More Zeros of Certain Bessel Functions of Fractional Order

The following table is a continuation of the one given by M. ABRAMOWITZ in *MTAC*, v. 1, 1945, p. 353–354. The quantities $j_{\nu,s}$, where $j_{\nu,s}$ denotes the sth zero of $J_\nu(x)$, are given below for $\nu = \pm\frac{1}{4}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{3}{4}$, and for $s = [8(1)30; 10D]$. These values are believed to be correct to well within a unit in the tenth decimal. Thus this table supplements the one given by Abramowitz up to $s = 7$ or 8. The roots $j_{\nu,s}$ were calculated from the well-known asymptotic expansion (see *MTAC* p. 354), carrying 12D in the work.

An interesting check was performed upon these roots by obtaining the first difference as a function of s and then obtaining the divided differences of those first differences as functions of ν. Apart from the first entry corresponding to $s = 8$ and which was checked independently, the divided differences (which were, in turn, also differenced as a function of s where they were largest, i.e. for s near 8), showed an error very much within 2000 units in the twelfth decimal place. Since the error in the roots was multiplied by a probable factor of 200 or more as a result of the process of taking the divided difference, the roots in the original manuscript are probably correct to about a unit in the 11th decimal place.

Most of the calculations on these roots were performed by Miss RUTH ZUCKER and Mrs. RUTH CAPUANO.

Tables of $j_{\nu,s}$

s	$\nu = \frac{1}{4}$	$\nu = -\frac{1}{4}$	$\nu = \frac{1}{3}$	$\nu = -\frac{1}{3}$
8	24.7438277961	23.9585534953	24.8737314228	23.8266562471
9	27.8849946034	27.0996936674	28.0150117149	26.9679102166
10	31.0262474761	30.2409276522	31.1563548787	30.1092347166
11	34.1675627296	33.3822290291	34.2977436760	33.2506098032
12	37.3089246363	36.5235804430	37.4391666407	36.3920224013
13	40.4503223437	39.6649700250	40.5806158496	39.5334635829
14	43.5917481224	42.8063893836	43.7220856531	42.6749270440
15	46.7331963179	45.9478324186	46.8635719141	45.8164082072
16	49.8746626986	49.0892945893	50.0050715339	48.9579036683
17	53.0161440346	52.2307724460	53.1465821455	52.0994108430
18	56.1576378181	55.3722633210	56.2881019100	55.2409277345
19	59.2991420721	58.5137651192	59.4296293768	58.3824527754
20	62.4406552173	61.6552761718	62.5711633865	61.5239847181
21	65.5821759767	64.7967951336	65.7127030012	64.6655225573
22	68.7237033066	67.9383209080	68.8542474541	67.8070654737
23	71.8652363457	71.0798525921	71.9957961121	70.9486127933
24	75.0067743771	74.2213894359	75.1373484480	74.0901639563
25	78.1483167988	77.3629308110	78.2789040192	77.2317184937
26	81.2898631020	80.5044761872	81.4204624512	80.3732760100
27	84.4314128537	83.6460251137	84.5620234254	83.5148361692
28	87.5729656827	86.7875772052	87.7035866686	86.6563986839
29	90.7145212696	89.9291321301	90.8451519455	89.7979633070
30	93.8560793373	93.0706896015	93.9867190522	92.9395298248

Tables of $j_{\nu,s}$—*Continued*

s	$\nu = \frac{2}{3}$	$\nu = -\frac{2}{3}$	$\nu = \frac{3}{4}$	$\nu = -\frac{3}{4}$
8	25.3907144312	23.2959758671	25.5193219366	23.1625059341
9	28.5327278949	26.4380633449	28.6615845511	26.3049025693
10	31.6746579988	29.5800458826	31.8037149655	29.4471278984
11	34.8165272733	32.7219536520	34.9457487864	32.5892313867
12	37.9583508024	35.8638062730	38.0877098894	35.7312451311
13	41.1001390646	39.0056170547	41.2296148817	38.8731908670
14	44.2418995658	42.1473953433	44.3714756721	42.0150838362
15	47.3836378232	45.2891478947	47.5133010223	45.1569350434
16	50.5253579832	48.4308797136	50.6550975224	48.2987526323
17	53.6670632220	51.5725945862	53.7968702239	51.4405427586
18	56.8087560137	54.7142954298	56.9386230642	54.5823101620
19	59.9504383142	57.8559845286	60.0803591576	57.7240585508
20	63.0921116899	60.9976636963	63.2220809997	60.8657908680
21	66.2337774103	64.1393343918	66.3637906137	64.0075094793
22	69.3754365150	67.2809978012	69.5054896572	67.1492163076
23	72.5170898635	70.4226548994	72.6471795012	70.2909129325
24	75.6587381727	73.5643064946	75.7888612893	73.4326006631
25	78.8003820451	76.7059532629	78.9305359832	76.5742805938
26	81.9420219908	79.8475957737	82.0722043975	79.7159536465
27	85.0836584448	82.9892345106	85.2138672265	82.8576206036
28	88.2252917800	86.1308698863	88.3555250659	85.9992821328
29	91.3669223180	89.2725022556	91.4971784295	89.1409388079
30	94.5085503377	92.4141319250	94.6388277630	92.2825911246

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RECENT MATHEMATICAL TABLES

299[A].—*Mathematical Cuneiform Texts*, edited by O. E. NEUGEBAUER and A. J. SACHS. (American Oriental Series, volume 29.) New Haven, Conn., Amer. Oriental So., 329 Sterling Memorial Library, and the Amer. Schools of Oriental Research, 1945. x, 177 p. + 49 plates. $5.00

In Quellen u. Studien zur Geschichte der Mathematik . . ., A. Quellen, v. 3, (MKT I–III) 1935–37, Otto Neugebauer, the outstanding authority on ancient mathematics and astronomy, published a monumental pioneer work on Mathematical Cuneiform Texts, tablets in collections of Berlin, Brussels, Istanbul, Jena, Leyden, London, New Haven, Oxford, Paris, Philadelphia, Strassburg, and Toronto. Nearly 4000 years ago the Babylonians knew how to find the positive roots of quadratic equations; how to solve simultaneous linear and quadratic equations; and how to calculate compound interest; and they knew that the angle in a semi-circle is a right angle, and our familiar relation between the hypotenuse and sides of a right-angled triangle.

The present work in English is a detailed study of about 200 tablets in the United States[1] and nearly a dozen in Europe. New material of great interest is presented.

In all Babylonian mathematical and astronomical work, tables were fundamental, especially tables for multiplication, squares, square roots, cubes and cube roots, and tables of reciprocals. Since 60 was basic in the sexagesimal notation of the Babylonians, if n were a number written in this notation, $\bar{n} = 1/n$ could be expressed in a finite number of terms (e.g. $\bar{9} = 0;\ 6, 40$), or in a never ending series of terms (e.g. $\bar{7} = 0;\ 8, 34, 17, 8, 34, 17 \ldots$, with 8, 34, 17 being repeated indefinitely). The necessary and sufficient condition for the first case is that n must be of the form $n = 2^\alpha 3^\beta 5^\gamma$, where α, β, γ are integers or zero. Such numbers are called regular numbers. In a Louvre tablet dating from about 350 B.C. there are