In addition to such functions as may be defined by subsidiary differential equations, the Analyzer can accept problems involving three arbitrary functions of one variable each. There is no means for dealing with functions of two or more variables, unless these functions can be reduced to the solutions of subsidiary differential equations, or to functions of one variable. There are in existence plans for four "function units" which will accept values tabulated at discrete intervals and interpolate continuously between them. These units are not yet in use.

Results of the computation are presented either graphically or in the form of typed tables.

Special techniques and schematic symbolisms have been developed for use in reducing mathematics to machine connections and are described by Bush and Caldwell. The symbols are simple and suggest the mechanical units they are intended to represent. While not logically necessary, the schematic or short-hand notation is important practically, as an aid to the rapid and easy reduction of problems to a form acceptable to the analyzer.

The design and construction of continuous computers having the accuracy and flexibility of the M.I.T. Analyzer, demands a high order of ingenuity and mechanical skill. Some of the outstanding problems met and solved by the Massachusetts Institute of Technology group and their associates are described by Bush and Caldwell. Among these are the design of data-transmission and servo units, the development of codes and related equipment for the rapid set-up of problems, and the conversion of continuous variables (shaft rotations) into digital form, suitable for printing.

The M.I.T. Analyzer is finding and will continue to find a field of usefulness in the solution of the differential equations of engineering and physics, especially as engineers delve further into situations which demand the solution of non-linear relations.

G. R. STIBITZ

20[Z].—INTERNATIONAL BUSINESS MACHINES CORP., *IBM Automatic Sequence Controlled Calculator*, New York, I.B.M., 590 Madison Avenue, 1945. 6 p. (21.5 × 28 cm.) + folding plate (83 × 28 cm.). Copies may be procured on application to the publisher.

This publication contains some notes on the development of this electro-mechanical calculator installed and constantly working in the Cruft Memorial Laboratory at Harvard University; it was officially turned over to the University on August 7, 1944, and has been, and still is, exclusively used by the U. S. Navy. The folding plate gives a complete view of the front side of the machine, and various statistics concerning the number of elements in the machine and the material used in its construction.

21[Z].—WILLIAM E. MORRELL, "A slide rule for the addition of squares," *Science*, n.s., v. 103, 25 Jan. 1946, p. 113–114.

The author describes a slide rule which he constructed, with appropriate square and square-root scales, for evaluating such an expression as $d = (x^2 + y^2 + z^2)^{\frac{1}{2}}$.

NOTES

For a Note on Harry Bateman see RMT **289**.

51. EARLY DECIMAL DIVISION OF THE SEXAGESIMAL DEGREE (see N **29**, p. 400f; N **41**, p. 454).—In our previous notes on this topic we listed 8 or 9 editions of *De Thiende*, 1585, by SIMON STEVIN; but we forgot to give a reference to the Dutch edition, 9 or 10, of EZECHIEL DE DECKER, *Eerste Deel van de Nievwe Telkonst . . . Noch is hier achter byghevoeght de Thiende* *van Symon Stevin van Brugghe.* Gouda, 1626. It occupies the last 27 p. of the volume and has its own title page.

R. C. A.

52. THE FUNCTION LN $F(\frac{1}{2}n, \frac{1}{2}, x)$ AND OTHER RELATED FUNCTIONS.—The NYMTP has carried out extensive computations on the function $\ln F(\frac{1}{2}n, \frac{1}{2}, x)$ and other related functions to be mentioned below for $n = 1(2)201$ and $x = 0(.01).6(.1)2(.2)7(1)45(5)100$. The confluent hypergeometric function $F(\frac{1}{2}n, \frac{1}{2}, x)$ is the solution of the differential equation

$$(1) \qquad 2xF'' + (1 - 2x)F' - nF = 0,$$

satisfying the boundary condition $F(\frac{1}{2}n, \frac{1}{2}, 0) = 1$. If we put $F = e^{\frac{1}{2}x}x^{-\frac{1}{4}}v$, then it is readily found that v satisfies the differential equation

$$16x^2v'' + [3 + (4 - 8n)x - 4x^2]v = 0;\ v(0) = 1.$$

For $n \gg x$, the Brillouin-Kramers-Wentzel approximation method yields

$$v \sim x^{\frac{1}{4}} \cdot e^{\sqrt{2nx}}, \text{ whence } F \sim e^{\frac{1}{2}x} \cdot e^{\sqrt{2nx}} \sim e^{\sqrt{2nx}},$$

and therefore $\ln F \sim \sqrt{2nx}$. The function $\ln F/\sqrt{2nx}$ is considerably smoother than either F or $\ln F$, and for this reason will be tabulated over the major range of n and x.

For x small, neither F nor $\ln F$ nor $\ln F/\sqrt{2nx}$ differences well for fixed n and running x, but the function $F/\cosh\sqrt{(2n - 1)x}$ does; it is therefore contemplated to tabulate the latter function for small values of x. That $F \sim \cosh\sqrt{(2n - 1)x}$ for small x may be shown as follows: If in (1) we make the substitution $x = y^2$ then

$$\frac{d^2F}{dy^2} - 2y\frac{dF}{dy} - 2nF = 0.$$

If we put $F = e^{\frac{1}{2}y^2}u$, then

$$(2) \qquad u'' + (1 - 2n - y^2)u = 0; \quad u(0) = 1.$$

For x (and therefore y) small, (2) yields

$$u'' + (1 - 2n)u = 0; \quad u(0) = 1;$$

whence $u = \cosh\sqrt{2n - 1}\,y$, and therefore $F \sim e^{\frac{1}{2}y^2}\cosh\sqrt{2n - 1}\,y$ or $F \sim \cosh\sqrt{2n - 1}\,y$.

ARNOLD N. LOWAN & WILLIAM HORENSTEIN

NYMTP

53. GUIDE (*MTAC*, no. 7), SUPPL. 3 (for Suppl. 1–2, see *MTAC*, v. 1, p. 403–404, v. 2, p. 59).—In *MTAC*, v. 1, p. 289, is listed C. C. FURNAS, "Evaluation of the modified Bessel function of the first kind and zeroth order," *Amer. Math. Mo.*, v. 37, 1930, with a table, p. 287, of $I_0(x)$, $x = [12(2)20(10)100(100)1000;\ 4S]$. On p. 492 of the same volume, S. A. SCHELKUNOFF gives a revised form of the table to 5S, from which the Furnas values differ by from -0.149 to $+0.111$ times the corresponding power of 10.

Since J. W. WRENCH, JR. had calculated $I_0(30)$ to 20S, *MTAC*, v. 1, p. 200, it was suggested to him that he might check the whole of Schelkunoff's table. He has now (26 Nov. 1945) made the following report: "I recalculated the entire table to 8 and 9S, and thereby proved Schelkunoff's values to be entirely free from error."

This calculation also enabled him to check 6 values [$x = 12(2)20, 30$] of $I_0(x)$ in the 6S table of H. E. H. WRINCH & D. M. WRINCH, *Phil. Mag.*, s. 6, v. 45, 1923, p. 847, and v. 47, 1924, p. 63. Compare *MTAC*, v. 1, p. 226. The values for $x = 12$ and 14 checked exactly; for $x = 16$ the last digit needs to be increased by unity; for $x = 18$ an increase of 3 units is necessary; for $x = 20$, a decrease of 3 units; and for $x = 30$ a decrease of 2 units.

R. C. A.

54. A HUGE NUMBER.—Introductory to comment on a recent paper involving tables, I shall reprint some of the items published in my note on "Huge Numbers," *Amer. Math. Mo.*, v. 28, 1921, p. 393–394. What is the largest number that we can express by three digits? One answer is $9^{(9^9)}$ or $9^{387420489}$. C. A. LAISANT drew attention to this number in his *Initiations Mathématiques*, Paris, 1906 (English edition, London, 1913, p. 101). He then remarks that in decimal numeration this number would have 369693100 figures. To write it on a single strip of paper, supposing that each figure occupied a space of one fifth of an inch, the length of the strip would be 1166 miles, 1690 yards, 1 foot, 8 inches.

Writing in 1913 A. C. D. CROMMELIN stated, Br. Astr. Assoc., *J.*, v. 23, p. 380–381, that he had come across the problem "in an old logarithm book." By the aid of 61-figure logarithms Crommelin found $\log N = 369{,}693{,}099.6315703587 \cdots$; whence the number of figures indicated above. He then found the first 28 figures of N to be 428,124,773, 175,747,048,036,987,115,9, and the last three to be 289. Crommelin then continues, "A knowledge of 30 figures out of 300 million may seem trifling, but in reality the error involved in taking all the remaining figures as zeros is only one part in a thousand quadrillions.[1] If the number were printed with 16 figures to an inch (about the lightest packing for decent legibility), it would extend over 364.7 miles. . . . If printed in a series of large volumes we might get 14,000 figures to a page, and with 800 pages to the volume it would fill 33 volumes. There are more than twice as many digits in the number as there are letters in the whole of the *Encyclopædia Britannica*. . . . We may take it as morally certain that we can write with three digits a number vastly exceeding the number of electrons in the whole of creation, which is a somewhat startling fact. Indeed, even the number 4^{4^4} (which is 13407813 followed by 147 other figures) probably exceeds the number of electrons in creation."

In Br. Astr. Assoc., *J.*, v. 26, 1915, p. 46–47, D. G. MCINTYRE gave the last eight figures as 17,177,289.

We now turn to *Matematisk Tidsskrift*, A, 1941, p. 63–70, and an article by CHR. WEISS, entitled "'Hu', Tallet $9^{(9^9)}$ og Endecifrene i Potenser af 9." Weiss found $\log N$ to 74D as follows: 369693099.631570 358743 543095 095482 929683 400246 855547 962405 328896 342699 975257 003407 00; whence he derived the first 60 figures of N to be (details of computation given) 4281 247731 757470 480369 871159 305635 213390 554822 414435 141747 53.

On p. 67 is a table of the last 26 figures for each of 35 values of 9^n, $n = 89,100,200,\cdots,387420489$. Thus the last 26 figures of N are found to be

$$24\ 178799\ 359681\ 422627\ 177289.$$

These results check with those quoted above, except in the case of the first of the McIntyre figures. Weiss gives also two tables and formulae for finding last figures of 9^n.

J. W. MEARES in Br. Astron. Assoc., *J.*, v. 31, 1921, p. 277–278, comments on $9!^{(9!^{9!})}$ and finds that its value is greater than 10 to the power $10^{2000000}$ but less than 10 to the power $10^{2000001}$.

R. C. A.

[1] In accordance with British usage, Crommelin here means 1000×10^{24}; in the United States this would be interpreted as 1000×10^{15}.

55. A NEW RESULT CONCERNING A MERSENNE NUMBER.—(Compare N. **23**, **33**, v. 1, p. 333, 404). On 9 February 1946 I finished testing the character of the Mersenne number $M_{229} = 2^{229} - 1 = 8627\ 18293\ 34882\ 04734\ 29344\ 48278\ 46281\ 81556\ 38862\ 15212\ 98319\ 39531\ 55279\ 74911$. Since the final residue, the 228th, was not zero the conclusion is that M_{229} is composite.

The Lucasian sequence used was 4,14,194,37634,1416317954, etc.

The 228th residue was found to be 1970 11660 94225 75309 56180 91126 86257 27776 96596 41856 06805 84362 68648 91891.

Thus, among these numbers M_p, up to and including $p = 257$, there are only three whose characters are unknown, namely: $p = 193,199,227$. There are, however, eleven M_p, known to be composite, but of which no factor is known.

I have begun a similar investigation of M_{199}.

HORACE S. UHLER

12 Hawthorne Avenue
Hamden 14, Connecticut

QUERIES

17. TABLES FOR CIRCLES.—In O. G. GREGORY, *Mathematics for Practical Men*, London, 1825, p. 406, after "A Table of Circles, from which knowing the diameters, the areas, circumferences, and sides of equal squares are found," by GOODWYN (see MTE **81**), Gregory remarks that this table was "to supersede the necessity of consulting some erroneous tables of the areas, &c. of circles recently put into circulation." What author and publication are here indicated? The *English Catalogue* lists the following anonymous item issued in the following year: *Tables of Areas and Circumferences of Circles*, 3 parts, London, 1826.

R. C. A.

QUERIES—REPLIES

21. BRIGGS' ARITHMETICA LOGARITHMICA (Q7, v. 1, p. 170).—In my library is a copy of this volume with the extra 12 pages containing the