and was obtained by inverse interpolation. The real part of the zero is then given by

$$x = \arccos(-y/\sinh y).$$

Zeros of $z + \sin z$ where $z = x + iy$

n	$\pm x$	$\pm y$
1	4.2123922	2.2507286
2	10.7125374	3.1031487
3	17.0733649	3.5510873
4	23.3983552	3.8588090.

If one considers the roots of $\sin z = z$ as functions of n and interpolates for the roots corresponding to $n = 4\frac{1}{2}, 5\frac{1}{2}, \cdots, 9\frac{1}{2}$, one obtains the zeros of $z + \sin z$ for $n = 5, 6, \cdots, 10$. Using the first ten roots of $\sin z = z$ as given in an earlier article by HILLMAN & SALZER,[1] the above mentioned roots of $z + \sin z$ can be obtained to at least four decimal places.

NYMPT

BEATRICE S. MITTELMAN & A. P. HILLMAN

[1] A. P. HILLMAN & H. E. SALZER, "Roots of $\sin z = z$," *Phil. Mag.*, s. 7, v. 34, 1943, p. 575. See *MTAC*, v. 1, p. 141.

EDITORIAL NOTE.—In *Ingenieur-Archiv*, v. 11, 1940, p. 129, J. FADLE gave the first five zeros of $\sin z \pm z$, to 5D. Comparing with the seven-place values listed above, it appears that six last-figure endings of Fadle should be increased by unity, namely in the real parts of the second and fourth zeros, and in the imaginary parts of the first four zeros. Comparing Fadle's zeros of $\sin z - z$ to 5D with those found by Hillman & Salzer[1] to 6D, we find that all of Fadle's end-figures in the first three zeros, as well as the end-figure in the real part of the fourth zero, should be increased by unity.

QUERIES

16. TABLES OF $\sin nx/\sin x$.—Has any table been calculated for the function $\sin nx/\sin x$, for large integral values of n, say up to 100, and for values of x in radians?

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EDITORIAL NOTE: In NYMTP, *Tables of Sines and Cosines for Radian Arguments*, 1940, are values of $\sin x$, $x = [0(1)100; 8D]$.

QUERIES—REPLIES

19. CUBE ROOTS (Q **11**, v. 1, p. 372; QR **15**, v. 1, p. 432). The answer to Q 11 seems to be a definite 'No.' The table required is equivalent to one giving 5 or 6D for $N = 1000(0.1)2000$, and a table at 10 times this interval is already interpolable linearly, so that a printed table at interval 0.1, although it might be very convenient, cannot be considered an urgent need. For a table at interval 1, the 1930 and 1941 editions of *Barlow's Tables* seem most convenient.

Use of linear interpolation when the second difference is about 2 units means, of course, that the last figure is subject to a maximum error of about $1\frac{1}{2}$ units, whereas tabular values are usually kept within half a unit.

If a full unit is not allowable, then an extra decimal must be kept; in this case linear interpolation may be inaccurate to the extent of 2 or 3 units in the extra decimal, and second differences may have to be used. If this is the case, the method of QR **15** is probably easier for isolated values, but for a succession of values with exact one-decimal arguments, subtabulation seems indicated, for example by the end-figure process described in the *Nautical Almanac* for 1931.

J. C. P. Miller

20. Tables of $\tan^{-1}(m/n)$ (Q **14**, p. 431; QR **18**, p. 460).—I know of no published table of $\tan^{-1}(m/n)$, but I am preparing one of this type as a preliminary to the preparation of a table of $\ln\Gamma(x+iy)$ with $x = 1/4$, $1/2$, $3/4$ for $y = 0(.1)2$ and beyond (compare RMT **234**). The need arises when applying the relation

$$\begin{aligned}\ln\Gamma(x+iy+1) &= \ln(x+iy) + \ln\Gamma(x+iy)\\ &= \tfrac{1}{2}\ln(x^2+y^2) + i\tan^{-1}(y/x) + \ln\Gamma(x+iy)\end{aligned}$$

in order to transfer the argument to a region where an expansion in series converges with sufficient rapidity. So far, my table gives $\tan^{-1}(m/n)$ for $n = 10(10)110$ and $m = [0(1)50;\ 12D]$, or, as it may be written, for $n = 1(1)11$, $m = 0(.1)5$. It is now being extended by Mr. S. Johnston.

An obvious way of producing or extending such a table is to interpolate in the NYMTP *Table of Arc Tan x*, 1942. This may be done by using the four-point Lagrange formula and it has seemed worth while to tabulate the coefficients when the interval is divided into 14, 18, 22, 26 or 30 parts. These are given in N **46**.

Nevertheless, interpolation in the table can be avoided and it may be of interest to others to have a description of the method I have used.

Firstly, we may note that

$$\tan^{-1}(m/n) = \tfrac{1}{2}\pi - \tan^{-1}(n/m)$$

so that the NYMTP tables may be used without interpolation whenever either m/n or n/m is a tabular argument. This accounts for almost all cases that normally arise except those in which m and n contain different primes (exclusive of 2 and 5) as factors.

Secondly, and more systematically, we may use relations of the type

$$\begin{aligned}\tan^{-1}(\lambda+1/3) &= \tan^{-1}\lambda + \tan^{-1}1/(3\lambda^2+\lambda+3)\\ &= \tfrac{1}{2}\pi + \tan^{-1}\lambda - \tan^{-1}(3\lambda^2+\lambda+3)\end{aligned}$$

which expresses $\tan^{-1}(\lambda+1/3)$, for example when λ is an integer (not too large), in terms of two tabular entries. Likewise, we have

$$\tan^{-1}(\lambda-1/3) = -\pi/2 + \tan^{-1}\lambda + \tan^{-1}(3\lambda^2-\lambda+3).$$

Thus

$$\begin{aligned}\tan^{-1}1/3 &= \pi/2 - \tan^{-1}3 &&= 0.32175\ 05543\ 97\\ \tan^{-1}2/3 &= -\pi/4 + \tan^{-1}5 &&= 0.58800\ 26035\ 48\\ \tan^{-1}4/3 &= 3\pi/4 - \tan^{-1}7 &&= 0.92729\ 52180\ 01\\ \tan^{-1}5/3 &= -\pi/2 + \tan^{-1}2 + \tan^{-1}13 &&= 1.03037\ 68265\ 24\end{aligned}$$

and so on.

With the NYMTP tables λ need not be an integer; multiples of 0.1, for example, will also serve. We may also use relations such as

$$\tan^{-1}(\lambda + 10/3) = \tfrac{1}{2}\pi + \tan^{-1}\lambda - \tan^{-1}(3\lambda^2 + 10\lambda + 3)/10.$$

Such relations have been found useful when $3\lambda^2 + \lambda + 3$ tends to be too big to be a tabular argument.

In some individual cases, it was found necessary to search rather carefully for suitable expressions, and in these circumstances, no saving of time results when comparison is made with straightforward methods of interpolation. It may be of interest, however, to note a few of these special cases,[1] suitable for use with the NYMTP tables:

$$\begin{aligned}
\tan^{-1}(4.9/9) &= \tan^{-1}0.5 - \tan^{-1}13 + \tan^{-1}23.88\\
\tan^{-1}(4.7/9) &= \tan^{-1}0.5 - \tan^{-1}25.5 + \tan^{-1}46.34\\
\tan^{-1}(3.1/6) &= \tfrac{1}{2}\pi + \tan^{-1}0.5 - \tan^{-1}75.5\\
\tan^{-1}(3.7/6) &= \pi - \tan^{-1}1.625 - \tan^{-1}1076\\
\tan^{-1}(4.1/7) &= -\tfrac{1}{2}\pi + \tan^{-1}0.6 + \tan^{-1}94.6
\end{aligned}$$

The first two of these cases are the only ones I have found so far which seem to need three tabular values; the others are typical of a fairly large number needing two tabular values together with a multiple of π. As a rule, I tried to express each value required in terms of two values only, one being a tabular value and the other a multiple of π if possible or, if not, a tabular value.[2]

J. C. P. Miller

[1] It has been found that

$$\begin{aligned}
\tan^{-1}(4.9/9) &= -\tfrac{1}{2}\pi + \tan^{-1}0.6 + \tan^{-1}23.88 \text{ (S.A.J.)},\\
\tan^{-1}(4.7/9) &= -\tfrac{1}{2}\pi + \tan^{-1}0.55 + \tan^{-1}46.34 \text{ (S.A.J.)},\\
&= \tfrac{1}{2}\pi + \tan^{-1}0.3 - \tan^{-1}5.205 \text{ (J.C.P.M.)};
\end{aligned}$$

we may also note that

$$\tan^{-1}(3.7/6) = \tfrac{1}{2}\pi + \tan^{-1}0.6 - \tan^{-1}\dot{8}2.2 \text{ (S.A.J.)}.$$

[2] Considerations in connection with this communication have suggested the question: What values of $\tan^{-1}N$ (N being an integer) are fundamental? For instance,

$$\begin{aligned}
\tan^{-1}3 &= 3\tan^{-1}1 - \tan^{-1}2\\
\tan^{-1}7 &= 2\tan^{-1}2 - \tan^{-1}1\\
\tan^{-1}8 &= 2\tan^{-1}1 + \tan^{-1}3 - \tan^{-1}5\\
\tan^{-1}13 &= 5\tan^{-1}1 - \tan^{-1}2 - \tan^{-1}4
\end{aligned}$$

So we certainly do not need $N = 3, 7, 8, 13$. But do we need all of 1, 2, 4, 5, 6, 9, 10, 11, 12?

CORRIGENDA

P. 54, l. -2, *for* e^{100} (117D), *read* e^{100} (116S); l. -3, *for* e^8 (255D), *read* e^8 (254D).

P. 55, l. 18, *for* ln 17 to 224D, *read* ln 17 to 274D.

P. 56, l. 1, *for* Recalculation of the modulus, *read* Recalculation and extension of the modulus; l. 16, *for* e^{10} to 289D, *read* e^{10} to 284D.

P. 59, l. -21, *for* The correct value, *read* The value.

P. 227, *delete* $\mathbf{A}_3$ 5, 13.

P. 229, *delete* $\mathbf{B}_3$ 3, 4.

P. 253, l. -4, *for* $2^{1/2}$, *read* $2^{-1/2}$; *for* $\mathrm{ber}'x$, *read* $-\mathrm{ber}'x$; l. -2, *for* $\mathrm{ker}'x$, *read* $-\mathrm{ker}'x$; *for* kei_2x, *read* $-\mathrm{kei}_2x$.