}{2}$ (1) $2\frac{1}{2}$ is the angular momentum quantum number for the substances considered, and x ranges from 0 to 6. Actually x = [0(.02).2(.04).28(.02).32(.04).4(.1)4(.2)6; 5S].

H. B.

199[E].—P. B. WRIGHT, "Resistive attenuator, pad and network, theory and design," part 3 of a 4-part paper, *Communications*, v. 25, Jan. 1945, p. 57, 58, 60. 19.5 × 27.1 cm. Compare RMT **174**, p. 358.

Tables for 20 log $e^{\theta}(e^{2\theta} = k^2 > 1)$ = 0(.01).2(.05).4(.1)4(.5)30(1)60(5)140, 150, of (a) $\sinh 2\theta = (k^4 - 1)/2k^2$; (b) $\tanh^2\theta = [(k^2 - 1)/(k^2 + 1)]^2$; (c) $(1 - e^{-\theta}) = (k - 1)/k$; (d) $\sinh^2\theta = (k^2 - 1)^2/4k^2$; (e)[1] $\cosh 2\theta = (k^4 + 1)/2k^2$; (f) $\tanh^2 \frac{1}{2}\theta = [(k - 1)/(k + 1)]^2$; (g) $\operatorname{csch} 2\theta$; (h) $\coth^2\theta$; (i) $1/[1 - e^{-\theta}]$; (j) $\operatorname{csch}^2\theta$; (k) $\operatorname{sech} 2\theta$; (l) $\coth^2 \frac{1}{2}\theta$. These tables are to 5–9S. In part 1, of this 4-part paper, Aug. 1944, v. 24, no. 8, p. 52, 54, 56 there are tables for the same range of argument for (a) $\ln k$, $\frac{1}{2}\ln k$, $2\ln k$; (b) k, $1/k$, k^2, $1/k^2$; (c) $(k - 1)$; $2(k - 1)$, $(k - 1)^2$; (d) $1/(k - 1)$, $\frac{1}{2}/(k - 1)$.

[1] The right-hand member is given incorrectly on each of the three pages as $(k^2 + 1)/2k^2$.

200[E, M].—F. STÄBLEIN & R. SCHLÄFER, "Numerische Berechnung von $y(x) = e^{-x^2}\int_0^x e^{t^2}dt$," *Z. angew. Math.*, v. 23, Feb. 1943, p. 59–61. 21.5 × 27.7 cm.

Here is a table of $y = e^{-x^2}\int_0^x e^{t^2}dt$, for $x = [0(.1)10; 4D]$, which satisfies the differential equation $y' + 2xy = 1$. There are no references to other tabulations of this function, such as by TERAZAWA, and MILLER & GORDON, that of the latter being considerably more extensive than the one under review; see *MTAC*, p. 323.

201[L].—R. P. BALDWIN, "Tables of functions used in determinations of stellar ionization temperatures," Northwestern Univ., Dearborn Observatory, *Annals*, v. 4, part 14, 1940, p. [3].

Here is a table of $\int_{x_0}^{\infty} x^2dx/(e^x - 1)$, mostly to 4 or 5S, for $x_0 = 1(.5)12.5$. Compare *MTAC*, p. 140, **46**, and p. 189, RMT **152**.

202[L].—K. E. BISSHOPP, "Stress coefficients for rotating disks of conical profile," Am. So. Mech. Engrs., *J. Appl. Mech.*, v. 11, 1944, A-8—A-9. 21.6 × 26.6 cm.

In the calculation of stresses it is found to be advantageous to use both the fundamental solutions of the hypergeometric equation

$$x(1 - x)P'' + [c - (a + b + 1)x]P' - abP = 0,$$

$a + b = 1$, $ab = \sigma - 1$, $c = 3$ for the point $x = 0$; and also those for the point $x = 1$. In equation (15) the power series for $t^2F(a + 2, b + 2; 3; t)$ is given incorrectly but the calculations for the tables seem to have been made with the correct formula. The tables give the values of P_1, $-P_1'$, P_2, $-P_2'$, for $\sigma = 0.3$ and 0.36, $x = [0(.01)1; 6S]$ and it is thought that they are accurate to within 5 parts in 2,000,000. The functions P are defined as follows:

$$P_1(x) = S_1(x) = F(a, b; 3; x) = C_2S_2(t) + C_1\bar{S}_2(t), t = 1 - x;$$

$$P_2(x) = S_2(t) = t^2F(a + 2, b + 2; 3; t) = C_3S_1(x) + C_1\bar{S}_1(x);$$

$$\bar{S}_1(x) = -\tfrac{1}{2}a(a + 1)b(b + 1)S_1(x)\ln x + x^{-2} - (a + 1)(b + 1)x^{-1} - \sum_{n=2}^{\infty}(a - 2)_n(b - 2)_n x^{n-2}\Phi_n/[n!(n - 2)!];$$

$$\Phi_n = \Psi(a + n - 3) + \Psi(b + n - 3) - \Psi(n - 2) - \Psi(n)$$

$$\bar{S}_2(t) = 1 - abt - \sum_{n=2}^{\infty} a_nb_nt^n\Psi_n/[n!(n - 2)!] - \tfrac{1}{2}a(a + 1)b(b + 1)S_2(t)\ln t;$$

$$\Psi_n = \Psi(a + n - 1) + \Psi(b + n - 1) - \Psi(n - 2) - \Psi(n)$$

$$C_1 = (2/\pi)\operatorname{Sin}(a\pi)/[a(a + 1)b(b + 1)],$$

$$C_2 = (2/\pi)\operatorname{Sin}(a\pi)\left[\frac{1}{a} - \gamma - \Psi(a)\right] - \operatorname{Cos}(a\pi),$$

$$C_3 = \frac{\operatorname{Sin} a\pi}{\pi}\left[\frac{1}{a-2} + \frac{1}{a-1} + \frac{1}{a} - \frac{1}{a+1} - 2\gamma - 2\Psi(a) - \pi\operatorname{Cot}(a\pi)\right]$$

γ = Euler's constant. $\Psi(x) = \Gamma'(x + 1)/\Gamma(x + 1)$.

Calculations were actually made for $\sigma = .24(.03).36$. Copies of the unpublished tables may be had on application to Fairbanks, Morse & Co., Beloit, Wisconsin.

The tables also include values of the stress coefficients p_1, p_2, p_3, q_1, q_2, q_3.

H. B.

203[L].—A. Bloch, On the temperature coefficient of air-cored self-inductances," *Phil. Mag.*, s. 7, v. 35, Oct. 1944, p. 704. 16.9 × 25.3 cm.

Tables of $F_1 = (x^2/8)(1 - \Phi_2)$, $G_{1B} = (x^2/8)(1 - \Phi_1)$, $x = [0(.1)10; 5D]$, where $-\Phi_2 - i\Psi_2 = J_3(x\sqrt{-i})/J_1(x\sqrt{-i})$, $-\Phi_1 - i\Psi_1 = J_2(x\sqrt{-i})/J_0(x\sqrt{-i})$, or $\Phi_1 = 1 - 2W(x)/[xX(x)]$, $\Phi_2 = 1 - 4Z(x)/[xV(x)]$. X, V, W, Z, W/X, and Z/V have been tabulated by H. G. Savidge, *Phil. Mag.*, s. 6, v. 19, 1910, p. 55f.; compare *MTAC*, p. 256.

204[L].—H. Buchholz, "Die Ausbreitung der Schallwellen in einem Horn von der Gestalt eines Rotationsparaboloides bei Anregung durch eine im Brennpunkt befindliche punktförmige Schallquelle," *Annalen der Physik*, s. 5. v. 42, Mar. 1943, p. 433–435. 12.9 × 20.5 cm.

$$\partial m_{i\tau}^{(0)}(i\zeta)/\partial\zeta = m_{i\tau}'^{(0)}(i\zeta).\ \text{For } \tau < 0,\ \zeta > 0, \text{ or for } \tau > 0,\ \zeta < 0,$$

$$m_{i\tau}'^{(0)}(i\zeta) = \tfrac{1}{2}i\,(\tfrac{1}{2}\pi)^{\frac{1}{2}}\cdot\left\{\frac{-4\tau J_1(2\sqrt{|\tau\zeta|})}{2\sqrt{|\tau\zeta|}} + \sum_{\lambda=0}^{\infty}\left(\frac{1}{4\tau}\right)^{\lambda}\cdot\gamma_\lambda^{(0)}(\zeta)(2\sqrt{|\tau\zeta|})^{\lambda}J_\lambda(2\sqrt{|\tau\zeta|})\right\};$$

for $\tau > 0$, $\zeta > 0$, or for $\tau < 0$, $\zeta < 0$,

$$m_{i\tau}'^{(0)}(i\zeta) = \tfrac{1}{2}i\,(\tfrac{1}{2}\pi)^{\frac{1}{2}}\cdot\left\{\frac{-4\tau I_1(2\sqrt{(\tau\zeta)})}{2\sqrt{(\tau\zeta)}} + \sum_{\lambda=0}^{\infty}\left(-\frac{1}{4\tau}\right)^{\lambda}\cdot\gamma_\lambda^{(0)}(\zeta)(2\sqrt{(\tau\zeta)})^{\lambda}I_\lambda(2\sqrt{(\tau\zeta)})\right\},$$

where $\gamma_\lambda^{(0)}(\zeta)$ is a given function of ζ and λ.

Of the function $(2/\pi)^{\frac{1}{2}}m_{i\tau}'^{(0)}(i\zeta)$ there is a perspective graph $0 < \zeta < 6$, $-3 < \tau < +3$; and a table $\zeta = 1(1)6$, $\tau = [-3(.5)+3; 5 \text{ or } 6S]$. Also there is a table of the first three zeros, τ_1', τ_2', τ_3' of $m_{i\tau}'^{(0)}(i\zeta)$, $\zeta = 0(1)6$; τ_1' to 5D; τ_2' to 4D, τ_3' to 2D. There are also graphs of the zeros $-4 < \tau' < +7$, $0 < \zeta < 6$.

205[L].—H. Buchholz, "Die Ausstrahlung einer Hohlleiterwelle aus einem kreisförmigen Hohlrohr mit angesetztem ebenen Schirm," *Archiv. f. Elektrotechnik*, v. 37, March 1943, p. 160, 161, 163. 19 × 27 cm.

There is a table of $J_\nu(j_{0,s})$ for $s = 1, 2, 3$ and $\nu = [.5(.5)20.5; 7D]$; for $s = 1$ and $\nu > 10.5$, $s = 2$ and $\nu > 16$, the values of the function are all zero. The graphs of $J_\nu(j_{0,s})$, $s = 1, 2, 3$, are for $0 < \nu < 13$. On p. 163 is a table of the exact values of $D_s^{(0)}(p)$, for $s = 0(1)7$, $p = 0(1)7$, $s + p \ngtr 7$; and of $C_s^{(0)}(p)$, for $s = 0(1)6$, $p = 1(1)7$, $s + p$ successively $\ngtr 1, 3, 5, \cdots, 13$, where

$$D_s^{(0)}(p) = \sum_{\lambda=0}^{s}\frac{(2p+1)(2p+2)\cdots(2p+\lambda)}{(2p+2\lambda+1)\lambda!},$$

$$C_s^{(0)}(p) = \sum_{\lambda=0}^{s}\frac{(-1)^{\lambda}(p+1)(p+2)\cdots(p+\lambda)}{(2\lambda+1)(p-1)(p-2)\cdots(p-\lambda)}.$$

The expression for $D_s^{(m)}$ as given by Buchholz in (5.5b) is incorrect; *for* $(2p+1)$, *read* $(2p+1)_\lambda$.

R. C. A.

206[L].—J. Cossar & A. Erdélyi, *Dictionary of Laplace Transforms.* Admiralty Computing Service, Department of Scientific Research and Experiment, London; Part 1, no. SRE/ACS.53, 1944, 42 leaves; Part 2A, no. SRE/ACS.68, Dec. 1944, 49 leaves; Part 2B, no. SRE/ACS.71, Feb. 1945, 56 leaves. 20.2 × 33 cm. Mimeographed on one side of each leaf. These publications are available only to certain Government agencies and activities.

Of this monograph, Part 1 contains an Introduction, and sections on: notes and abbreviations, general formulae, short table of Laplace transforms, bibliography. Only the unilateral Laplace transformation is considered and the *Dictionary* consists of pairs of functions $f(t)$, $\phi(p)$, connected by the relation

$$\phi(p) = \int_0^\infty e^{-pt}f(t)dt = L\{f(t);p\}$$

called the Laplace transform of $f(t)$. In early days $\phi(p)$ was called the generating function of $f(t)$ and later writers spoke of Abel's generating functions. The extension of the term generating function from series to integrals was a natural development of the theory of generating functions of Lagrange and Laplace. The functions $f(t)$, $\phi(p)$ are classified according to $f(t)$ in Part 2, and according to $\phi(p)$ in Part 3 (which has not yet been published). In Part 2A $\phi(p)$ is given for each $f(t)$ of successive groups of functions: rational, algebraic, powers with arbitrary index, jump- and step-, exponential, logarithmic, trigonometric, hyperbolic, and composite elementary functions. In Part 2B, $f(t)$ includes the following forms: Bessel functions, modified Bessel functions, products of Bessel functions, Bessel integral functions and Fresnel integrals; Kelvin and Struve functions; sine, cosine, exponential and logarithmic integrals; Legendre, Gegenbauer, Jacobi, Hermite, and Laguerre polynomials; parabolic cylinder functions, Bateman k-function, and Whittaker functions; Legendre functions, Gauss's series, general hypergeometric series, functions of two or more variables; and theta functions. In many cases $\phi(p)$is expressed by means of the various types of Bessel functions, confluent hypergeometric functions, the incomplete gamma function and other functions of modern analysis.

No attempt is made at completeness. Not all available sources have been consulted; and even from the works consulted not all formulae have been included. Formulae which need much explanation, or which involve rare functions, have been omitted. Sometimes only a few typical examples are selected from an extensive list of related formulae. Then reference is made to the list where more involved formulae are found.

It is pointed out in the Bibliography that the most extensive published list of Laplace transforms in existence is in N. W. McLachlan & P. Humbert, *Formulaire pour le Calcul symbolique* (*Mémorial des Sci. Mathématiques*, v. 100), Paris, 1941, 67 p.

Each of these Dictionaries should be very useful.

H. B. & R. C. A.

207[L].—NYMTP, *Jacobi Elliptic Functions*, Washington, D. C. 1942 34 hektographed sheets and six mimeographed sheets, printed on one side only 21.5 × 35.5 cm. Not available for general distribution.

These tables were described *MTAC*, p. 125–126.

208[L].—NYMTP, Table in J. R. Whinnery & H. W. Jamieson, "Equivalent circuits for discontinuities in transmission lines," Inst. Radio Engrs., *Proc.*, v. 32, 1944, p. 114. 21.8 × 28 cm.

The table is of Hahn's function

$$S_0(a) = \sum_{n=1}^{\infty} \sin^2(n\pi a)/(a^2n^3),$$

$a = [.01(.01)1; 4\text{–}5S]$. There are also graphs, $0 < a < 1$, of $S_0(a)$, and of the associated function

$$S_p(a) = \sum_{n=1}^{\infty} p^2 \sin^2(n\pi a)/[n(n^2a^2 - p^2)],\ p = 1(1)8.$$

H. B.

209[L].—L. Schwarz, "Zur Theorie der Beugung einer ebenen Schallwelle an der Kugel," *Akustische Z.*, v. 8, 1943, p. 91–117.

The pressure on the surface of a rigid sphere under the influence of a plane wave of sound can be calculated by the known theory of Rayleigh. Calculations were made by the author with the aid of Miss E. Friedrichs of the real and imaginary parts of the functions ψ and

$$\psi e^{-i\omega} = -(2/\pi\omega)^{\frac{1}{2}} \sum_{n=0}^{\infty} \frac{(2n+1)i^{n+1}}{nH^2_{n+\frac{1}{2}}(\omega) - \omega H^2_{n+3/2}(\omega)} P_n(\cos\theta)e^{-i\omega},\ \omega = 2\pi a/\lambda = ka$$

for $\theta = 0(5°)180°$ and $\omega = 1(1)10$, 5D being given. Table I gives $|\psi|$ and T. II gives arc ψ for the same ranges. The first quantity represents the absolute value of the ratio $|p/p_0|$ on the spherical surface of the amplitudes of the diffracted and incident waves, the second quantity decreased by $\omega \cos\theta$ represents the difference in phase of these waves and is given in both radians and degrees. Tables are given also for the real functions $f_n(\omega)$, $g_n(\omega)$ defined by equation

$$H^2_{n+\frac{1}{2}}(\omega) = (2/\pi\omega)^{\frac{1}{2}} e^{-i\omega}\omega^{-n}[f_n(\omega) + ig_n(\omega)]$$

The ranges are $n = 0(1)15$ and $\omega = 1(1)10$. The tables of the quantities

$$A_n(\omega) = (2n+1)\frac{\omega^n}{\hat{f}_n^2 + \hat{g}_n^2}\cdot\hat{f}_n,\ B_n(\omega) = (2n+1)\frac{\omega^n}{\hat{f}_n^2 + \hat{g}_n^2}\cdot\hat{g}_n,$$

$$\hat{f}_n = nf_n(\omega) - f_{n+1}(\omega),\ \hat{g}_n = ng_n(\omega) - g_{n+1}(\omega),$$

are for $\omega = 1(1)10$ and for various values of n ranging from 0 to 22, 6D being given. A table with 5D is also given for $|p/p_0|^2$. Diagrams illustrate the numerical results. In fig. 1 relating to $|p/p_0|$ it is noted that the maximum at the south pole denotes 'optically speaking' that there is a bright spot on the face of the sphere away from the source. Plots of $|p/p_0| + \cdot 6\omega - 1$, $|p/p_0| + .04\theta°$, $\phi° - \cdot 4\theta°$, $\phi° - 2\theta°$ against θ or ω are given to elucidate the properties of the functions tabulated.

H. B.

210[L].—H. Stenzel, "Über die Berechnung des Schallfeldes unmittelbar vor einer kreisförmigen Kolbenmembran," *Annalen d. Physik*, s. 5, v. 41, 1942, p. 256–259. 14 × 21.6 cm.

$$S_m(x) = (\tfrac{1}{2}\pi x)^{\frac{1}{2}} J_{m+\frac{1}{2}}(x),\ C_m(x) = (-1)^m(\tfrac{1}{2}\pi x)^{\frac{1}{2}} J_{-m-\frac{1}{2}}(x);$$

T. 1 gives the values of

I. $\dfrac{1\cdot3\cdot5\cdot\ \ldots\ 2m-1}{2\cdot4\cdot6\cdot\ \ldots\ 2m}[S_{m-1}(10) + iC_{m-1}(10)]$, for $m = [1(1)12; 4D]$;

II. $(.4)^m \cdot J_m(4)$, for $m = [1(1)5; 4D]$;

I × II, for $m = [1(1)5; 4D]$;

III. $(.8)^m J_m(8)$, for $m = [1(1)12; 4D]$;

I × III, for $m = [1(1)12; 4D]$.

Table 2 gives the values of

I. $\dfrac{1\cdot3\cdot5\cdot\ \ldots\ 2m-1}{2\cdot4\cdot6\cdot\ \ldots\ 2m}\cdot J_{m+1}(10)$, for $m = [0(1)15; 4D]$;

II. $(10/12)^{m+1}[S_m(10) + iC_m(10)]$, for $m = [0(1)14; 4D]$;

I × II, for $m = [0(1)14; 4D]$;

III. $(10/16)^{m+1}[S_m(10) + iC_m(10)]$, for $m = [0(1)9; 4D]$;

I × III, for $m = [0(1)9; 4D]$. Compare Stenzel 1, *MTAC*, p. 233.

Table 3 gives the values of $p_a + ip_m$, to 3D, for $x = 10, 20, 40, 60$. $y = 0(1)20$. For $y < x$

$$p_a + ip_m = \sum_{m=1}^{\infty} \frac{1\cdot3\cdot5\cdot\ \ldots\ 2m-1}{2\cdot4\cdot6\cdot\ \ldots\ 2m}(y/x)^{m+1}J_{m+1}(y)[S_m(x) + iC_m(x)];\ \text{and for } y > x$$

$$p_a + ip_m = 1 - e^{-iy}J_0(x) - \sum_{m=1}^{\infty} \frac{1\cdot3\cdot5\cdot\ \ldots\ 2m-1}{2\cdot4\cdot6\cdot\ \ldots\ 2m}(x/y)^m J_m(x)[S_{m-1}(y) + iC_{m-1}(y)].$$

There are also graphs of p_a, p_m for $y = 10, 20, 40, 60$; $0 < x < 20$.